



SE2IR Model Considering the Influence of the Exposed Person on Investment Sentiment in a Heterogeneous Network

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Abstract: In today's financial market system, more and more people are pouring into the financial market, and they are also motivating people around them to join in. In this article, in order to study the impact of the investment sentiment of people gradually entering the financial market on those exposed to their words and deeds, we propose an SE2IR model and divide the infected people into two parts. They may represent investment. People in different fields use this to discuss the various results they bring. We prove our results by calculating the basic reproduction number and verifying the stability of the established model.

Keywords: Emotion spread; Investor; Infectious disease model

1 Introduction

Investor sentiment has now become an important object of studying financial markets. A very common example is that people's sentiments are high during a bull market, and they are very low during a bear market. Nowadays, more and more scholars incorporate this subject into their own research projects, through the use of a series of mathematical methods to make an expectation of the future trend of the market, and prevent the occurrence of financial crises. Most people use traditional epidemic spread models. We improve and analyze the traditional models to create a new path for investor sentiment research.

In the financial market, the behavior of investors tends to show irrational behavior, and the spread of investor sentiment will play a very important role among investors. Spyrou's investigation verified the impact of investor sentiment on equity returns in the European market.[1] Kelly Nianyun Cai stated in the article that investor sentiment and capital market conditions play an important role in explaining the temporal changes in the number of DIPOs.[2] .Liu used the infectious disease model SEIR in a heterogeneous network to discuss the propagation threshold of rumors in Weibo.[3]Wang et al. discuss the spread of emotions in complex networks in the article, and use models and simulations to more deeply verify their own ideas.[4]With the vigorous development of infectious disease models in recent years, in addition to being applied to the medical field,[5-6] it can also be used in many other areas, such as rumor control,[7-12] financial risk prevention[13-14], and information dissemination.[15-16]

In this article, investors are divided into four categories, representing those who don't know, those who are exposed to investment information, those who are infected with investment information in two states, and those who no longer listen to information. We put these four categories together. In a heterogeneous network, each person represents a node and sets parameters for their propagation rate. And for those infected with investment information, we have set two states: active recipients and passive recipients, and finally verify their relationship through proof. I hope that this article can be helpful to investment decision-making in the financial market.

The rest of this paper is organized as follows. In section 2, we describe a mathematical model of *SE2IR* and there is an explanation in general. In the third section, In the third part, we calculate the basic living numbers for the model proposed in the second part. In section 4, we discuss the local stability of the model proposed in the second part. In section 5, we analysis the existence and attractiveness of balance point. Some conclusions and discussions are given in the rest part.

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2 Mathematic model description

In this section, we propose a new SE2IR emotion communication model.

First of all, we assume that investors in the market are in one of the following states: active and susceptible people, these people are equivalent to newcomers, and their attitudes towards the market are easily affected by others; exposed people, these people are susceptible and spread. They are easily transformed into susceptible persons; communicators, these people are divided into two categories, one is the active communicator (the one who actively transmits information to other people), the other is the passive communicator (the passive communicator. Information is passed to other people); healers, these people say that they no longer spread the news and no longer receive the news.

The established emotion communication model is formulated for each degree class k , and the dynamic meanfield equation is presented by:

$$\begin{aligned}
 \frac{dS_k(t)}{dt} &= p - \lambda(k)S_k(t)(\theta_1(t) + \theta_2(t)) - qS_k(t) \\
 \frac{dE_k(t)}{dt} &= \lambda(k)S_k(t)(\theta_1(t) + \theta_2(t)) - \beta E_k(t) - qE_k(t) \\
 \frac{dI_{1k}(t)}{dt} &= \beta h_1 E_k(t)\theta_1(t) + r_2 I_{2k}(t) - r_1 I_{1k}(t) - \alpha_1 I_{1k}(t) - qI_{1k}(t) \\
 \frac{dI_{2k}(t)}{dt} &= \beta h_2 E_k(t)\theta_2(t) + r_1 I_{1k}(t) - r_2 I_{2k}(t) - \alpha_2 I_{2k}(t) - qI_{2k}(t) \\
 \frac{dR_k(t)}{dt} &= \alpha_1 I_{1k}(t) + \alpha_2 I_{2k}(t) + (1 - h_1\theta_1(t) - h_2\theta_2(t))\beta E_k(t) - qR_k(t)
 \end{aligned} \tag{1}$$

There is a normalization condition for any time t :

$$S_k(t) + E_k(t) + I_{1k}(t) + I_{2k}(t) + R_k(t) = 1$$

Assume that the system's immigration rate is equal to the immigration rate, that is, $p=q$. Where $\langle k \rangle$ represents the average degree of nodes in the network. And contact rate of nodes in the network:

$$\theta_1(t) = \frac{1}{\langle k \rangle} \sum_k \varphi(k)p(k)I_{1k}(t), \theta_2(t) = \frac{1}{\langle k \rangle} \sum_k \varphi(k)p(k)I_{2k}(t)$$

Here the $p(k)$ is stand for connectivity distribution function, that is probability that an individual has k contacts. And the values of all parameters in the equation are within $(0, 1)$.

3 The basic reproduction number R_0

In this section, we will calculate the basic reproduction number R_0

Lemma 1 Assume that:

$$M = \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & K \end{bmatrix}$$

among them, A, E, K are square matrices of order n , order m , and order s , respectively. And B, C, D, F, G, H are square matrices of order $n * m$, order $n * s$, order $s * n$ and order $s * m$. Let E and $E - DA^{-1}B$ are reversible, so M is reversible, and at this time $K - GA^{-1}C - (H - GA^{-1}B)(E - DA^{-1}B)^{-1}(F - DA^{-1}C)$ is reversible, we can obtain:

$$M^{-1} = \begin{bmatrix} A^{-1} + PT^{-1} & PT^{-1} \\ T^{-1}Q & T^{-1} \end{bmatrix}$$

where

$$P = (-A^{-1}B - A^{-1}C), Q = \begin{bmatrix} -DA^{-1} \\ -GA^{-1} \end{bmatrix}, T = \begin{bmatrix} E - DA^{-1}B & F - DA^{-1}C \\ H - GA^{-1}B & K - GA^{-1}C \end{bmatrix}$$

We can know from the system that only $E_k(t), I_{1k}(t), I_{2k}(t)$ are involved in the calculation of R_0 . So consider the system:

$$\begin{aligned}
 \frac{dE_k(t)}{dt} &= \lambda(k)S_k(t)(\theta_1(t) + \theta_2(t)) - \beta E_k(t) - qE_k(t) \\
 \frac{dI_{1k}(t)}{dt} &= \beta h_1 E_k(t)\theta_1(t) + r_2 I_{2k}(t) - r_1 I_{1k}(t) - \alpha_1 I_{1k}(t) - qI_{1k}(t) \\
 \frac{dI_{2k}(t)}{dt} &= \beta h_2 E_k(t)\theta_2(t) + r_1 I_{1k}(t) - r_2 I_{2k}(t) - \alpha_2 I_{2k}(t) - qI_{2k}(t)
 \end{aligned}$$

Where:

$$F(x) = \begin{pmatrix} \lambda(k)S_k(t)[\theta_1(t) + \theta_2(t)] \\ \beta h_1 E_k(t)\theta_1(t) \\ \beta h_2 E_k(t)\theta_2(t) \end{pmatrix}, \quad V(x) = \begin{pmatrix} \beta E_k(t) + qE_k(t) \\ r_1 I_{1k}(t) + \alpha_1 I_{1k}(t) + qI_{1k}(t) - r_2 I_{2k}(t) \\ r_2 I_{2k}(t) + \alpha_2 I_{2k}(t) + qI_{2k}(t) - r_1 I_{1k}(t) \end{pmatrix}$$

The Jacobian matrices of $F(x)$ and $V(x)$ at the non-emotional balance point of R_0 are:

$$F = \begin{pmatrix} 0 & F_{12} & F_{13} \\ F_{21} & F_{22} & 0 \\ F_{31} & 0 & F_{33} \end{pmatrix}, \quad V = \begin{pmatrix} \beta + q & 0 & 0 \\ 0 & r_1 + \alpha_1 + q & -r_2 \\ 0 & r_2 + \alpha_2 + q & -r_1 \end{pmatrix}$$

From the lemma 1 of the block inverse matrix:

$$V^{-1} = \begin{pmatrix} \frac{1}{\beta+q} & 0 & 0 \\ 0 & \frac{1}{\alpha_1+r_1+q} - \frac{r_1}{r_2(\alpha_2+r_2+q)} & \frac{1}{\alpha_2+r_2+q} - \frac{r_1}{r_1(\alpha_1+r_1+q)} \\ 0 & \frac{r_2+\alpha_2+q}{r_1(\alpha_1+r_1+q)} - \frac{1}{r_2} & -\frac{r_2(r_2+\alpha_2+q)}{r_1^2(\alpha_1+r_1+q)} \end{pmatrix}$$

Among them:

$$F_{12} = \frac{1}{\langle k \rangle} \begin{pmatrix} \lambda(1)\varphi(1)p(1) & \lambda(1)\varphi(2)p(2) & \dots & \lambda(1)\varphi(n)p(n) \\ \lambda(2)\varphi(1)p(1) & \lambda(2)\varphi(2)p(2) & \dots & \lambda(2)\varphi(n)p(n) \\ \vdots & \vdots & \ddots & \vdots \\ \lambda(n)\varphi(1)p(1) & \lambda(n)\varphi(2)p(2) & \dots & \lambda(n)\varphi(n)p(n) \end{pmatrix} = F_{13}$$

$$F_{21} = \frac{\beta h_1}{\langle k \rangle} \begin{pmatrix} \lambda(1)\varphi(1)p(1) & \lambda(1)\varphi(2)p(2) & \dots & \lambda(1)\varphi(n)p(n) \\ \lambda(2)\varphi(1)p(1) & \lambda(2)\varphi(2)p(2) & \dots & \lambda(2)\varphi(n)p(n) \\ \vdots & \vdots & \ddots & \vdots \\ \lambda(n)\varphi(1)p(1) & \lambda(n)\varphi(2)p(2) & \dots & \lambda(n)\varphi(n)p(n) \end{pmatrix} = F_{22} = \beta h_1 F_{12}$$

$$F_{31} = \frac{\beta h_2}{\langle k \rangle} \begin{pmatrix} \lambda(1)\varphi(1)p(1) & \lambda(1)\varphi(2)p(2) & \dots & \lambda(1)\varphi(n)p(n) \\ \lambda(2)\varphi(1)p(1) & \lambda(2)\varphi(2)p(2) & \dots & \lambda(2)\varphi(n)p(n) \\ \vdots & \vdots & \ddots & \vdots \\ \lambda(n)\varphi(1)p(1) & \lambda(n)\varphi(2)p(2) & \dots & \lambda(n)\varphi(n)p(n) \end{pmatrix} = F_{33} = \beta h_2 F_{12}$$

Next order:

$$A = \frac{1}{\alpha_1+r_1+q} - \frac{r_1}{r_2(\alpha_2+r_2+q)}, \quad B = \frac{1}{\alpha_2+r_2+q} - \frac{r_2}{r_1(\alpha_1+r_1+q)} \\ C = \frac{r_2+\alpha_2+q}{r_1(\alpha_1+r_1+q)} - \frac{1}{r_2}, \quad D = -\frac{r_2(r_2+\alpha_2+q)}{r_1^2(\alpha_1+r_1+q)}$$

So we can get:

$$FV^{-1} = \begin{pmatrix} 0 & AF_{12} + CF_{13} & BF_{12} + DF_{13} \\ \frac{1}{B+q}F_{21} & AF_{22} & BF_{22} \\ \frac{1}{B+q}F_{31} & CF_{33} & DF_{33} \end{pmatrix} \\ = \begin{pmatrix} 0 & (A+C)F_{12} & (B+D)F_{12} \\ \frac{1}{B+q}\beta h_1 F_{12} & A\beta h_1 F_{12} & B\beta h_1 F_{12} \\ \frac{1}{B+q}\beta h_2 F_{12} & C\beta h_2 F_{12} & D\beta h_2 F_{12} \end{pmatrix} \\ = \begin{pmatrix} \frac{1}{B+q}\beta h_1 F_{12} & A\beta h_1 F_{12} & B\beta h_1 F_{12} \\ 0 & (A-C)\beta h_1 h_2 F_{12} & (B-D)\beta h_1 h_2 F_{12} \\ 0 & 0 & (2AD - 2BC)\beta h_1 h_2 F_{12} \end{pmatrix}$$

where

$$A - C = \frac{(r_1 - r_2 - \alpha_2 - q)[r_2(\alpha_2 + r_2 + q) - r_1(\alpha_1 + r_1 + q)]}{r_1 r_2 (\alpha_1 + r_1 + q) (\alpha_2 + r_2 + q)},$$

$$2AD - 2BC = \frac{1}{r_2(\alpha_2 + r_2 + q)} - \frac{1}{r_1(\alpha_1 + r_1 + q)}.$$

So we can calculate that:

$$R_0 = \rho(FV^{-1}) = \text{Max} \left(\frac{\langle k \rangle^2 \beta h_1}{\langle k \rangle (B + q)}, \frac{\langle k \rangle^2}{\langle k \rangle} (A - C) \beta h_1 h_2, \frac{\langle k \rangle^2}{\langle k \rangle} (2AD - 2BC) \beta h_1 h_2 \right)$$

In its: $\langle k \rangle^2 = \sum k^2 p(k)$.

4 The local stability analysis

Theorem 2 When the basic reproduction number $R_0 < 1$, the system's non-emotional balance point is gradually stable locally.

Proof. The Jacobian matrix of the above system at the disease-free equilibrium point is:

$$M = \begin{pmatrix} M_{11} & M_{12} & \cdots & M_{1n} \\ M_{21} & M_{22} & \cdots & M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{n1} & M_{n2} & \cdots & M_{nn} \end{pmatrix}$$

In it:

$$M_{jj} = \begin{pmatrix} -\beta - q & \frac{1}{\langle k \rangle} \lambda(k) \varphi(k) p(k) & \frac{1}{\langle k \rangle} \lambda(k) \varphi(k) p(k) \\ \beta h_1 \frac{1}{\langle k \rangle} \lambda(k) \varphi(k) p(k) & -(r_1 + \alpha_1 + q) & r_2 \\ \beta h_2 \frac{1}{\langle k \rangle} \lambda(k) \varphi(k) p(k) & r_1 & -(r_2 + \alpha_2 + q) \end{pmatrix}$$

$$M_{ij} = \begin{pmatrix} 0 & \frac{\lambda(k) \varphi(k) p(k)}{\langle k \rangle} & \frac{\lambda(k) \varphi(k) p(k)}{\langle k \rangle} \\ 0 & \beta h_1 \frac{\lambda(k) \varphi(k) p(k)}{\langle k \rangle} & 0 \\ 0 & 0 & \beta h_2 \frac{\lambda(k) \varphi(k) p(k)}{\langle k \rangle} \end{pmatrix}$$

By mathematical induction, at $R_0 < 1$, all the characteristic roots of M have negative real parts, so that the disease-free equilibrium point of the system is locally stable.

■

5 The existence and attractiveness of balance point N^*

First calculate the equilibrium point N^* in this system. We let:

$$\begin{aligned} p - \lambda(k) S_k(t) (\theta_1(t) + \theta_2(t)) - q S_k(t) &= 0 \\ \lambda(k) S_k(t) (\theta_1(t) + \theta_2(t)) - \beta E_k(t) - q E_k(t) &= 0 \\ \beta h_1 E_k(t) \theta_1(t) + r_2 I_{2k}(t) - r_1 I_{1k}(t) - \alpha_1 I_{1k}(t) - q I_{1k}(t) &= 0 \\ \beta h_2 E_k(t) \theta_2(t) + r_1 I_{1k}(t) - r_2 I_{2k}(t) - \alpha_2 I_{2k}(t) - q I_{2k}(t) &= 0 \\ \alpha_1 I_{1k}(t) + \alpha_2 I_{2k}(t) + (1 - h_1 \theta_1(t) - h_2 \theta_2(t)) \beta E_k(t) - q R_k(t) &= 0 \end{aligned}$$

It can be calculated from the above formula:

$$S^* = \frac{p}{\lambda(k) (\theta_1 + \theta_2) + q} \quad E^* = \frac{\lambda(k) (\theta_1 + \theta_2) p}{\lambda(k) (\theta_1 + \theta_2) (\beta + q) + q(\beta + q)}$$

$$I_1^* = \frac{\lambda(k) (\theta_1 + \theta_2) p \cdot \beta h_1 \theta_1 [r_2 \theta_2 h_2 + (r_2 + \alpha_2 + q) \theta_1 h_1]}{[\lambda(k) (\theta_1 + \theta_2) (\beta + q) + q(\beta + q)] \cdot [(r_2 \theta_2 h_2 + (r_2 + \alpha_2 + q) \theta_1 h_1) (r_1 + \alpha_1 + q)] - U}$$

$$I_2^* = \frac{r_1 \theta_1 h_1 + (r_1 + \alpha_1 + q) \theta_2 h_2}{r_2 \theta_2 h_2 + (r_2 + \alpha_2 + q) \theta_1 h_1} I_1^*$$

$$R^* = \frac{\alpha_1}{q} I_1^* + \frac{\alpha_2 r_1' \theta_1 h_1 + (r_1 + \alpha_1 + q) \theta_2 h_2 \alpha_2}{r_2 \theta_2 h_2 q + (r_2 + \alpha_2 + q) q \theta_1 h_1} I_1^* + \frac{(1 - h_1 \theta_1 - h_2 \theta_2) \beta \cdot \lambda(k) (\theta_1 + \theta_2) p}{q \lambda(k) (\theta_1 + \theta_2) (\beta + q) + q^2 (\beta + q)}$$

where

$$U = r_1 r_2 \theta_1 h_1 + (r_1 + \alpha_1 + q) \theta_2 h_2 r_2$$

Theorem 3 Suppose that $(S_k, E_k, I_{1k}, I_{2k}, R_k)$ is a solution of system, satisfy with the initial condition. If $R_0 > 1$, then

$$\liminf_{t \rightarrow \infty} \{S_k(t), I_{1k}(t), I_{2k}(t), E_k(t), R_k(t)\} = (S_k^*(t), E_k^*(t), I_{1k}^*(t), I_{2k}^*(t), R_k^*(t)) = N^*$$

, where $(S_k^*, E_k^*, I_{1k}^*, I_{2k}^*, R_k^*)$ is a positive equilibrium point of system (1).

Proof. Hypothesis that when $R_0 > 1$, exist a $\xi > 0$, let

$$\liminf_{t \rightarrow \infty} \{S_k(t), I_{1k}(t), I_{2k}(t), E_k(t), R_k(t)\} \geq \xi$$

Let k be fixed to any element in the set $\{1, 2, 3 \dots n\}$, Suppose there are $0 < \xi < \frac{1}{4}$ and a sufficiently large constant $T > 0$, there are: $E_k(t) \geq \xi, I_{1k}(t) \geq \xi, I_{2k}(t) \geq \xi, R_k(t) \geq \xi, (t > T)$. So have $\xi < \theta_1 \leq 1, \xi < \theta_2 \leq 1$. The first formula substituted into the original system (1) is obtained:

$$S'_k(t) \leq p - \lambda(k) \cdot 2\xi \cdot S_k(t) - qS_k(t), (t > T)$$

Comparison principle by differential equation: $\forall 0 < \xi_1 < \frac{2\lambda(k)\xi}{2\lambda(k)\xi+q}, \exists t_1 > T$, that make $S_k(t) \leq X_k^{(1)} - \xi_1, X_k^{(1)} = \frac{p}{2\lambda(k)\xi+q} + 2\xi_1 < 1$.

From the second formula of the original system (1):

$$E'_k(t) \leq \lambda(k)2\xi(1 - E_k(t)) - \beta E_k(t) - qE_k(t), (t > t_1)$$

For $\forall 0 < \xi_2 < \min\left\{\frac{1}{2}, \xi_1, \frac{2\lambda(k)\xi}{2\lambda(k)\xi+q}\right\}, \exists t_2 > t_1$, that make $E_k(t) \leq Y_k^{(1)} - \xi_2, (t > t_2), Y_k^{(1)} = \frac{2\lambda(k)\xi}{2\lambda(k)\xi+\beta+q} + 2\xi_2 < 1$.

From the third formula of the original system (1):

$$I'_{1k}(t) \leq \beta h_1 \xi (1 - I_{1k}(t)) + r_2 (1 - I_{1k}(t)) - r_1 I_{1k}(t) - \alpha_1 I_{1k} - qI_{1k}$$

For $\forall 0 < \xi_3 < \min\left\{\frac{1}{3}, \xi_2, \frac{r_1+\alpha_1+q}{2(\beta h_1 \xi+r_2+r_1+\alpha_1+q)}\right\}, \exists t_3 > t_2$, that make $I_{1k}(t) \leq Z_k^{(1)} - \xi_3, (t > t_3)$,

$$Z_k^{(1)} = \frac{\beta h_1 \xi}{\beta h_1 \xi+r_1+\alpha_1+q+r_2} + 2\xi_3 < 1.$$

From the fourth formula of the original system (1):

$$I'_{2k}(t) \leq \beta h_2 \xi (1 - I_{2k}(t)) + r_1 (1 - I_{2k}(t)) - r_2 I_{2k}(t) - \alpha_2 I_{2k} - qI_{2k}$$

For $\forall 0 < \xi_4 < \min\left\{\frac{1}{4}, \xi_3, \frac{r_2+\alpha_2+q}{2(\beta h_2 \xi+r_2+r_1+\alpha_2+q)}\right\}, \exists t_4 > t_3$, that make $I_{2k}(t) \leq W_k^{(1)} - \xi_4, (t > t_4), W_k^{(1)} = \frac{\beta h_2 \xi}{\beta h_2 \xi+r_1+\alpha_2+q+r_2} + 2\xi_4 < 1$.

From another aspect, We substitute $\theta_1 \leq 1, \theta_2 \leq 1$ into the first formula of the original system (1) to get:

$$S'_k(t) \geq p - 2\lambda(k)S_k(t) - qS_k(t), t > T$$

For $\forall 0 < \xi_5 < \min\left\{\frac{1}{5}, \xi_4, \frac{p}{2(2\lambda(k)\xi+q)}\right\}, \exists t_5 > t_4$ that make $S_k(t) \geq x_k^{(1)} + \xi_5, (t > t_5), x_k^{(1)} = \frac{p}{2\lambda(k)+q} - 2\xi_5 > 0$.

The second formula for the same reason,

$$E'_k(t) \geq 2\lambda(k)\xi_k^{(1)} - \beta E_k(t) - qE_k(t), t > T$$

For $\forall 0 < \xi_6 < \min\left\{\frac{1}{6}, \xi_5, \frac{2\lambda(k)\xi}{2(2\lambda(k)\xi+\beta+q)}\right\}, \exists t_6 > t_5$, that make $E_k(t) \geq y_k^{(1)} + \xi_6, (t > t_6), y_k^{(1)} = \frac{2\lambda(k)\xi}{2\lambda(k)\xi+\beta+q} - 2\xi_6 > 0$. The third formula for the same reason,

$$I'_{1k}(t) \geq \beta h_1 \xi y_k^{(1)} - r_1 I_{1k}(t) - \alpha_1 I_{1k}(t) - qI_{1k}(t), t > T$$

For $\forall 0 < \xi_7 < \min\left\{\frac{1}{7}, \xi_6, \frac{\beta h_1 \xi y_k^{(1)}}{2(r_1+\alpha_1+q)}\right\}, \exists t_7 > t_6$, that make $I_{1k}(t) \geq z_k^{(1)} + \xi_7, (t > t_7), z_k^{(1)} = \frac{\beta h_1 \xi y_k^{(1)}}{r_1+\alpha_1+q} - 2\xi_7 > 0$.

The fourth formula for the same reason,

$$I'_{2k}(t) \geq \beta h_2 \xi_k^{(1)} - r_2 I_{2k}(t) - \alpha_2 I_{2k}(t) - qI_{2k}(t), t > T$$

For $\forall 0 < \xi_8 < \min \left\{ \frac{1}{8}, \xi_7, \frac{\beta h_2 \xi y_k^{(1)}}{2(r_2 + \alpha_2 + q)} \right\}$, $\exists t_8 > t_7$, that make $I_{2k}(t) \geq w_k^{(1)} + \xi_8$, ($t > t_8$), $w_k^{(1)} = \frac{\beta h_2 \xi y_k^{(1)}}{r_2 + \alpha_2 + q} - 2\xi_8 > 0$.

Because ξ is a small enough constant, there are:

$$0 < x_k^{(1)} < X_k^{(1)} < 1, 0 < y_k^{(1)} < Y_k^{(1)} < 1, 0 < z_k^{(1)} < Z_k^{(1)} < 1, 0 < w_k^{(1)} < W_k^{(1)} < 1$$

Let:

$$\begin{aligned} m^{(j)} &= \frac{1}{\langle k \rangle} \sum \varphi(k)p(k)x_k^{(j)}M^{(j)} = \frac{1}{\langle k \rangle} \sum \varphi(k)p(k)X_k^{(j)} \\ n^{(j)} &= \frac{1}{\langle k \rangle} \sum \varphi(k)p(k)y_k^{(j)}N^{(j)} = \frac{1}{\langle k \rangle} \sum \varphi(k)p(k)Y_k^{(j)} \\ q^{(j)} &= \frac{1}{\langle k \rangle} \sum \varphi(k)p(k)z_k^{(j)}Q^{(j)} = \frac{1}{\langle k \rangle} \sum \varphi(k)p(k)Z_k^{(j)} \\ r^{(j)} &= \frac{1}{\langle k \rangle} \sum \varphi(k)p(k)w_k^{(j)}R^{(j)} = \frac{1}{\langle k \rangle} \sum \varphi(k)p(k)W_k^{(j)} \end{aligned}$$

And

$$v^{(j)} = \frac{1}{\langle k \rangle} \sum \lambda(k)p(k) \left(z_k^{(j)} + w_k^{(j)} \right), \quad V^{(j)} = \frac{1}{\langle k \rangle} \sum \lambda(k)p(k) \left(Z_k^{(j)} + W_k^{(j)} \right)$$

Apparently $0 < v^{(j)} \leq \theta \leq V^{(j)}$. So from the first equation of the original system (1),

$$S'_k(t) \leq p - \lambda(k)S_k(t) \cdot v^{(1)} - qS_k(t), (t > t_8)$$

For $\forall 0 < \xi_9 < \min \left\{ \frac{1}{9}, \xi_8 \right\}$, $\exists t_9 > t_8$, let $S_k(t) \leq X_k^{(2)}$, and $X_k^{(2)} = \min \left\{ X_k^{(1)} - \xi_1, \frac{p}{\lambda(k)v^{(1)+q}} + \xi_9 \right\}$.

From the second equation of the original system (1),

$$E'_k(t) \leq \lambda(k)X_k^{(2)} \cdot v^{(1)} - \beta E_k(t) - qE_k(t), (t > t_9)$$

For $\forall 0 < \xi_{10} < \min \left\{ \frac{1}{10}, \xi_9 \right\}$, $\exists t_{10} > t_9$, let $E_k(t) \leq Y_k^{(2)}$, and $Y_k^{(2)} = \min \left\{ Y_k^{(1)} - \xi_2, \frac{\lambda(k)X_k^{(2)}v^{(1)}}{\beta+q} + \xi_{10} \right\}$.

From the third equation of the original system (1),

$$I'_{1k}(t) \leq \beta h_1 Y_k^{(2)} \cdot v^{(1)} + r_2 W^{(1)} - r_1 I_{1k}(t) - \alpha_1 I_{1k}(t) - qI_{1k}(t), (t > t_{10})$$

For $\forall 0 < \xi_{11} < \min \left\{ \frac{1}{11}, \xi_{10} \right\}$, $\exists t_{11} > t_{10}$, let $I_{1k}(t) \leq Z_k^{(2)}$, and $Z_k^{(2)} = \min \left\{ Z_k^{(1)} - \xi_3, \frac{\beta h_1 Y_k^{(2)}v^{(1)} + r_2 W^{(1)}}{r_1 + \alpha_1 + q} + \xi_{11} \right\}$.

From the fourth equation of the original system (1),

$$I'_{2k}(t) \leq \beta h_2 Y_k^{(2)} \cdot v^{(1)} + r_1 Z^{(1)} - r_2 I_{2k}(t) - \alpha_2 I_{2k}(t) - qI_{2k}(t), (t > t_{11})$$

For $\forall 0 < \xi_{12} < \min \left\{ \frac{1}{12}, \xi_{11} \right\}$, $\exists t_{12} > t_{11}$, let $I_{2k}(t) \leq W_k^{(2)}$, and $W_k^{(2)} = \min \left\{ W_k^{(1)} - \xi_4, \frac{\beta h_1 Y_k^{(2)}v^{(1)} + r_1 Z^{(1)}}{r_2 + \alpha_2 + q} + \xi_{12} \right\}$.

Back to the first formula again,

$$S'_k(t) \geq p - \lambda(k)S_k(t) \cdot V^{(2)} - qS_k(t) (t > t_{12})$$

so $\forall 0 < \xi_{13} < \min \left\{ \frac{1}{13}, \xi_{12}, \frac{p}{2(\lambda(k)V^{(2)}+q)} \right\}$, $\exists t_{13} > t_{12}$, let $S_k(t) \geq x_k^{(2)} + \xi_{13}$, ($t > t_{13}$),

and $x_k^{(2)} = \max \left\{ x_k^{(1)} + \xi_5, \frac{p}{\lambda(k)V^{(2)}+q} - 2\xi_{13} \right\}$.

Back to the second formula,

$$E'_k(t) \geq \lambda(k)X_k^{(2)} \cdot V^{(2)} - \beta E_k(t) - qE_k(t), (t > t_{13})$$

so $\forall 0 < \xi_{14} < \min \left\{ \frac{1}{14}, \xi_{13}, \frac{\lambda(k)V^{(2)}X_k^{(2)}}{2(\beta+q)} \right\}$, $\exists t_{14} > t_{13}$, let $E_k(t) \geq y_k^{(2)} + \xi_{14}$, ($t > t_{14}$),

and $y_k^{(2)} = \max \left\{ y_k^{(1)} + \xi_6, \frac{\lambda(k)V^{(2)}X_k^{(2)}}{\beta+q} - 2\xi_{14} \right\}$.

Back to the third formula,

$$I'_{2k}(t) \geq \beta h_1 Y_k^{(2)} \cdot V^{(1)} + r_2 W^{(1)} - r_i I_{1\bar{k}}(t) - \alpha_1 I_{1k}(t) - qI_{1k}(t), (t > t_{15})$$

so $\forall 0 < \xi_{15} < \min \left\{ \frac{1}{15}, \xi_{14}, \frac{\beta h_1 V^{(1)} Y_k^{(2)} + r_2 W^{(1)}}{2(r_1 + \alpha_1 + q)} \right\}$, $\exists t_{14} > t_{13}$, let $I_{1k}(t) \geq z_k^{(2)} + \xi_{15}$, ($t > t_{15}$),

and $z_k^{(2)} = \max \left\{ z_k^{(1)} + \xi_7, \frac{\beta h_1 V^{(1)} Y_k^{(2)} + r_2 W^{(1)}}{r_1 + \alpha_1 + q} - 2\xi_{15} \right\}$.

Back to the fourth formula,

$$I'_{2k}(t) \geq \beta h_2 Y_k^{(2)} \cdot V^{(1)} + r_1 Z^{(1)} - r_2 I_{2k}(t) - \alpha_2 I_{2k}(t) - q I_{2k}(t), (t > t_{16})$$

so $\forall 0 < \xi_{16} < \min \left\{ \frac{1}{16}, \xi_{15}, \frac{\beta h_2 V^{(1)} Y_k^{(2)} + r_1 Z^{(1)}}{2(r_2 + \alpha_2 + q)} \right\}$, $\exists t_{15} > t_{14}$, let $I_{2k}(t) \geq w_k^{(2)} + \xi_{16}$, ($t > t_{16}$)

and $w_k^{(2)} = \max \left\{ w_k^{(1)} + \xi_8, \frac{\beta h_2 V^{(1)} Y_k^{(2)} + r_1 Z^{(1)}}{r_2 + \alpha_2 + q} - 2\xi_{16} \right\}$.

After $h (h \geq 3)$ iterations, we can get the sequence:

$$\left\{ x_k^{(h)} \right\}, \left\{ y_k^{(h)} \right\}, \left\{ z_k^{(h)} \right\}, \left\{ w_k^{(h)} \right\}, \left\{ X_k^{(h)} \right\}, \left\{ Y_k^{(h)} \right\}, \left\{ Z_k^{(h)} \right\}, \left\{ W_k^{(h)} \right\}$$

obviously the first four sets of numbers are monotonically increasing, and the last four sets of numbers are monotonically decreasing.

Thus there is a sufficiently large integer L , at $h > L$ has:

$$\begin{aligned} X_k^{(h)} &= \frac{p}{\lambda(k)v^{(h-1)}+q} + \xi_{8h-7} & x_k^{(h)} &= \frac{p}{\lambda(k)V^{(h-1)}+q} - 2\xi_{8h-3} \\ Y_k^{(h)} &= \frac{\lambda(k)X_k^{(h)}v^{(h-1)}}{\beta+q} + \xi_{8h-6} & y_k^{(h)} &= \frac{\lambda(k)x_k^{(h)}V^{(h-1)}}{\beta+q} - 2\xi_{8h-2} \\ Z_k^{(h)} &= \frac{\beta h_1 V^{(h-1)} Y_k^{(h)} + r_2 W^{(h)}}{r_1 + \alpha_1 + q} + \xi_{8h-5} & z_k^{(h)} &= \frac{\beta h_1 v^{(h-1)} y_k^{(h)} + r_2 w^{(h)}}{r_1 + \alpha_1 + q} - 2\xi_{8h-1} \\ W_k^{(h)} &= \frac{\beta h_2 V^{(h-1)} Y_k^{(h)} + r_1 Z^{(h)}}{r_2 + \alpha_2 + q} + \xi_{8h-4} & w_k^{(h)} &= \frac{\beta h_2 v^{(h-1)} y_k^{(h)} + r_1 z^{(h)}}{r_2 + \alpha_2 + q} - 2\xi_{8h} \end{aligned}$$

And the above formula has a limit $\lim_{h \rightarrow \infty} \delta^{(k)} = \delta_k$, where $\delta_k^{(h)} = \left\{ X_k^{(h)}, Y_k^{(h)}, Z_k^{(h)}, W_k^{(h)}, x_k^{(h)}, y_k^{(h)}, z_k^{(h)}, w_k^{(h)} \right\}$,

and $\delta_k = \{X_k, Y_k, Z_k, W_k, x_k, y_k, z_k, w_k\}$.

Because $0 < \xi_h < \frac{1}{h}$, when $h \rightarrow \infty$, $\xi_h \rightarrow 0$, $h \rightarrow \infty$, we have:

$$\begin{aligned} X_k &= \frac{p}{\lambda(k)v+q} & x_k &= \frac{p}{\lambda(k)V+q} \\ Y_k &= \frac{\lambda(k)X_k v}{\beta+q} & y_k &= \frac{\lambda(k)x_k V}{\beta+q} \\ Z_k &= \frac{\beta h_1 V Y_k + r_2 W_k}{r_1 + \alpha_1 + q} & z_k &= \frac{\beta h_1 v y_k + r_2 w_k}{r_1 + \alpha_1 + q} \\ W_k &= \frac{\beta h_2 V Y_k + r_1 Z_k}{r_2 + \alpha_2 + q} & w_k &= \frac{\beta h_2 v y_k + r_1 z_k}{r_2 + \alpha_2 + q} \end{aligned}$$

also because:

$$v^{(j)} = \frac{1}{\langle k \rangle} \sum \lambda(k)p(k) \left(z_k^{(j)} + w_k^{(j)} \right), \quad V^{(j)} = \frac{1}{\langle k \rangle} \sum \lambda(k)p(k) \left(Z_k^{(j)} + W_k^{(j)} \right)$$

So we can get:

$$\begin{aligned} V &= \frac{1}{\langle k \rangle} \sum \lambda(k)p(k) \left(\frac{\beta h_1 V Y_k + r_2 W_k}{r_1 + \alpha_1 + q} + \frac{\beta h_2 V Y_k + r_1 Z_k}{r_2 + \alpha_2 + q} \right) \\ v &= \frac{1}{\langle k \rangle} \sum \lambda(k)p(k) \left(\frac{\beta h_1 v y_k + r_2 w_k}{r_1 + \alpha_1 + q} + \frac{\beta h_2 v y_k + r_1 z_k}{r_2 + \alpha_2 + q} \right) \end{aligned}$$

From the limit theory and $x_k^{(h)} \leq X_k^{(h)}, y_k^{(h)} \leq Y_k^{(h)}, z_k^{(h)} \leq Z_k^{(h)}, w_k^{(h)} \leq W_k^{(h)}$, we can know that:

$\lim_{h \rightarrow \infty} (V - v) = 0$, so $V = v$, then there is $z_k = Z_k, w_k = W_k, k = 1, 2, 3 \dots$.

Therefore

$$\begin{aligned} \lim_{h \rightarrow \infty} S_k(t) &= X_k = x_k, \lim_{h \rightarrow \infty} E_k(t) = Y_k = y_k \\ \lim_{h \rightarrow \infty} I_{1k}(t) &= Z_k = z_k, \lim_{h \rightarrow \infty} I_{2k}(t) = W_k = w_k \end{aligned}$$

From the previous formula: $R_k(t) = 1 - S_k(t) - E_k(t) - I_{1k}(t) - I_{2k}(t)$, so $\lim_{t \rightarrow \infty} R_k(t) = R_k^*$. Thereby knowing: $N^* (S_k^*(t), E_k^*(t), I_{1k}^*(t), I_{2k}^*(t))$ is globally attractive. ■

Conclusion

All in all, in this article, we have improved the traditional epidemic model and proposed a new SEIR model with the presence of exposed persons, and analyzed its behavior in the financial market, and discussed their different acceptance and The ability to spread emotions.

Through analysis and proof, we come to the conclusion that when $R_0 > 1$, the equilibrium point of emotion exists and is globally attractive. In order to prevent and control the spread of malicious emotions, we can educate people to keep them as sensible as possible to control their ability to spread emotions; we can control the occupation of various groups of people in a stable state by controlling the behaviors and concepts of people with greater influence. In contrast, giving full play to the influence of authority figures to stabilize the stock market dynamics; for new entrants, increasing the proportion of active individuals can speed up the spread of emotions, but on the contrary, it will inhibit them.

In the future ,we will consider more factors in our model such as media coverage,time delay and so on.

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