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Hybrid Evolutionary Algorithm with Inexact Line Search for Solving Non-linear Stock-Bond Portfolio Optimization Problems

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Abstract: Portfolio optimization is an effective way for investors to disperse investment risk and increase portfolio return. Markowitz introduces the mean-variance (M-V) portfolio model by using the variance of security return as the investment risk measure. In this paper, the risk measure entropy function of stock-bond is defined firstly based on the concept of information entropy. Secondly, a new non-linear stock-bond portfolio optimization model with transaction cost is given, and a hybrid evolutionary algorithm with inexact line search for solving the non-linear portfolio problem is designed. Finally, numerical simulations indicate that the proposed algorithm can not only avoid the evolutionary algorithm's defect of easily plunging into local optimization but also can find the optimal solution of the portfolio problem. The results can also provide optimal portfolio plans and investment strategies for investors to allocate and manage assets effectively.

Keywords: Portfolio optimization; risk measure entropy function; evolutionary algorithm; inexact line search

1 Introduction

Security investment [1] is a complex and high-risk activity, which requires investors to reasonably allocate assets and obtain the maximum portfolio return in various complex and uncertain investment environments. Due to the restriction from politics, economy, law and the company itself, the return of security investment has some uncertainty, and sometimes even suffers devastating losses. The probability of such uncertainty is usually called investment risk [2]. In fact, portfolio optimization problem is very complicated as it depends on many factors such as the preferences of the decision makers, resource allocation, transaction cost, no short selling and the market investment environment, etc [3, 4]. Generally speaking, the higher the expected return of portfolio, the greater the risk. In order to avoid or diversify portfolio risk and achieve satisfactory return, investors need to choose multiple assets from lots of available options to form a single effective portfolio.

In recent years, many researches are interested in the advancement of Markowitz's mean-variance model by incorporating more real-word constraints, and an enormous amount of studies have been published by extending or modifying the model and developing new effective portfolio optimization algorithm. For example, R. Azmi and M. Tamiz [5] reviewed lexicographic, weighted minimizing and fuzzy goal programming models and discussed the issues concerning multi-period returns, extended factors and measurement of risk. R. Mansini, W. Ogryczak, and M. G. Speranza [6] focused on linear programming solvable models in the portfolio optimization and classified the models according to decision variables in the integration of real features. V. Dallagnol, J. Van and L. Mous [7] showed the application of PSO and genetic algorithm for portfolio optimization with VaR objective. M. Masmoudi and F. B. Abdelaziz [8] focused on deterministic and stochastic multi-objective programming models by comparing the different assumptions. R. Ruiz-Torrubiano, A. Su¨¢rez [9] proposed a memetic algorithm

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with transaction cost for cardinality-constrained portfolio optimization. J. Li and J. P. Xu [10] proposed a multi-objective portfolio selection model with fuzzy random returns. K. Lwin, R. Qu and G. Kendall [11] proposed a learning-guided multi-objective evolutionary algorithm for constrained portfolio optimization; K. P. Anagnostopoulos and G. Mamanis [12] built a portfolio optimization model with three objectives and discrete variables. P. A. Rezaei, M. Solimanpur and M. J. Rezaee [13] used the invasive weed optimization method to solve multi-objective portfolio optimization problem, and so on [14–17].

In this paper, the risk measure entropy function of stock-bond is defined based on the concept of information entropy, and a constraint portfolio optimization model with transaction cost is given. In order to obtain the optimal solution of the model, a hybrid evolutionary algorithm with inexact line search is designed based on the Levy mutation operator and the inexact line search operator. Furthermore, the performance of the proposed algorithm is verified by seven security portfolio optimization problems. The experimental study demonstrates that the proposed hybrid evolutionary algorithm can obtain faster and better convergence compared with two state-of-the-art evolutionary algorithms.

The paper is organized as follows. In section 2, the stock-bond portfolio optimization model with transaction cost is given. The hybrid evolutionary algorithm with inexact line search for solving the non-line portfolio problem is proposed in section 3. The simulation results are shown in section 4, After that the conclusions and acknowledgments are made in section 5 and section 6 respectively.

2 Stock-bond portfolio optimization model

In Markowitz M-V model, the variance used to measure the risk of security investment mainly lies in the random volatility of deviation from the mean. To find the minimum risk of security investment is essentially to find the probability of the minimum volatility of deviation from the mean of return. In fact, the greater the probability of such volatility, the greater the uncertainty of the actual investment return. The variance generally represents two deviations with plus or minus from the mean. For security investors, they do not want the actual portfolio return to be less than the expected return, but they do not reject the fact that the actual portfolio return is higher than the expected return. The concept of entropy was originally derived from thermodynamics, and then developed in statistical mechanics, information theory and other disciplines. Entropy is also used to measure the probability of uncertainty; thus, a risk measure entropy function of stock-bond used to measure the risk of security investment is proposed as following.

2.1 Risk measure entropy function of stock-bond

Suppose that $\Theta = (x_1, x_2, \cdots, x_n, y_1, y_2, \cdots, y_m)^T$ is a portfolio including n stocks and m bonds. $\varepsilon > 0$ is an arbitrarily small positive number. For $\forall x_i, y_j \in [\epsilon, 1-\epsilon]$, the risk measure entropy function $\mathcal{F}_{n+m}$ of stock-bond for portfolio Θ is defined as following,

$$\mathcal{F}_{n+m}(\Theta) = -\left(\sum_{i=1}^{n} x_i \ln x_i + \sum_{j=1}^{m} y_j \ln y_j\right) \tag{1}$$

where $\sum_{i=1}^{n} x_i + \sum_{j=1}^{m} y_j = 1$. x_i is the investment proportional weight of the i-th stock for $i = 1, 2, \cdots, n$ and y_j is the investment proportional weight of the j-th bond for $j = 1, 2, \cdots, m$.

2.2 Property of the function $\mathcal{F}_{n+m}(\Theta)$

According to the definition of above the risk measure entropy function of stock-bond, we can easily obtain four properties of the risk measure entropy function as follows.

(1) $\mathcal{F}_{n+m}(\Theta) > 0$.

(2) Swap any two investment variable states, the entropy value of $\mathcal{F}_{n+m}(\Theta)$ doesn't change.

(3) When each of the stock-bond states is an equal probability event, the risk measure entropy value will reach to maximize, i.e.,

$$\mathcal{F}_{n+m}(\Theta) \leq \mathcal{F}_{n+m}\left(\overbrace{\frac{1}{n+m},\cdots,\frac{1}{n+m}}^{n+m}\right) = \ln(n+m) \tag{2}$$

where $x_i = x_j = \frac{1}{n+m}$ for $i = 1, 2, \cdots, n$, and $j = 1, 2, \cdots, m$.

(4) $\mathcal{F}_{n+m}(\Theta)$ is a concave function for the proportion coefficients of all the investment variables.

Remark 1 In security investment, we can obtain the entropy value of portfolio optimization by calculating the portfolio proportion (i.e. probability of the portfolio return) according to formula (1); then, the value of risk measure entropy can present the portfolio's indeterminacy. When various investment proportion is of equal probability, the portfolio's indeterminacy will research maximum. Therefore, risk measure entropy function $\mathcal{F}_{n+m}(\Theta)$ can be used a measure of the portfolio risk.

2.3 Assumption of transaction cost

Transaction cost is a common friction factor in security market. Each efficient investment incurs a certain percentage of transaction fees in practical transaction, and these transaction fees have a great impact on the optimal portfolio for investor. So, the transaction cost should be considered into the portfolio optimization model.

Suppose that c_i and $\hat{c}_j$ represent the transaction cost of the i-th stock and the j-th bond, respectively. p_{si} and $\hat{p}_{bj}$ are the transaction price of the i-th stock and the j-th bond respectively. λ_s and $\hat{\lambda}_b$ are transaction cost ratio of stock and bond under a certain condition, where the transaction fee ratio of different stocks and bonds is same. Furthermore, we assume that $\hat{\Theta} = (\hat{x}_1, \hat{x}_2 \cdots, \hat{x}_n, \hat{y}_1, \hat{y}_2, \cdots, \hat{y}_m)^T$ presents the investment proportion of the initial stock-bond; then, the sum of the portfolio transaction costs is equal to

$$c = \sum_{i=1}^{n} c_i + \sum_{j=1}^{m} \hat{c}_j \tag{3}$$

where, $c_i = \lambda_s p_{si}|x_i - \hat{x}_i|$, and $\hat{c}_j = \hat{\lambda}_b \hat{p}_{bj}|y_j - \hat{y}_j|$.

2.4 New model of the portfolio optimization

Suppose that $\Theta = (x_1, x_2, \cdots, x_n, y_1, y_2, \cdots, y_m)^T$ is a column vector of portfolio including n stocks and m bonds, μ_i and ω_j are the expected return rates of the i-th stock and the j-th bond respectively. σ is the minimum requirement rate of return of portfolio Θ. Then, the new model of the portfolio optimization with transaction cost is proposed as following.

$$\begin{cases} \min\limits_{\Theta\in[\epsilon,1-\epsilon]^{n+m}} \mathcal{F}(\Theta) = \left(\sum\limits_{i=1}^{n} x_i \ln x_i + \sum\limits_{j=1}^{m} y_j \ln y_j\right) \\ s.t. \quad \sum\limits_{i=1}^{n} (\mu_i p_{si} x_i - c_i) + \sum\limits_{j=1}^{m} (\omega_j \hat{p}_{bj} y_j - \hat{c}_j) \geq \sigma \end{cases} \tag{4}$$

where $[\epsilon, 1-\epsilon]^{n+m}$ is the search space, $\epsilon \leq x_i, y_j \leq 1-\epsilon$ for $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, m$, and $\epsilon > 0$, $\sum\limits_{i=1}^{n} x_i + \sum\limits_{j=1}^{m} y_j = 1$.

3 Hybrid evolutionary algorithm with inexact line search

3.1 Levy mutation operator

As we know, mutation operator is one of the key evolutionary operators in evolutionary computation. Since it not only increases the diversity of evolution population but also makes the evolution population approach the optimal solution. In this section, a Levy mutation operator is given as follows. Suppose that individual $\Theta = (x_1, x_2, \cdots, x_n, y_1, y_2, \cdots, y_m)^T$ is chosen to take part in mutation operator, $\bar{\Theta} = (\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n, \bar{y}_1, \bar{y}_2, \cdots, \bar{y}_m)^T$ is the mutation offspring generated by individual Θ, then:

$$\bar{x}_i = x_i + \sigma_i L_i(\alpha), i = 1, 2, \cdots, n \tag{5}$$

$$\bar{y}_j = y_j + \sigma_j L_j(\beta), j = 1, 2, \cdots, m \tag{6}$$

where, $\sigma_i = \exp(\tau N(0,1))$, $\sigma_j = \exp(\kappa N(0,1))$. $N(0,1)$ is the Gaussian distribution with mean 0 and variance 1. $\tau = \frac{1}{\sqrt{2m}}$, and $\kappa = \frac{1}{\sqrt{2\sqrt{m}}}$. $L_i(\alpha)$ and $L_j(\beta)$ are random numbers satisfying with Levy distribution for $i = 1, 2, \cdots, n$ and $j = 1, 2, \cdots, m$, where $\alpha = 0.5$ and $\beta = 0.7$.

3.2 Inexact line search operator

At the beginning of evolution, evolutionary algorithm converges quickly, and at the later stage, it is easy for the evolutionary algorithm to fall into the local optimal of the problem. In order to overcome this defect of evolutionary algorithm and achieve the global portfolio optimal solution effectively, a inexact line search operator based on the condition of quasi-Newton is discussed in this subsection.

Step1: Give the initial individual $\Theta^{(0)}$ in the search space and the permissible error $\eta > 0$.

Step2: Compute $\|g_k\| = \|\nabla\mathcal{F}(\Theta^{(k)})\|$, if $\|g_k\| \leq \eta$, algorithm stop; otherwise, compute inexact line search direction $d_k = -H_k g_k$, where H_k satisfies the condition of quasi-Newton in following.

$$p_{k-1} = H_k q_{k-1} \tag{7}$$

$$p_{k-1} = \Theta^{(k)} - \Theta^{(k-1)} \tag{8}$$

$$q_{k-1} = \nabla\mathcal{F}(\Theta^{(k)}) - \nabla\mathcal{F}(\Theta^{(k-1)}) \tag{9}$$

Step3: Regard individual $\Theta^{(k)}$ as the the initial point of the iteration in the k-th generation, carry out inexact line search according to the direction d_k, and generate the new iteration point $\Theta^{(k+1)}$; then, the optimal step length λ_k is obtained, i.e.,

$$\mathcal{F}(\Theta^{(k+1)}) = \mathcal{F}(\Theta^{(k)} + \lambda_k d_k) = \min_{\lambda \geq 0} \mathcal{F}(\Theta^{(k)} + \lambda d_k) \tag{10}$$

Step4: utilize the correction formula of rank 1 [18] to correct the H_k, and generate its correction value H_{k+1},

$$H_{k+1} = H_k + \frac{(p_k - H_k q_k)(p_k - H_k q_k)^T}{q_k^T (p_k - H_k q_k)} \tag{11}$$

let $k = k + 1$, go to Step2.

3.3 Strategy of keeping the optimal individual

For intelligence algorithm, the best individuals not always appear till the last evolution generation. Therefore, in each evolution step, we always keep the best individuals found in current generation. And if the global optimal individual is obtained in current evolution population; then, we use the global optimal individual to replace the previously reserved best individuals.

3.4 Hybrid evolutionary algorithm with inexact line search

Based on aforementioned discussion, a hybrid evolutionary algorithm with inexact line search to solve the nonlinear stock-bond portfolio optimization problem (4), denotes in HNPEA, is described as following:

Step1: Give initial population size N, crossover probability p_c, mutation probability p_m, permissible error η, and minimum rate of return σ.

Step2: Generate initial evolution population $P(0)$ based on the orthogonal design method [19] in the search space, let $t = 0$.

Table 1: The data of the history return rate (H. R. R.) and the history risk value (H. E. V.) of the seven securities

	Security						
Name	S. C.	O. E.	Z. T.	G. T.	N. S.	CDB 1401	06 N. D. 19
H. R. R.	0.0230	0.0068	0.0021	0.0023	0.0026	0.0025	0.0021
H. E. V.	0.0308	0.1365	0.1579	0.2402	0.0048	0.0039	0.0032

Table 2: The value of risk entropy (V. R. E.), the optimal proportion of investment (O. P. I.) and the optimal rate of return (O. R. R.) obtained by the proposed algorithms HNPEA for seven securities in different minimum requirement rate of return σ

		O. P. I.							
Value of σ	V. R. E.	S. C.	O. E.	Z. T.	G. T.	N. S.	CDB 1401	06 N. D. 19	O. R. R.
0.003	0.00224	0.102	0.013	0.012	0.002	0.217	0.444	0.210	0.00295
0.005	0.00317	0.103	0.021	0.010	0.001	0.197	0.463	0.205	0.00338
0.009	0.00458	0.136	0.008	0.028	0.006	0.235	0.356	0.236	0.00920
0.013	0.00597	0.139	0.002	0.013	0.005	0.246	0.371	0.224	0.01862

Step3: Select crossover individuals based on the crossover probability p_c from $P(k)$, and use the arithmetic operator to generate their crossover offspring. All the crossover offsprings along with those individuals not taking part in the crossover operator constitute the crossover offspring set $C(k)$.

Step4: Choose the individuals based on the mutation probability p_m from $C(k)$, and utilize the Levy operator in section 3.1 to generate their offspring. Then all the mutation offsprings along with those individuals in $C(k)$ not taking part in mutation constitute a new solution set $M(k)$.

Step5: Use the inexact line search operator to generate the offspring for each of the individuals including in the set $M(k)$, all of the offsprings constitute the next evolution population $pop(t+1)$, and keep the optimal individual Θ^* belonging to $P(t)\bigcup C(t)\bigcup M(t)$ in the next evolution population $P(t+1)$.

Step6: If the maximum number of the cycles has been reached, output the optimal portfolio solution in the new set $P(t)$; otherwise, let $k = k+1$, go to Step2.

4 Numerical simulation

In order to evaluate the efficiency of the proposed algorithm HNPEA, we choose five stocks (denoted as S. C., O. E., Z. T., G. T. and N. S. respectively) and two bonds (denoted as CDB 1401 and 06 N. D. 19) in which the data of the history return rate (H. R. R.) and the history risk value (H. E. V.) come from the Web of Shenzhen Stock Exchange (http://www.szse.cn/index/index.html) and Construction Bank of China, respectively. The name of five stocks and two bonds, the history return rate and the history risk value are described in Table 1.

In the simulation, suppose a million capital is used for investment, we will measure the performance of the proposed algorithm with two state-of-the-art algorithms MCCGA [9] and ANOGA [17]. Here, we just assume that all of the portfolio weights satisfy the constraint condition with $0.001 \leq x_i, y_j \leq 0.999$, i.e. $\epsilon = 0.001$. Furthermore, since it's difficult to make an absolute fair comparison with the existing algorithms directly. Thus, we adopt the same parameters for both the proposed algorithm and the compared algorithms in the numerical simulation. Each algorithm was implemented by using MATLAB 7.0 on an Intel Pentium IV 2.8-GHz personal computer and was executed 10 runs. In the simulation, the initial population size $N = 200$, crossover probability $p_c = 0.65$, mutation probability $p_m = 0.4$, permissible error $\varepsilon = 10^{-5}$, and minimum requirement rate of return $\sigma = 0.003, 0.005, 0.009, 0.013$, the number of maximum generation is 300.

We record the value of risk entropy (V. R. E.), the optimal proportion of investment (O. P. I.) and the optimal rate of return (O. R. R.) of each security obtained by our algorithms HNPEA for problem (4) in different minimum requirement rate of return σ in Table 2. In order to further illustrate the effectiveness of the proposed algorithm, we record the worst results (W. R.), best results (B. R.) and standard deviation (Std.) of the V. R. E. obtained by three algorithms for the problem (4) in different minimum requirement rate of return σ at different number of iterations (N. I.) in Table 3.

Comparing the results of V. R. E., O. P. I. and O. R. R. obtained by our algorithm HNPEA in different minimum requirement rate of return for each security in Table 2, we can see that a smaller (hardly any)

Table 3: The worst results (W. R.), best results (B. R.) and standard deviation (Std.) of the V. R. E. obtained by three algorithms in different minimum requirement rate of return σ at different number of iterations (N. I.).

Value of σ	Algorithm	N. I.								
		50			200			350		
		W. R.	B. R.	Std.	W. R.	B. R.	Std.	W. R.	B. R.	Std.
0.003	MCCGA	0.00713	0.00416	0.01068	0.00611	0.00433	0.01098	0.00558	0.00371	0.01011
	ANOGA	0.00721	0.00407	0.01123	0.00629	0.00439	0.01102	0.00566	0.00364	0.01023
	HNPEA	0.00678	0.00375	0.01021	0.00601	0.00427	0.01081	0.00542	0.00353	0.01002
0.005	MCCGA	0.00724	0.00532	0.01211	0.00699	0.00531	0.01178	0.00703	0.00522	0.01067
	ANOGA	0.00730	0.00547	0.01219	0.00719	0.00532	0.01214	0.00714	0.00512	0.01101
	HNPEA	0.00693	0.00482	0.01183	0.00683	0.00520	0.01123	0.00691	0.00499	0.01028
0.009	MCCGA	0.00698	0.00558	0.01114	0.00725	0.00544	0.01205	0.00698	0.00626	0.01199
	ANOGA	0.00712	0.00563	0.01127	0.00722	0.00552	0.01217	0.00702	0.00623	0.01207
	HNPEA	0.00674	0.00521	0.01103	0.00711	0.00502	0.01181	0.00612	0.00592	0.01167
0.013	MCCGA	0.00733	0.00527	0.01122	0.00732	0.00535	0.01301	0.00698	0.00494	0.01203
	ANOGA	0.00742	0.00528	0.01181	0.00757	0.00532	0.01328	0.00711	0.00599	0.01218
	HNPEA	0.00739	0.00509	0.01138	0.00723	0.00527	0.01260	0.00684	0.00483	0.01127

proportion of investment is selected for stock G. T. in different minimum requirement rate of return σ, because the history risk value of stock G. T. is big and its history return rate is relatively not too high. Also, the stock O. E. basically isn't selected to invest when the minimum rate of return is greater than 0.009 for the reason that the history return rate (0.0068) is not too high relative to the history risk value (0.1365). The history return rate of bond CDB 1401 is bigger that the stock Z. T., and the history risk value relatively lower, we will increase the proportion of investment for bond CDB 1401 in different minimum requirement rate of return σ. Furthermore, the history risk value of the stock S. C., N. S. and bond 06 N. D. 19 are not high relatively the history return rate respectively, we keep a relatively stable investment proportion in the whole process of investment, and maintain a small amount of investment in each period.

It can be seen from Table 3, our algorithm HNPEA can find a better best results (B. R.) of the V. R. E. compared with the two algorithms MCCGA, ANOGA in different minimum requirement rate of return σ at different generations $k = 50, 200, 350$. In addition, our algorithm can find better Std. and worst results (W. R.) of V. R. E. in different minimum requirement rate of return at different number of iterations. compared with ANOGA but not better than MCCGA in minimum requirement rate of return $\sigma = 0.013$ at generation 50.

The results in Table 2 and Table 3 verify that our algorithm HNPEA has the capability of converging faster, and the compared results in Table 3 reflect the fact that our algorithm is capable of performing a robust and stable search.

5 Conclusions

In this paper, a novel portfolio optimization model with transaction cost is given based on the risk measure entropy function of stock-bond, and a hybrid evolutionary algorithm with inexact line search for solving the non-linear portfolio problem is also introduced. In order to improve the diversity and convergence, a Levy mutation operator and a inexact line search based the condition of quasi-Newton are designed. Finally, numerical simulations indicate that the proposed algorithm can not only avoid the evolutionary algorithm's defect of easily plunging into local optimization but also can find the optimal solution of the problem. The results can also provide optimal portfolio plans and investment strategies for investors to allocate and manage assets effectively. The future research opportunities for interested authors are as follows:

(1) Computational complexity of the portfolio optimization algorithms should be concerned.

(2) Constraint handling methodologies of the nonlinear constrained portfolio optimization algorithms that may lead algorithmic enhancements in optimal solution require a careful attention and should be further investigated.

(3) Dynamic portfolio problem with the time variable combining exact solution techniques or successful local search methodologies would be another important research problem in the future.

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