



A New Type of Green Factor Decomposition Model for Regional Carbon Productivity

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Abstract: Carbon productivity is an important index to measure economic growth and low carbon emission reduction by combining economic indicators with carbon emission indicators. In order to study the change of carbon productivity in a certain region over a certain period, a new production technology function model was built, and the knowledge stock input of the production process was added. Combining the distance function based on energy input and the distance function based on knowledge stock input, the LMDI-PDA decomposition method is introduced to decompose the regional carbon productivity by four factors, and then three intensity indicators were further decomposed to investigate the driving force of change, and a new green factor decomposition model of the regional carbon productivity was established.

Keywords: Carbon productivity; Knowledge stock; Distance function; Decomposition model

1 Introduction

For a long time, carbon productivity has been concerned by policy makers and researchers. In production activities, economic entities obtain expected output (industrial added value) and unwanted output (waste water, carbon dioxide emissions, etc.) through the comprehensive effects of various factors capital, energy, labor, etc.). After Kaya and Yokobori put forward the concept of defining carbon productivity as the ratio of unit GDP to carbon dioxide emissions in the same period [1], domestic and foreign scholars began to study the measurement and influencing factors of carbon productivity [2-5], and decomposed them from the global level [6] and the national level [7]. On the basis of existing literature, this paper further considers the knowledge stock input, combines the distance function, adopts LMDI decomposition method and constructs a new decomposition model of regional carbon productivity green factors through two steps.

The innovation and contribution of this paper are mainly reflected in the following aspects. Firstly, put forward the input of knowledge stock in the production process, construct a new production technology function and linear programming model, and preliminarily decompose the regional carbon productivity. Secondly, the LMDI model is used to decompose the regional carbon productivity into four factors, and the three intensity factors are further expanded and decomposed, so as to test the driving influence of the primary and secondary indicators.

This paper is arranged as follows. In the second part, the new production process function and linear programming model are introduced. The third part introduces the choice and application of decomposition method. In the fourth part, the contribution rate of indicators at all levels in the decomposition results is introduced and the model is summarized.

2 The model

2.1 New production technology function and its properties

In the process of economic production in any region, after inputting a certain type and quantity of production factors, we can obtain both expected output: industrial added value and unexpected output: waste water, waste gas, solid waste, etc.

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For a certain region, this paper considers M industry sectors ($i=1\sim M$), and constructs the new production technology function of industry sector i at time t as:

$$S_i^t = \{ (K_i^t, L_i^t, E_i^t, H_i^t, Y_i^t, C_i^t) : (K_i^t, L_i^t, E_i^t, H_i^t) \text{ can produce } (Y_i^t, C_i^t) \} \quad (1)$$

Among them, K represents capital input; L represents labor input; E represents energy input; H represents knowledge stock input; Y represents expected output, that is, industrial added value; C represents undesired output, including $CO_2, SO_2, NO_2, \text{etc.}$, this paper focus on carbon dioxide emissions, so here C represents carbon dioxide emissions.

According to the actual social production process and Farrell's description of production technology, this function satisfies the following two properties [8]:

(1) Null-jointness assumption: to achieve $C = 0$, it can only be achieved by stopping all production activities, at this time $Y = 0$. That is, if $(Y, C) \in S$, and $C = 0$, then $Y = 0$;

(2) Weakly disposable assumption: it shows that the reduction of carbon dioxide emissions from undesired output must be at the expense of expected output, and the reduction ratio of both of them may be the same. That is, if $(K, L, E, H, Y, C) \in S$ and $0 \leq \theta \leq 1$ exists, then there is $(K, L, E, H, \theta Y, \theta C) \in S$.

2.2 Linear programming model of the production process

The above-mentioned new production technology function has effectively combined four factors of production: capital input K , labor input L , energy input E , and knowledge stock input H ; two outputs: expected output Y and undesired output C . Applying this function to the subsequent decomposition calculations, the DEA method can be used to model the above concepts.

In this paper, using non-parametric method, under the premise of constant returns to scale [9], denote the input, expected output, and undesired output vectors of the industry sector i at time t as $(K_i^t, L_i^t, E_i^t, H_i^t, Y_i^t, C_i^t)$, then the linear programming model of the production process of industry sector i is as follows:

$$S_i : \left\{ \begin{array}{l} (K_i, L_i, E_i, H_i, Y_i, C_i) : \sum_i \lambda_i K_i \leq K; \sum_i \lambda_i L_i \leq L; \sum_i \lambda_i E_i \leq E; \sum_i \lambda_i H_i \leq H; \\ \sum_i \lambda_i Y_i \geq Y; \sum_i \lambda_i C_i = C; \lambda_i \geq 0, i = 1 \sim M \end{array} \right\} \quad (2)$$

Which λ_i represents the intensity variable (observations value, decision-making power).

3 Decomposition method

3.1 Preliminary decomposition of Regional carbon productivity

According to the definition of carbon productivity, carbon productivity in a certain area can be decomposed into:

$$P = \frac{Y}{C} = \sum_{i=1}^M \frac{Y_i}{C} = \sum_{i=1}^M \frac{Y_i}{E_i} \cdot \frac{E_i}{H_i} \cdot \frac{H_i}{C_i} \cdot \frac{C_i}{C} = \sum_{i=1}^M EE_i \cdot GEE_i \cdot HCR_i \cdot SC_i \quad (3)$$

Among them, P represents carbon productivity of this region, that is, the ratio of industrial added value to the total carbon dioxide emissions; $\frac{Y_i}{E_i}$ represents the energy efficiency factor EE_i of the industrial sector i in the region, that is, the output value created by unit energy input; $\frac{Y_i}{H_i}$ represents the green efficiency of energy input factor GEE_i of the industrial sector i in the region, that is, the energy consumption per unit knowledge stock; $\frac{H_i}{C_i}$ represents the knowledge emission ratio factor HCR_i of the industry sector i in the region, that is, the knowledge stock input consumed per unit of carbon emissions; $\frac{C_i}{C}$ represents the spatial structure factor of carbon emissions SC_i of the industry sector i in the region, that is, the ratio of the carbon emissions of the industry sector i to the total carbon emissions in the region.

3.2 Selection and application of LMDI decomposition model

Ang and Zhang pioneered the IDA index decomposition analysis method [10]. Since 2000, the most commonly used IDA method is LMDI. The LMDI model includes three different dimensions of choice.

(1) Aggregate indicator: quantity indicator and intensity indicator. The quantity indicator is an absolute quantity, three factors are obtained after decomposition: activity effect, structure effect and intensity effect. The intensity indicator is a relative quantity, two factors are obtained after decomposition: structure effect and intensity effect.

(2) Decomposition process: additive decomposition and multiplicative decomposition. Additive decomposition is the analysis of the arithmetic changes of the comprehensive index, the total change and the decomposition result are given in the form of physical quantities. Multiplication decomposition is the analysis of the change of the comprehensive index ratio, the total change and decomposition result are given in the form of indexes.

(3) Decomposition method: LMDI-i and LMDI-ii. The weight formula used by the two decomposition methods is different, compared with LMDI-ii, the formula of LMDI-i is more concise.

According to the above three classification methods, there are 8 LMDI decomposition models. The aggregate indicator used to measure regional performance in this paper, regional carbon productivity, represents the ratio of industrial added value to carbon dioxide emissions in a certain period of time, that is, the economic benefits generated by unit carbon dioxide emissions [11], which is a relative quantity, so it is an intensity index. It is additively decomposed and the LMDI-i method is used. Therefore, model 5 in the LMDI decomposition model is used to decompose the change in regional carbon productivity from period 0 to period T into [12] :

$$\Delta P_{\text{total}} = \Delta P_{EE} + \Delta P_{GEE} + \Delta P_{HCR} + \Delta P_{SC} \tag{4}$$

Among them: ΔP_{EE} is the energy utilization efficiency effect; ΔP_{GEE} is the green efficiency of energy input effect; ΔP_{HCR} is the knowledge emission ratio effect; ΔP_{SC} is the spatial structure effect of carbon emissions. The decomposition results of each effect are as follows:

$$\Delta P_{EE} = \sum_{i=1}^M L \left(\frac{Y_i^T}{C^T}, \frac{Y_i^0}{C^0} \right) \cdot \ln \left(\frac{EE_i^T}{EE_i^0} \right) \tag{5}$$

$$\Delta P_{GEE} = \sum_{i=1}^M L \left(\frac{Y_i^T}{C^T}, \frac{Y_i^0}{C^0} \right) \cdot \ln \left(\frac{GEE_i^T}{GEE_i^0} \right) \tag{6}$$

$$\Delta P_{HCR} = \sum_{i=1}^M L \left(\frac{Y_i^T}{C^T}, \frac{Y_i^0}{C^0} \right) \cdot \ln \left(\frac{HCR_i^T}{HCR_i^0} \right) \tag{7}$$

$$\Delta P_{SC} = \sum_{i=1}^M L \left(\frac{Y_i^T}{C^T}, \frac{Y_i^0}{C^0} \right) \cdot \ln \left(\frac{SC_i^T}{SC_i^0} \right) \tag{8}$$

Where $L(.,.)$ is the logarithmic average function. $L(a, b) = \begin{cases} \frac{a-b}{\ln a - \ln b}, & a \neq b \\ 0, & a = b \end{cases}$.

3.3 Further decomposition of strength factors

From the above decomposition, it can be seen that the first three decomposition factors are intensity factors, which will be further decomposed later.

(1) Energy efficiency

Here we introduce a Shephard distance function based on energy input of the industry sector i at time t as follows [13]:

$$D_{ei}^t (K_i^t, L_i^t, E_i^t, H_i^t, Y_i^t, C_i^t) = \sup \left\{ \theta : \left(K_i^t, L_i^t, \frac{E_i^t}{\theta}, H_i^t, Y_i^t, C_i^t \right) \in S_i^t \right\} \tag{9}$$

This is the maximum feasible shrinkage ratio of energy input under the premise of a given input vector $(K_i^t, L_i^t, E_i^t, H_i^t)$ and production technology S_i^t , where θ represents the distance between the energy input in the actual state and the energy input in the optimal ideal state.

Energy efficiency is the ratio of industrial added value to energy input. It is an intensity indicator. Combining the distance function based on energy input, the energy utilization efficiency can be decomposed to obtain:

$$EE = \frac{Y}{E} = \sum_{i=1}^M \frac{Y_i}{E} = \sum_{i=1}^M \frac{Y_i}{E_i/D_{ei}} \cdot \frac{1}{D_{ei}} \cdot \frac{E_i}{E} = \sum_{i=1}^M PEE_i \cdot \frac{1}{EFF_i} \cdot SE_i \tag{10}$$

Among them, $\frac{Y_i}{E_i/D_{ei}}$ represents the potential energy utilization efficiency factor PEE_i of the industry sector i in the region, that is, the output value created by each unit of energy input in the optimal state; $\frac{1}{D_{ei}}$ represents the reciprocal of

the energy technical efficiency factor EFF_i of the industry sector i in the region, that is, the reciprocal of the Shephard distance function based on energy input; $\frac{E_i}{E}$ represents the spatial structure factor of energy input of SE_i the industry sector i in the region, that is, the ratio of the energy input of the industrial sector i to the total energy input of the region.

In the following, the LMDI-i method is also selected for additive decomposition of energy utilization efficiency, and the LMDI decomposition model 5 is used to decompose the energy utilization efficiency changes from period 0 to period T into:

$$\Delta EE_{total} = \Delta EE_{PEE} + \Delta EE_{\frac{1}{EFF}} + \Delta EE_{SE} \quad (11)$$

Among them: ΔEE_{PEE} is the potential energy utilization efficiency effect; $\Delta EE_{\frac{1}{EFF}}$ is the reciprocal effect of energy technology efficiency; ΔEE_{SE} is the spatial structure effect of energy input. The decomposition results of each effect are as follows:

$$\Delta EE_{PEE} = \sum_{i=1}^M L \left(\frac{Y_i^T}{E^T}, \frac{Y_i^0}{E^0} \right) \cdot \ln \left(\frac{PEE_i^T}{PEE_i^0} \right) \quad (12)$$

$$\Delta EE_{\frac{1}{EFF}} = \sum_{i=1}^M L \left(\frac{Y_i^T}{E^T}, \frac{Y_i^0}{E^0} \right) \cdot \ln \left(\frac{EFF_i^0}{EFF_i^T} \right) \quad (13)$$

$$\Delta EE_{SE} = \sum_{i=1}^M L \left(\frac{Y_i^T}{E^T}, \frac{Y_i^0}{E^0} \right) \cdot \ln \left(\frac{SE_i^T}{SE_i^0} \right) \quad (14)$$

(2) Green efficiency of energy input

The green efficiency of energy input is the ratio of energy input to knowledge stock input. It is an intensity indicator and can be decomposed to obtain:

$$GEE = \frac{E}{H} = \sum_{i=1}^M \frac{E_i}{H} = \sum_{i=1}^M \frac{E_i/D_{ei}}{H_i} \cdot D_{ei} \cdot \frac{H_i}{H} = \sum_{i=1}^M PGE E_i \cdot EFF_i \cdot SH_i \quad (15)$$

Among them, $\frac{E_i/D_{ei}}{H_i}$ represents the green efficiency of potential energy input factor $PGE E_i$ of the industry sector i in the region, that is, the energy consumption accompanying each unit of knowledge stock input in the optimal state; D_{ei} represents the energy technology efficiency factor EFF_i of the industry sector i in the region, that is, the Shephard distance function based on energy input; $\frac{H_i}{H}$ represents the spatial structure factor of the input of knowledge stock SH_i in the industry sector i in the region, that is, the ratio of the input of knowledge stock in the industrial sector i to the total input of knowledge stock in the region.

In the following, the LMDI-i method is also selected for the additive decomposition of the green efficiency of energy input, and the LMDI decomposition model 5 is used to decompose the energy efficiency change from the period 0 to the period T as:

$$\Delta GEE_{total} = \Delta GEE_{PGE} + \Delta GEE_{EFF} + \Delta GEE_{SH} \quad (16)$$

Among them: ΔGEE_{PGE} is the green efficiency effect of potential energy input; ΔGEE_{EFF} is the energy technology efficiency effect; ΔGEE_{SH} is the spatial structure effect of the knowledge stock input. The decomposition results of each effect are as follows:

$$\Delta GEE_{PGE} = \sum_{i=1}^M L \left(\frac{E_i^T}{H^T}, \frac{E_i^0}{H^0} \right) \cdot \ln \left(\frac{PGE E_i^T}{PGE E_i^0} \right) \quad (17)$$

$$\Delta GEE_{EFF} = \sum_{i=1}^M L \left(\frac{E_i^T}{H^T}, \frac{E_i^0}{H^0} \right) \cdot \ln \left(\frac{EFF_i^T}{EFF_i^0} \right) \quad (18)$$

$$\Delta GEE_{SH} = \sum_{i=1}^M L \left(\frac{E_i^T}{H^T}, \frac{E_i^0}{H^0} \right) \cdot \ln \left(\frac{SH_i^T}{SH_i^0} \right) \quad (19)$$

(3) Knowledge emission ratio

Here we introduce a Shephard distance function based on knowledge input of the industry sector i at time t as follows:

$$D_{hi}^t (K_i^t, L_i^t, E_i^t, H_i^t, Y_i^t, C_i^t) = \sup \left\{ \theta : \left(K_i^t, L_i^t, E_i^t, \frac{H_i^t}{\theta}, Y_i^t, C_i^t \right) \in S_i^t \right\} \quad (20)$$

This is the maximum feasible shrinkage ratio of the knowledge input under the premise of a given input vector $(K_i^t, L_i^t, E_i^t, H_i^t)$ and production technology S_i^t , where θ is the distance between the knowledge input in the actual state and the knowledge input in the optimal ideal state.

The knowledge emission ratio is the ratio of the knowledge input to the carbon dioxide emission, which is an intensity index. Combining the distance function based on knowledge input, the knowledge emission ratio can be decomposed to obtain:

$$HCR = \frac{H}{C} = \sum_{i=1}^M \frac{H_i}{C} = \sum_{i=1}^M \frac{H_i/D_{hi}}{C_i} \cdot D_{hi} \cdot \frac{C_i}{C} = \sum_{i=1}^M PHCR_i \cdot HFF_i \cdot SC_i \quad (21)$$

Among them, $\frac{H_i/D_{hi}}{C_i}$ represents the potential knowledge emission ratio factor $PHCR_i$ of the industry sector i in the region, that is, the knowledge stock input consumed per unit of carbon emission in the optimal state; D_{hi} represents the knowledge stock technical efficiency factor HFF_i of the industry sector i in the region, that is, the Shephard distance function based on the knowledge input; $\frac{C_i}{C}$ represents the spatial structure factor of carbon emissions SC_i of the industry sector i in the region, that is, the ratio of the carbon emissions of the industry sector i to the total carbon emissions in the region.

In the following, the LMDI-i method is also selected for the additive decomposition of the knowledge emission ratio, and the LMDI decomposition model 5 is used to decompose the change of the knowledge emission ratio from the period 0 to the period T as:

$$\Delta HCR_{total} = \Delta HCR_{PHCR} + \Delta HCR_{HFF} + \Delta HCR_{SC} \quad (22)$$

Among them: ΔHCR_{PHCR} is the potential knowledge emission ratio effect; ΔHCR_{HFF} is the knowledge stock technical efficiency effect; ΔHCR_{SC} is the spatial structure effect of carbon emission. The decomposition results of each effect are as follows:

$$\Delta HCR_{PHCR} = \sum_{i=1}^M L \left(\frac{H_i^T}{C^T}, \frac{H_i^0}{C^0} \right) \cdot \ln \left(\frac{PHCR_i^T}{PHCR_i^0} \right) \quad (23)$$

$$\Delta HCR_{HFF} = \sum_{i=1}^M L \left(\frac{H_i^T}{C^T}, \frac{H_i^0}{C^0} \right) \cdot \ln \left(\frac{HFF_i^T}{HFF_i^0} \right) \quad (24)$$

$$\Delta HCR_{SC} = \sum_{i=1}^M L \left(\frac{H_i^T}{C^T}, \frac{H_i^0}{C^0} \right) \cdot \ln \left(\frac{SC_i^T}{SC_i^0} \right) \quad (25)$$

(4) The spatial structure factor of carbon emissions is a structure effect in itself, so it will not decompose.

4 Conclusion

In the above model, the regional carbon productivity is decomposed twice. It is decomposed into four primary indicators, and the contribution rate of primary indicators to changes of carbon productivity is obtained.

In the regional carbon productivity $\Delta P_{total} = \Delta P_{EE} + \Delta P_{GEE} + \Delta P_{HCR} + \Delta P_{SC}$: the contribution rate of energy utilization efficiency effect is $\frac{\Delta P_{EE}}{\Delta P_{total}} \cdot 100\%$; the contribution rate of green efficiency of energy input effect is $\frac{\Delta P_{GEE}}{\Delta P_{total}} \cdot 100\%$; the contribution rate of knowledge emission ratio effect is $\frac{\Delta P_{HCR}}{\Delta P_{total}} \cdot 100\%$; the contribution rate of carbon emission spatial structure effect is $\frac{\Delta P_{SC}}{\Delta P_{total}} \cdot 100\%$.

Then, the first three primary indicators representing the intensity effect are decomposed for the second time, and the contribution rate of the second-level indicators to the corresponding first-level indicators is obtained:

(1) In the energy utilization efficiency effect $\Delta EE_{total} = \Delta EE_{PEE} + \Delta EE_{\frac{1}{E_{FF}}} + \Delta EE_{SE}$: the contribution rate of potential energy utilization efficiency effect is $\frac{\Delta EE_{PEE}}{\Delta EE_{total}} \cdot 100\%$; the contribution rate of energy technology efficiency reciprocal effect is $\frac{\Delta EE_{\frac{1}{E_{FF}}}}{\Delta EE_{total}} \cdot 100\%$; the contribution rate of energy input spatial structure effect is $\frac{\Delta EE_{SE}}{\Delta EE_{total}} \cdot 100\%$.

(2) In the green efficiency effect of energy input $\Delta GEE_{total} = \Delta GEE_{PGEE} + \Delta GEE_{EFF} + \Delta GEE_{SH}$: the contribution rate of green efficiency of potential energy input effect is $\frac{\Delta GEE_{PGEE}}{\Delta GEE_{total}} \cdot 100\%$; the contribution rate of energy technology efficiency effect is $\frac{\Delta GEE_{EFF}}{\Delta GEE_{total}} \cdot 100\%$; the contribution rate of knowledge stock input spatial structure effect is $\frac{\Delta GEE_{SH}}{\Delta GEE_{total}} \cdot 100\%$.

(3) In the knowledge emission ratio effect $\Delta HCR_{total} = \Delta HCR_{PHCR} + \Delta HCR_{HFF} + \Delta HCR_{SC}$: the contribution rate of potential knowledge emission ratio effect is $\frac{\Delta HCR_{PHCR}}{\Delta HCR_{total}} \cdot 100\%$; the contribution rate of knowledge stock technical efficiency effect is $\frac{\Delta HCR_{HFF}}{\Delta HCR_{total}} \cdot 100\%$; the contribution rate of carbon emission spatial structure effect is $\frac{\Delta HCR_{SC}}{\Delta HCR_{total}} \cdot 100\%$.

In the above contribution rate, if the numerical value is positive, it has a positive effect. The closer the numerical value is to 1, the greater the contribution rate and the more important the influence of the index. If this value is negative, it will have a negative impact, so that the production process can be improved.

Among the eight secondary indicators, the energy technology efficiency effects appear twice in the original form and the reciprocal form. The industrial structure effect of carbon emissions not only used as a primary indicator, but also participates in other primary indicators in the form of a secondary indicator. And innovatively put forward the decomposition factor effect related to the knowledge stock. Therefore, in the actual research process, it is necessary to focus on the influence of these decomposition factors, which play an important role in the research of carbon productivity driving factors.

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References

- [1] Y. Kaya, K. Yokobori. Environment, Energy and Economy: Strategies for Sustainability. *Delhi: Bookwell Publications*, 1999.
- [2] X. Hu, C. Liu. Carbon productivity: a case study in the Australian construction industry. *Journal of Cleaner Production*, 112(2016):2354-2362.
- [3] Z. Lu, Y. Yang, J. Wang. The impact of changes in carbon structure on industrial system carbon productivity: Empirical analysis based on the Laspeyres decomposition model. *Science and Technology Management Research*, 35(2015):234-238.
- [4] G. Liobikiene, M. Butkus. Environmental Kuznets Curve of greenhouse gas emissions including technological progress and substitution effects. *Energy*, 135(2017):237-248.
- [5] C. Ma, David Ian Stern. Long-run estimates of interfuel and interfactor elasticities. *Resource and Energy Economics*, 46(2016):114-130.
- [6] J. He, M. Su. Analysis of carbon productivity in response to global climate change. *China Soft Science*, 10(2019):42-47.
- [7] C. Zhang, J. Wang, W. Shi, Y. Li. Factor decomposition of China's regional carbon productivity fluctuations. *China Population, Resources and Environment*, 24(2014):41-47.
- [8] H. Wang, P. Zhou. Multi-country comparisons of CO2 emission intensity: The production-theoretical decomposition analysis approach. *Energy Economics*, 74(2018):310-320.
- [9] M. Bostian et al. Environmental investment and firm performance: A network approach. *Energy Economics*, 57(2016):243-255.
- [10] Beng Wah Ang, F. Zhang. A survey of index decomposition analysis in energy and environmental studies. *Energy*, 25(2000):1149-1176.
- [11] Z. Wang, J. Su, K. Peng. Measurement and factor decomposition of regional differences in carbon productivity in my country. *Statistics and Decision*, 8 (2007): 116-120.
- [12] Beng Wah Ang. LMDI decomposition approach: A guide for implementation. *Energy Policy*, 86 (2015): 233-238.
- [13] S. Li, L. Luo. The factor decomposition and growth momentum of China's carbon productivity during the "Twelfth Five-Year Plan" period. Based on the LMDI-PDA decomposition method. *Technoeconomics*, 37 (2018): 77-86.