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A Prisoner's Dilemma Game in a Two-layer Network with Random Comparisons

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Abstract: This paper studies the problem of prisoner's dilemma game on a two-layer network. Three different type of two-layer network structures are constructed. Stochastic selectivity is added to the prisoner's dilemma game and we further explore the influence of it on individual cooperative behavior. Finally, we get the result that random comparison can inhibit cooperative behavior. The probability of random selection is bigger, the proportion of collaborators in the network is smaller. The results of this study provide further theoretical insights for different levels of cooperation in human social activities.

Keywords: prisoner's dilemma game; two-layer network; random selection

1 Introduction

Understanding cooperative behavior among social individuals is a great challenge to people today. Of course, the interaction between individuals [1] and the change of cooperative behavior [2] has always been the focus of people's research. Evolutionary games can be used to describe the cooperative evolution behavior between individuals [3], so many types of games are considered as simplified social behaviors [4], especially the prisoner's dilemma game [5] and the public goods game [6].

In order to understand the existence of cooperative behavior, Novak proposed five well-known mechanisms. They are direct reciprocity, indirect reciprocity, spatial game [7], group selection and kin selection, which can support the evolution of cooperation. At present, network reciprocity has become a hot topic in people's research. Under different network structures [8, 9], such as small-world network, regular network, ER network, BA network and so on, we have made certain progress and results in different research directions. A series of studies have shown that the prisoner's dilemma game model and other game types can survive and even dominate in regular networks, complex networks [10, 11] and multi-layer networks [12–16]. In addition, by improving game rules[17, 18], introducing transaction cost [19], link weight[20, 21], reward asymmetry [22], punishment mechanism [23] and other factors to research the influence of different factors on cooperative behavior. Some people modified the basic game model [24] and studied the second-order reputation evaluation model of four typical spatial prisoner's dilemma games [25]. Considering spatial reciprocity in the prisoner's dilemma games, it was proved that the longer the time, the better the reputation, the higher the step length, and the higher the cooperation level. Others have introduced the trickle-down effect of economics, complementing the standard options of "co-operation" and "weakness" with a new bait-and-switch "reward". It's usability stimulates a significant increase in overall cooperation and improves the probability of success of cooperative individuals in the game [26]. As a typical cooperative evolutionary game [27], the prisoner's dilemma game is often used to discuss many social behaviors in the world [28–30]. As a social situation created in the laboratory [31], it gives reasonable explanations and suggestions based on a lot of practical situations. For example, by replicating prisoner's dilemma games in drug-dependent people, it's found that individuals who prefer delayed rewards in time-discounting tasks are more likely to choose cooperation strategy in prisoner's dilemma games [32]. The applicability of game theory in water resources management and conflict resolution is reviewed through a series of non-cooperative water resources games. That expounds the dynamic structure of water resources problems and the importance of considering game evolution path when researching water resources problems [33].

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In the traditional prisoner's dilemma model, individuals only play games with their neighbors and affected by them. However, as for the constantly developing social relations and forms, people have more and more access to information, so there are a plenty of useful information for reference. The traditional strategy change model has great limitations in the current situation. Therefore, in this paper, strategy selection can be optimized by selecting not only adjacent individuals for comparison, but also any individual in the network for comparison(in a certain probability). This suggests that an individual can be influenced by other individuals besides those connecting with him. Considering the increasing complexity and vastness of current social network, choosing a two-layer network to explore will be more similar to the phenomenon in the world. Therefore, random network and scale-free network are selected to explore new regulars and conclusions, and we also explore the influence of various factors on individual behaviors in the network. Since the factors that influence our strategies come from various aspects in social, it's entirely reasonable to consider the influence of other individuals when changing strategies. The significance of this research subject is that it can explain the reasons for making choices in numerous cases.

2 Model

In the two-layer network we considered, both the upper network and the lower network have n nodes, and the structure of the upper and lower networks is different but stable. It is represented by the dashed line in Fig.1 that each node in the upper layer has only one node in the lower layer corresponding to it, which constitutes the connection between the two layers of the network and represents the information exchange and interaction between two layers. But the corresponding nodes of different layer networks don't play games. On the basis of two typical networks, ER network and BA network, this paper discusses three different situations. One is that the upper and lower layers are both BA networks (abbreviated $BA * BA$). One is that both the upper and lower layers are ER networks ($ER * ER$) and the upper layers are BA networks. Another type is the BA network at the top and ER network at the bottom (abbreviated $BA * ER$).

Take the $BA * ER$ network as an example, and the model of the two-layer network is shown in Fig.1.

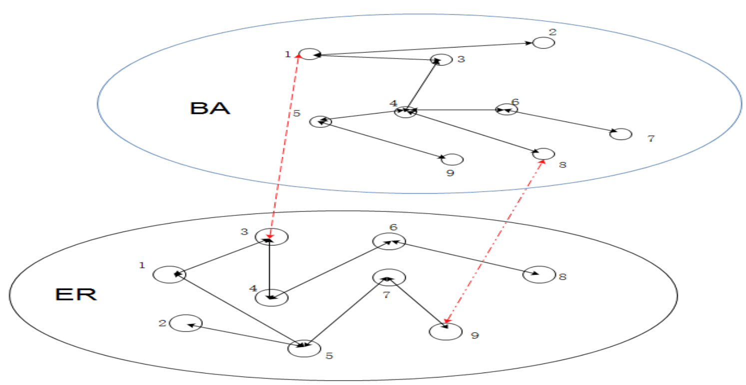

Fig. 1. BA network and ER network constitute a two-layer network. Each layer of the network is connected, and each node of the upper layer is only connected to one node of the lower layer.

Based on the above two-layer network structure, the individuals in the network will play games with the connected individuals in the same layer and calculate their respective benefits. Moreover, each individual has the opportunity to randomly select individuals for comparison, it can imitate the strategy of the other party for the next round of game or remain unchanged. When the whole network presents a relatively stable state after sufficient games, we can summarize the rules and draw a conclusion.

First of all, in the traditional prisoner's dilemma model, two connected individuals play games, and their respective strategies determine their respective benefits. Before the first game, each individual randomly chooses the cooperative strategy(C) or the betrayal strategy(D). In the prisoner's dilemma game, both players decide the strategy at the same time. If they are both collaborators, benefits are equal, denoted as R. If two individuals are both traitors, the return is P, where it is less than the return when they are both collaborators($P < R$). When two sides of the game have different strategies, the income of the collaborator is recorded as S, while the income of the betrayer is recorded as T. Obviously, the betrayer gains more at this time. The general form of its return matrix can be expressed as

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array} \tag{1}$$

The order of their size is $T > R > P > S$. When the two strategies are different, the betrayer benefits the most. When they both are collaborators, the two sides will gain the second most. When the opponent of the game chooses the betrayal strategy, but chooses the cooperation strategy himself, the profit will be the lowest. When both sides of the game are collaborators, the total revenue reaches the maximum, namely $2R > T + S$. Weak prisoner's dilemma model can show the most characteristics of the prisoner's dilemma model, and in a weak prisoner's dilemma model we only need to consider a change of earnings that defectors, remember to b ($b \geq 1$). On the basis of this paper research content, weak prisoner's dilemma can satisfy the need completely, so we will take the simple payoff matrix to represent

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 0 \\ P & 0 \end{pmatrix} \end{array} \tag{2}$$

Supposing the number of individuals connected to node x is N_x, then the number of individuals who connected to node x in the same layer network is $N_x - 1$. Games are only played between connected individuals at the same level. For the corresponding node x' of the other layer, it is connected to node x but not played with it. All $N_x - 1$ adjacent individuals of node x are denoted as V_x. y represents the individual playing with node x, and p_{xy} represents the revenue generated by the game between node x and node y (calculated according to the revenue matrix and the two-node strategy). When the game is over with all the adjacent individuals, the sum of all the benefits is calculated to obtain the total benefits of that individual. Let S_x and S'_x represent the revenue of individual in the upper layer and the lower layer, so the total revenue of nodes can be calculated by eq.(3,4)

$$S_x = \sum_{y \in V_x} p_{xy} \tag{3}$$

$$S'_x = \sum_{y' \in V'_x} p_{x'y'} \tag{4}$$

The interaction between networks is expressed by bias utility function, and the utility of node x is determined by the total revenue obtained by itself and the total revenue of corresponding node x' . Give a variable coefficient q ($0 \leq q \leq 1$), the utility U_x of node x and the utility U'_x of the corresponding node x' is calculated according to eq.(5,6),

$$U_x = S_x * q + S'_x * (1 - q) \tag{5}$$

$$U'_x = S_x * q + S'_x * (1 - q) \tag{6}$$

where q ($0 \leq q \leq 1$) represents the deviation between node x and node x' in calculating utility. The larger the value of q is, the greater the proportion of S_x is. In other words, the network in which node x is located occupies a dominant position and has a greater influence on the change of node strategy. When $q = 0.5$, the two-layer network is of equal importance. When $q = 0$ or $q = 1$, the function of one layer of the network becomes 0, and then the two-layer network is equivalent to the single-layer network.

At the end of each game, the individual has the opportunity to choose a neighbor to compare and decide the next game strategy. You can take the other side's strategy in the round, or you can keep your own strategy unchanged. In the previous model, adjacent individuals are selected for comparison. In this study, selection can be made from adjacent individuals or any individual in the network. Before an individual plays the next game, a parameter $e(0 \leq e \leq 1)$ is given as the probability of selecting adjacent individuals for comparison. That is to say, the probability that an individual chooses a comparison object from its neighbors is e, and the probability that chooses any comparison object from this layer is $1 - e$. After selecting individual for comparison, Fermi function is used to calculate the probability of strategy change according to eq.(7),

$$W(c_y \to c_x) = \frac{1}{1 + \exp[(U_x - U_y)/K]} \tag{7}$$

where c_x represents the strategy of node x in the round, and $W(c_y \to c_x)$ represents the probability that node x adopts the strategy of node y. According to eq.(7), it can be clearly seen that when U_x is larger than U_y, the probability of node x adopting the strategy of node y is smaller, but ultimately, the strategy change is still random. Here K is the noise factor, and it describes how rational an individual is. The greater the value of the noise coefficient, the more the individual is subjected to other interference, and the more random the probability of learning the other party's strategy is. When $K \to 0$, it means that the policy update is deterministic. If its rival's returns exceed its own, it must learn, otherwise it will stick to its strategy.

3 Results

Firstly, in order to discuss the influence of different factors on the evolution of network individual cooperative behavior, it is necessary to establish a benchmark. Then, change one or more variables, and discuss the influence of the change of this variable on network cooperative behavior. Based on this idea, we set the number of nodes in each layer as 1000. In the initial state, each individual has a probability of 50% to choose cooperation strategy(C) or betrayal strategy(D), and ensure that the structure and initial state of each layer remain unchanged. Set the influence coefficient $q = 0.5$ between the two-layer networks, the upper and lower networks are equally important. Set $e = 0.9$, that is, it has the probability of 90% to select individuals from neighbors for strategy learning, so the results obtained are less different from the traditional selection method. The degree distribution of the BA network is extremely asymmetric. The degree of most nodes is small, and only a few nodes have relatively large degree. Therefore, the BA network is highly robust. When nodes turn into traitors randomly, the cooperative behavior of the whole network will not be greatly affected. Compared with BA network, ER is vulnerable to the influence of betrayers, which is not conducive to the generation of cooperative behaviors, and the temptation of betrayal will also inhibit the cooperative behaviors in the network. Therefore, for $BA * BA$ network and $BA * ER$ network, set betrayal temptation $b = 2$. However, the exploration result of $ER * ER$ network setting $b = 1.03$ is more significant. Set $K = 0.5$ in the fermi function. Two groups of changes in different network states are obtained.

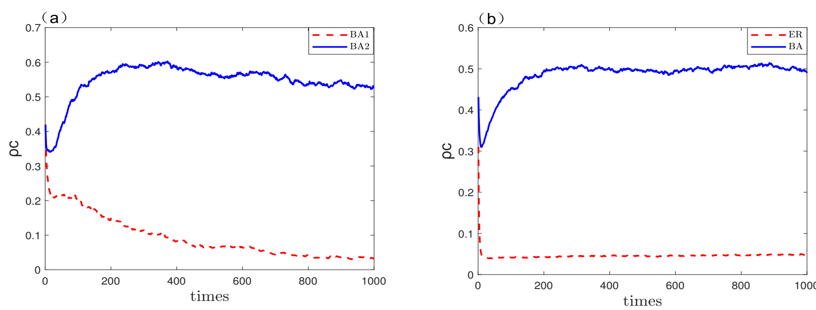

Fig. 2. The upper and lower networks have different structures and different initial states. (a) is the $BA * BA$ network, and (b) is the $BA * ER$ network. Under the determined conditions $q = 0.5$ and $e = 0.9$, the cooperation strategy changes.

Fig.2 shows the co-evolution of $BA * BA$ network and $BA * ER$ network when $b = 2$. ρc represents the proportion of partners in the network. $ER * ER$ networks end up with no collaborators, even when the betrayal of temptation is small. It can be seen that in Fig.2(a), for $BA * BA$ network, in the case of equal mutual influence, although there are eventually cooperators, the proportion of cooperators in the two-layer network is greatly different, which is related to the initial state of the network. In Fig.2(b), there is a large difference in the proportion of two-layer network collaborators in the $BA * ER$ network. It can be concluded that the influence of network structure on the final cooperative behavior is relatively large by comparison. Although the results shown in the figure are not completely stable, they all have slight fluctuations around certain values. If the game continues, the result will be more stable, but according to the results in Fig.2, we can get the regular. In the following part, we mainly discuss the influence of the mutual constraint between two-layer networks on the cooperative behavior under the condition that the random comparison probability remains unchanged. First of all, the probability of random selection is guaranteed to remain unchanged and the influence coefficient between networks is changed. Taking the results of Fig.2 as the benchmark, $q = 0.3$ and $e = 0.9$ are selected respectively to obtain the following results.

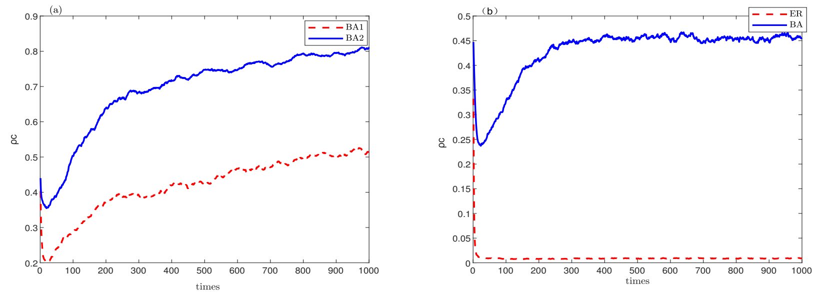

Fig. 3. The evolution process of the strategy under the conditions of $b = 2$,$q = 0.3$,$e = 0.9$.

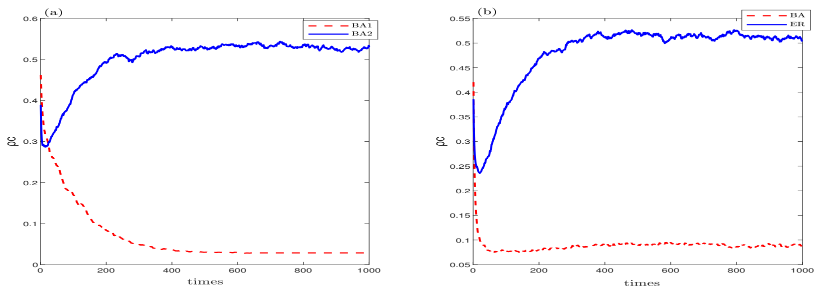

Fig. 4. The evolution process of the strategy under the conditions of $b = 2$,$q = 0.8$,$e = 0.9$.

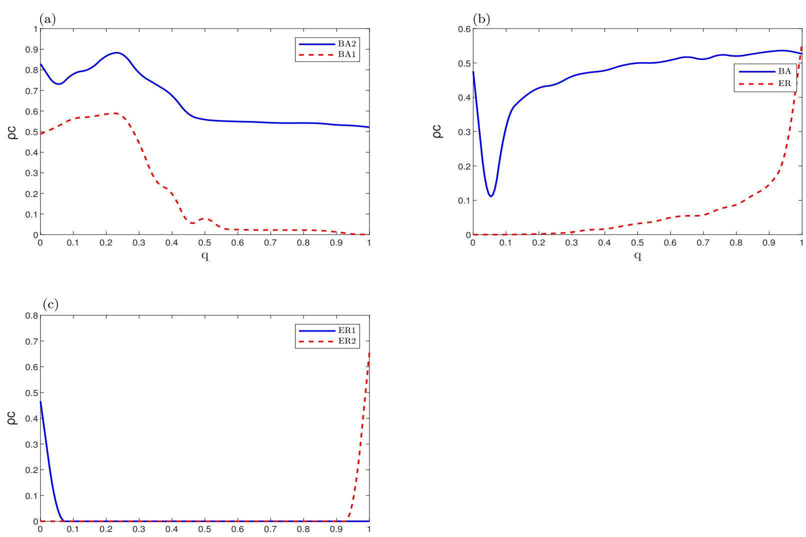

Fig. 5. For $BA * BA$ network and $BA * ER$ network, let $b = 2$ and $e = 0.9$. Considering that the $ER * ER$ network is not conducive to the development and continuation of cooperative behaviors, the above results are obtained when $b = 1.03$ and $e = 0.9$.

For Fig.3 and Fig.4, take $q = 0.3$ and $q = 0.8$ respectively for discussion. It can be clearly seen that when $q = 0.3$, the network with a large proportion of cooperators dominates and the proportion of cooperators in both layers increases. However, when $q = 0.8$, the network with a small proportion of cooperators dominates, and the proportion of cooperators in both layers decreases. Similarly, the same conclusion is reached for the $BA * ER$ network. This shows that the dominant position of the cooperator can promote cooperative behavior. However, for the $ER * ER$ network, there is no cooperator in the above two states. In order to more accurately see the changing trend of interaction between networks on cooperative behavior, the final results of multiple specific values of influence coefficient of two-layer networks from 0 to

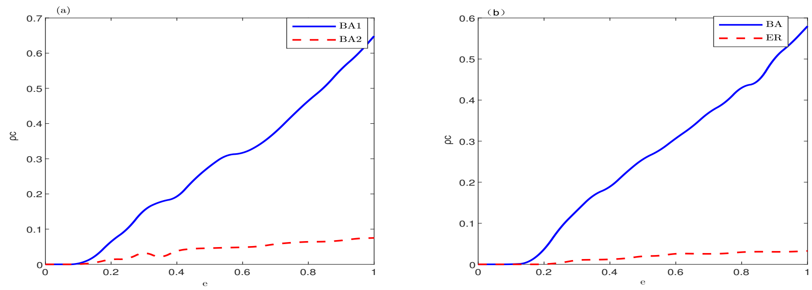

Fig. 6. Under the conditions of $q = 0.5$ and $b = 2$, the change of the cooperation ratio of $BA * BA$ network and $BA * ER$ network.

1 are calculated.

From the results shown in Fig.5(a), for the $BA * BA$ network, the larger the coefficient in the bias utility function, the more unfavorable the cooperative behavior of the network. On the contrary, in Fig.5(b), the cooperative behavior is obviously promoted in the $BA * ER$ network. For $ER * ER$ network, observe the results in Fig.5(c). Only when $q = 0$ or $q = 1$, can a collaborator appear. In other words, when the ER network is not affected by the other layer of network and is equivalent to a single-layer network, there has chance for collaborators to survive. After that, the effect of random comparison on cooperative behavior is studied. Firstly, the influence coefficient $q = 0.5$ between networks is determined. In line with the previous idea, multiple groups of different values of e are taken for exploration. The results in Fig.6 provide a good explanation.

Except that there is no cooperator in $ER*ER$ network, the other two networks show the same trend. With the increase of e, there are more and more cooperators in the network. In other words, cooperative behaviors can be promoted when a high probability of selection is compared among neighboring individuals. The greater the randomness of selection, the more unfavorable the development of individual cooperative behaviors in the network. So what happens when these two things work together? Would the conclusion be different? To solve this problem, the answer is given in Fig.7. Fig.7(a1) and Fig.7(a2) represent the cooperation in the upper and lower layer of the $BA * BA$ network. Fig.7(b1) represents the cooperation of BA network in $BA * ER$ network, and Fig.7(b2) represents the cooperation of ER network in $BA * ER$ network. Fig.7(c1) and Fig.7(c2) represent the cooperation in the upper and lower layer of the $ER * ER$ network. As can be seen from Fig.7, when the two factors act simultaneously, the previous conclusion is still valid. For $BA * BA$ network, the overall trend is that q is smaller, e is larger, and the proportion of network partners is larger. For the $BA * ER$ network, the larger q and e are, the more cooperative behaviors can be promoted in the network. The $ER * ER$ network is not easy to exist under the influence of its own network structure. It only exists when $q = 0$ or $q = 1$, and only exists in the single-layer ER network and under certain conditions. In other words, for the inter-affecting $ER * ER$ network, there is no collaborator.

4 Conclusion

The increase of random selectivity is not conducive to the generation of cooperative behavior, which is consistent with the structural nature of the network. For example, in the BA network, collaborators generally appear and survive in the form of groups, because of the influence and containment of individual neighbors. Random selection, on the other hand, reduces the influence of surrounding individuals and makes it more difficult for cooperative groups to form, thus reducing the proportion of collaborators in the whole network. As for the two-layer network with mutual influence, when the network with a large proportion of collaborators takes the dominant position, it can promote the cooperation in the other layer network, otherwise inhibit the generation and development of cooperative behaviors. This is consistent with the rule of strategy selection in our life. The more information we receive from the outside, the lower the probability of cooperation will be, because we are tempted by the high yield of vast traitors. However, if a cooperative group has greater influence on it, it will be more confident to choose a cooperative strategy.

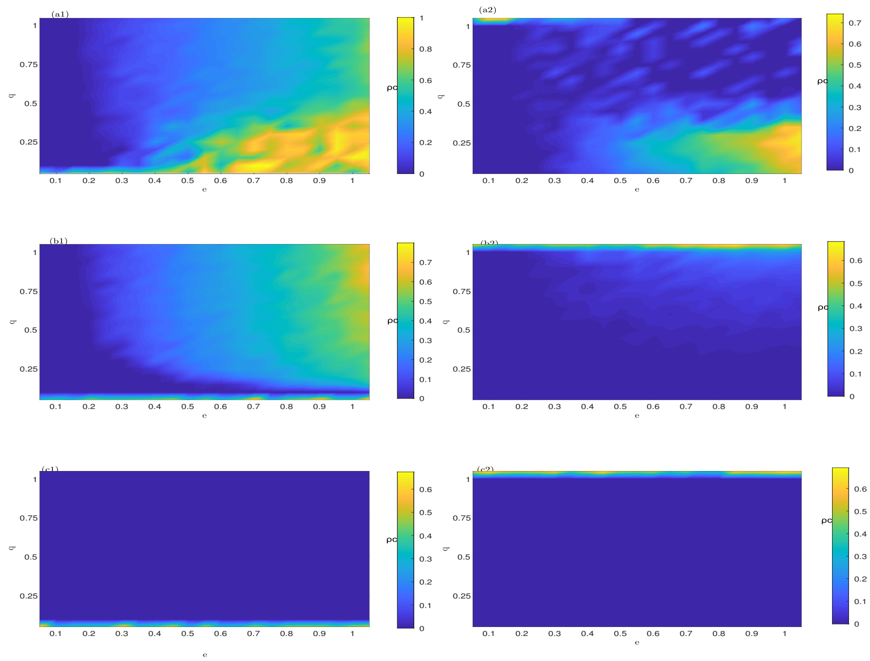

Fig.7. $BA * BA$ network, $BA * ER$ network and $ER * ER$ network correspond to the proportion of collaborators when different values of e and q are taken.

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