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The Application and Thinking of Mathematical Methods in the Teaching of Political Economy

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Abstract: This article mainly discusses the specific application of mathematical methods in the teaching of political economy by the method of case analysis. The application of mathematical methods can concretize the basic principles of political economy, and the mathematical methods can help students better understand socialist economic theory and practice. In specific teaching practice, the principles of moderation and practicability should be followed to evaluate and adjust the use of mathematical methods in teaching.

Keywords: Mathematical method; Political economy; Teaching reform

1 Introduction

The research object of political economy is a certain mode of production and its relations of production. All categories of production modes and production relations have qualitative and quantitative regulations. Therefore, the combination of qualitative analysis and quantitative analysis can deeply understand the nature of economic phenomena and reveal the objective laws of economic development. As an important tool to describe the relationship between economic variables, mathematical modes occupy a very important position in economic research. In fact, Marx himself attaches great importance to the use of mathematical methods in economics [1]. He believes that a science can only be truly perfected when it is successfully used in mathematics. Economists such as Sweezy [2], Nobuo Okishio [3], Tosuo Morishima [4], and Romer [5] continued to advance the quantitative analysis of political economy, and gradually established a more systematic and comprehensive mathematical system of political economy. However, mathematical methods and some mature mathematical models have not developed synchronously in the teaching practice. In fact, flexible use of mathematical expression in the teaching process can make relevant problems precise, simple and clear, improve the ability of students to use theory to analyze practical problems, which is conducive to the improvement of the quality and effect of political economy teaching.

2 The Concrete Application of Mathematical Methods in the Teaching of Political Economy

2.1 Using Mathematical Methods in Teaching to Express and Demonstrate the Basic Principles and Theories of Political Economy

Mathematical methods can be used to deduce and prove the changes and relations of various quantities in economic relations, and the theoretical expression of the process of economic movement can be simplified with formal mathematical language, which is helpful for students to better understand the basic categories and principles. For example, the factors and laws that affect the amount of surplus value can be expressed in the form of a function. M is used to represent the amount of surplus value, m is used to represent the surplus value provided by a worker every day, v is the variable capital that a labor power pays in advance every day, V is used to represent the total amount of variable capital, and k is used

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to represent the value of an average labor power, the degree to which an average labor power is exploited is expressed by $\alpha^{'}/\alpha$ (surplus labor / necessary labor), and the number of workers we use is expressed by n. The amount of surplus value can be expressed by mathematical equations:

$$M = \left(\frac{m}{v}\right) \times V \text{ or } M = k \times \left(\frac{\alpha^{'}}{\alpha}\right) \times n \tag{1}$$

so that the factors that affect the amount of surplus value and the law of change can be clearly displayed.

Let's take the law of wage movement in Marx's "Das Kapital" as an example to explore how to use mathematical models in teaching to present wage theory and its changing laws more intuitively and concretely.

Basic Content of the Law of Wage: Marx discussed the change of labor price and surplus value in Chapter 15 of Volume 1 of Das Kapital. Wage is the price of labor, that is, the value of labor. The value of labor power is determined by the value required to reproduce the labor power, or the value of the means of subsistence usually required by average worker (in fact, the development cost of the labor power and the natural difference between the labor power are also influencing factors.).

Basic Assumption: For the sake of simplicity, we set aside the development costs of labor force and the natural differences between labor power, and assume that the prices and values of all commodities (including labor) are indistinguishable in quantity, that is, the prices of all commodities (including labor) are equal to the value of labor. The prices of labor, that is, the relative amount of wages, depends on four factors: (i) the length of the working day, that is, the amount of labor; (ii) the intensity of the working day and the complexity of labor, that is, the quality of labor; (iii) labor productivity, that is, the amount of value of products provided by a unit of labor in a unit of time; (iv) the rate of surplus value, that is, the degree of exploitation of labor by capital. These four factors can have multiple combinations, and several factors change at the same time because of the different magnitude and direction of the changes, which makes the changes more diverse.

The Model: In order to facilitate analysis, we first make algebraic settings for each related quantity and relationship. It is represented by mathematical letters and symbols as follows:

w: average daily wage, that is, constant capital v, unit: yuan;

i: coefficient of labor intensity and labor complexity;

r: the proportion of surplus labor time in total labor time;

l: average daily working time, that is, the length of the working day, unit: hour;

p: labor productivity, unit: yuan/hour;

π: rate of surplus value;

m: surplus value;

(a) $0 \leqslant w < +\infty, 0 < r < 1, 0 < p < +\infty, 0 < \pi < +\infty$;

(b) Due to the basic living needs of workers, we assume that the minimum necessary living time is 8, then $0 \leqslant l \leqslant 24 - 8$;

(c) Set $i = e^h, -\infty < h < +\infty$, then $0 < i < +\infty$.When $h = 0, i = 1$. It means the labor intensity and labor complexity when the labor is simple labor. In this article, we call it the standard labor coefficient, and the corresponding labor is called standard labor.

(d) Since $\pi = \frac{n}{w}, r = \frac{m}{w+m}$, then $\frac{1}{\pi} + 1 = \frac{1}{r}$, when π is unchanged, r is obviously also unchanged.

We use the following function to express the relationship between factors that affect wage changes and wages: $w = w(i, l, r, p)$. We get:

$$w = w(i, l, r, p) = i(1 - r)lp = e^h(1 - r)lp \tag{2}$$

Mathematical Economic Analysis: Marx examines four main change combinations, and the following is an analysis of the mathematical characteristics of these four combinations through the wage function:

Combination 1. The length of the working day, labor intensity and labor complexity, the rate of surplus value remain unchanged, and the labor productivity changes, that is, i, l, r, are constants, and p is the independent variable.

The function $w = w(\bar{i}, \bar{l}, \bar{r}, p)$ can be abbreviated as: $w = w(p) = \bar{i}(1 - \bar{r})\bar{l}p$. The first derivative of w with respect to p is: $w^{'} = \frac{dw}{dp} = \bar{i}(1 - \bar{r})\bar{l} > 0$, which means the increase in wages brought about by a unit increase in labor productivity.

Combination 2. The length of the working day, labor productivity, and the rate of surplus value remain unchanged, and the complexity of labor and labor intensity change, that is, l, p, r, are constants, and i is the independent variable.

The function $w = w(i, \bar{l}, \bar{r}, \bar{p})$ can be abbreviated as: $w = w(p) = i(1 - \bar{r})\bar{l}\bar{p}$. The first derivative of w with respect to i is: $w^{'} = \frac{dw}{di} = (1 - \bar{r})\bar{l}\bar{p}$, which means the increase in wages brought about by a unit increase in the complexity and intensity of work.

Assuming that the function of labor intensity i of a particular economic sector with respect to the historical time point t is $i = i(t)$, the image of this function can be roughly depicted as shown in Figure 1.

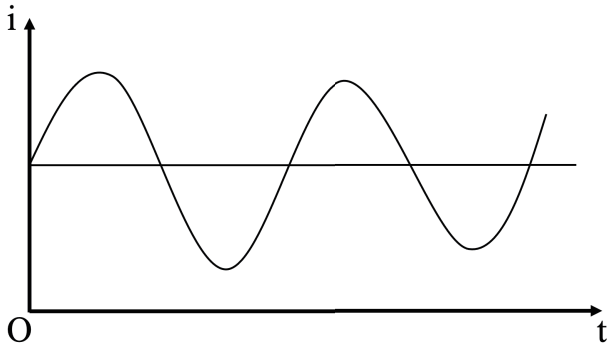

Figure 1: The Curve of Labor Intensity As a Function of Time

Combination 3. Labor intensity and labor complexity, labor productivity, and the rate of surplus value remain unchanged, and the length of the working day changes, that is, i, p, r, are constants, and l is the independent variable.

The function $w = w(\bar{i}, l, \bar{r}, \bar{p})$ can be abbreviated as: $w = w(p) = \bar{i}(1-\bar{r})\bar{p}l$.The first derivative of w with respect to l is:$w^{'} = \frac{dw}{di} = \bar{i}(1-\bar{r})\bar{p}$, which means the increase in wages brought about by each unit of labor time. The working day can be changed in two directions, that is, it can be extended and shortened.

Combination 4. Labor intensity and labor complexity, the length of working day, the rate of surplus value and the labor productivity all change, that is, i, p, r, and l are all independent variables.

The functional relation of w is: $w = w(i, l, r, p) = i(1-r)lp = e^h(1-r)lp$. Since $\frac{1}{\pi} + 1 = \frac{1}{r}$, we get:

$$1 - r = \frac{1}{1+\pi} \tag{3}$$

Substitute (2) in $w = i(1-r)lp$ gives the following expression:

$$w = i\frac{1}{1+\pi}lp \tag{4}$$

$$i = w^{-1}l^{-1}p^{-1}(1+\pi) \tag{5}$$

If w, p, and l remain constant, then i is inversely proportional to $(1-r)$, and labor intensity is positively correlated with the rate of surplus value. By $w = w(i, l, r, p) = i(1-r)lp$, it's easy to get:

$$pl = w(1-r)^{-1}i^{-1} \tag{6}$$

If w, r, and i remain constant, that is, $pl = w(1-r)^{-1}i^{-1} = constant\ C$, p and l are inversely proportional. That is, when labor productivity and working days change in opposite directions at the same ratio (for example, labor productivity decreases while working days are extended), their effects on wages can cancel each other out.

$MRS_{i\to l}$ is used to denote the marginal substitution rate of i to l, that is, in order to keep w constant (p and r are fixed), the amount of i that needs to be increased to replace one unit of l. It reflects the degree of substitution of labor intensity to the length of labor time.

$$MRS_{i\to l} = \frac{di}{dl}$$

From the formula(5),we can get:

$$\frac{di}{dl} = -w^{-1}p^{-1}(1+\pi)l^{-2} < 0 \tag{7}$$

$$MRS_{i\to l} = -w^{-1}p^{-1}(1+\pi)l^{-2} \tag{8}$$

$MRS_{i\to l} < 0$ indicating that labor intensity and labor time length are mutually replaceable, that is, the effect of a decrease in one quantity can be compensated by an increase in the other quantity.

$$\frac{d^2 i}{dl^2} = 2w^{-1}p^{-1}(1+\pi)l^{-3} > 0 \tag{9}$$

From the formulas (7) and (9), the approximate shape of the indifference curve of the wage w between the labor intensity i and the length of working time l (or the equal wage line of i and l) is shown in Figure 2.

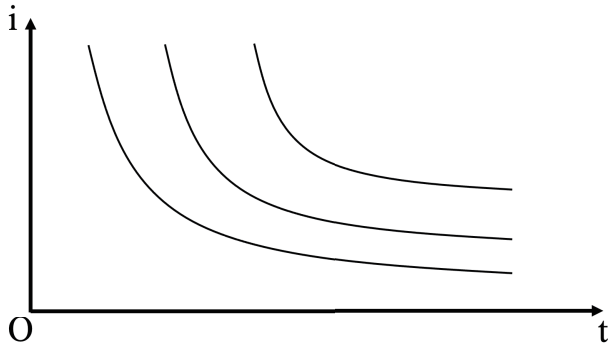

Figure 2: The Indifference Curve of Labor Intensity and Length of Labor Time

In summary, we can make a quantitative analysis of Marx's wage theory on the basis of qualitative analysis by using the method of mathematical function which show the changing law of wage more intuitively. For other principles and laws in political economy, mathematical models can also be used to deepen the understanding of the theory, such as the law of surplus value, the law of reproduction of total social capital, and so on.

2.2 Using Mathematical Methods in Teaching to Help Analyze the Theory and Practice of Socialist Political Economy.

Socialist political economy should thoroughly study the system of socialist production relations, reveal its complicated internal connections and movement mechanisms, and serve the practical problems in socialist economic construction. Therefore, the mathematical methods not only have the meaning of expression and explanation, it can also play a role in the study of socialist economic activities and economic movements. As a branch of mathematical statistics used in economics, econometrics has a unique and important position in modern economics. It combines mathematical methods with economic theory to establish and develop quantitative analysis methods and tools for economic relations. Samuelson even called the 20th century the century of econometrics [6]. These methods and tools can be used in teaching and research to analyze socialist production, exchange, distribution and consumption, improve students' ability of theoretical analysis of reality, and help to strengthen the scientific understanding of macroeconomic control policies. The following takes the practice of socialist political economy"poverty alleviation" as an example to explore how to use econometric methods to analyze economic problems.

Case Content: Eradicating poverty and achieving common prosperity are the essential requirements of socialism. China has achieved a comprehensive victory in the fight against poverty, and it has also submitted China's experience and plan for poverty eradication to the world. Understanding the main causes of poverty of low-income groups and putting forward effective measures to eliminate poverty from the aspects of system and policy is an important part to help students understand the advantages of socialist system in the teaching of socialist political economy.

Question: From the perspective of income mobility, the migration from low income to middle and high income is an important way to eliminate poverty. However, not all individuals or groups in society have the same income mobility. Some workers (or families) will be at the bottom of the income distribution for a long time, and their weak position is difficult to change, and they fall into a "low-income trap" [7].So was there a low-income trap in China? What were the influencing factors leading to the low-income trap? How did China establish a long-term mechanism of poverty eradication in terms of systems and policies?

Data Source and Sample Characteristics: The database to be used in this case is the "China Nutrition and Health Survey Data" (CHNS) provided by the Population Research Center of the University of North Carolina in the United States. According to the classification criteria of Scott and Pressman [8], all the samples are divided into three levels: low-income, middle-income and high-income. Taking the median income as the standard, those with a median income of 75%-125% are middle-income, those with a median income of 125% are high-income, and the rest are low-income. Among them, the low-income group is in a state of relative poverty, and if this low-income state persists for a long time, it can be regarded as a "low-income trap". We can observe the mobility of low-income groups through the income conversion matrix (see Table 1).From Table 1, we can see the existence of the low-income trap.

Table 1: Income Mobility of Workers at Different Income Levels (2000/2015)

		2015		
	income level	low-income	middle-income	high-income
2000	low-incone	64.74	19.29	15.97
	middle-income	50.59	25.37	24.05
	high-income	31.73	21.50	46.77

Descriptive Statistical Analysis: Who are more likely to get out of the low-income trap, and what factors have affected poor individuals to get out of poverty? The current literature research mainly focuses on human capital (see [9] and [10]), career choice [11], nutrition [12] and so on. Combining literature and experience, we think that factors such as education level, medical and health status, household registration, gender, age, regional distribution and other factors will affect the mobility of residents' income.

We consider those with a disease diagnosis and self-evaluation of health as "poor" as unhealthy, and the value is 0; for those with no disease diagnosis or the answer is "don't know", the value is 1, which is deemed healthy. When there is medical insurance (including public medical insurance, commercial insurance, basic medical insurance for urban employees, new rural cooperative medical care, etc.), the value is 1, and there is no medical insurance with a value of 0. Except for education level and age, the other variables are virtual variables. Gender is a natural feature of the population, with a value of 1 for males and 0 for females. Household registration is an important social feature. Urban household registration is 1 and rural household registration is 0. Because of the imbalance between regions in China's economic development, we need to consider the regional variables of the sample distribution. We identify Jiangsu, Shandong, Beijing, Shanghai and Zhejiang as the eastern region with a value of 1, while the remaining 10 provinces (municipalities and autonomous regions) are central and western regions with a value of 0.

Econometric Analysis: We use Logit model to analyze our dataset empirically:

$$logit_i = \beta_1 + \beta_2 H_i + \beta_3 X_i + \mu_i \tag{10}$$

The value of $logit_i$ is 0 or 1. For urban and rural workers whose income were in the low-income class in 2000, if they were still in the low-income class in 2015, it means that they are in a low-income trap, with a value of 0. Those who entered the middle-income or high-income class in 2015 can be considered to get out of the low-income trap, with a value of 1. β_1 is a constant term, β_2, β_3 are the parameter vectors to be estimated. H_i is the explanatory variable of human capital factors, including education years and medical and health status. X_i are other control variables such as age, gender, household registration, regional distribution, etc. and μ_i is a random error term. Since individuals of the same age have intra-group correlation, age is used as a cluster variable for regression analysis.

Result analysis: According to Table 2, we can get the conclusion:

(i) More human capital investment for low-income groups will help them to get out of the low-income trap. The three variables of education years, health status, and medical insurance all represent the human capital investment of the poor. Except for the insignificant influence of medical insurance, the education level and health status play a positive role in getting rid of the low-income trap.

(ii) Urban household registration has a positive effect on eliminating poverty and getting out of the low-income trap.

(iii) In terms of the natural characteristics of low-income samples, age is negatively correlated with the probability of getting rid of the low-income trap, and correlated with gender.

(iv) Regional distribution also affects poverty reduction. For the eastern region, it is easier to break the low-income trap.

Robustness test: In order to illustrate the rationality of the model setting, we use the data of the CHNS database in 2006 and 2015 to test the stability of the measurement model. In the balance panel, among all 3158 samples, we will

Table 2: Regression Result

variables	Regression coefficients	P value	Marginal effect
Education years	0.0477**	0.017	0.0104**
Health condition	0.4023*	0.055	0.0823**
Medical insurance	0.2016	0.668	0.0451
Age	-0.0372***	0.000	-0.0081***
Gender	0.3306**	0.011	0.0722**
Household registration	1.0382***	0.000	0.2469***
eastern coastal area	0.3521**	0.025	0.0793**
constant term	0.6392**	0.0357	-
R^2		0.1293	

Note: ***,**,* shows respectively the significance on the significance level of 0.01, 0.05 and 0.1.

focus on testing and analyzing the changes in income levels of 1,306 samples in the low-income level in 2006. The model is analyzed by robust cluster regression, and the results are shown in Table 3.

The test results show that, in terms of significance, education years, household registration, age and health status are still significant factors, while the impact of medical insurance is still not significant.

Through the empirical analysis by using econometric model, we find the main factors that affect people to get out of the low-income trap, among which medical insurance and household registration are the two most important factors. Education is also an important factor affecting income, and the poverty of poor residents comes from the poverty of education to a large extent. Therefore, in the process of teaching, it can help students to understand the orientation and effects of China's public policies in the process of poverty alleviation. By establishing a multi-level medical insurance system to increase the coverage of medical insurance, optimizing the fair allocation of educational resources, deepening the reform of the household registration system, promoting population urbanization and other measures, China has established a long-term mechanism for poverty alleviation in terms of systems and policies.

3 Thoughts on the Teaching Reform of the Application of Mathematical Methods in the Teaching of Political Economy

3.1 Some Concrete Ways for Teaching Reform

To strengthen the application of mathematical methods in the teaching of political economy, it is first required to do the basic work before teaching, that is, to systematically sort out and select the research results of mathematical models in the current political economy teaching, and eliminate complex and formal mathematical models. Secondly, it is necessary to complete the transformation of academic research into teaching thinking, turn academic language into teaching language,

Table 3: Robustness test

variables	Regression coefficients	P value	Marginal effect
Education years	0.0599**	0.008	0.0124**
Health condition	0.3043*	0.085	0.0599**
Medical insurance	0.2700	0.693	0.0527
Age	-0.0343***	0.000	-0.0071***
Gender	0.2403*	0.065	0.0503**
Household registration	0.9534***	0.000	0.2207***
eastern coastal area	0.0782	0.2531	0.0160
constant term	0.0602	0.942	-
R^2		0.1301	

Note: ***,**,* shows respectively the significance on the significance level of 0.01, 0.05 and 0.1.

and form teaching materials into teaching content after conversion. Finally, in specific teaching practice, the relevant theories of political economy can be organically integrated with mathematical methods and models. For example, the theoretical model of labor value, value transformation model, capital accumulation model and so on can be applied to the teaching of political economy to deepen students' understanding and mastery of basic theories.

3.2 Practical Examination and Adjustment of the Teaching Reform Plan

The application of mathematical methods to the teaching reform of political economy, and the effect of the reform, need to be tested in practice. We can collect feedback information and data by means of questionnaire survey and panel discussion, and we can also solicit the opinions and suggestions of students and relevant experts. The purpose of using mathematical methods is to simplify complex knowledge points, better reflect the logic and integrity of political economy theory, improve teaching efficiency, and make it easier for students to learn and teachers to teach. In the process of teaching practice, if the use of mathematical methods makes students' understanding of relevant knowledge points more complex and fuzzy, then it is necessary to evaluate and adjust the teaching reform plan and related mathematical methods and models.

4 Summary

The use of mathematical methods in the teaching of political economy can not only improve teaching efficiency, but also express the theoretical characteristics and conclusions of the subject more clearly, and speed up the accumulation and dissemination of economics knowledge. However, the application of mathematical methods in teaching should also follow the principles of moderation and practicability. The principle of moderation means that in the process of teaching, it is not that more is better, but moderation. Because the application of mathematical methods in economics is also restricted by factors such as economic system changes, power transformation, and social relations which can't be fully expressed through mathematical methods. The principle of practicability means that the use of mathematical methods in teaching is to make teaching simpler and clearer, not the more complex the better. The use of mathematical methods must be based on the methodology of political economy.

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References

[1] Marx, K. Capital(3 vols).*Harmondsworth:Penguin*.1976.
[2] Sweezy,P.M. The Theory of Capitalist Development: Principles of Marxian Political Economy. *New York: Oxford University Press*.1942.
[3] Okishio,N. Technical changes and the rate of profit. *kobe university economic review*.1961.
[4] Morishima,M. Marx's economics. *Cambridge University Press*.1973.
[5] Romer,P.M. Increasing Returns and Long-Run Growth. *Journal of Political Economy*.(5)1986:1002-1037.
[6] Samuelson, P. A. Foundations of Economic Analysis. *Harvard University Press*.1948.
[7] Azariadis,C. and J. Stachurski. Poverty traps. *Handbook of Economic Growth, North-Holland*.(1A)2005:295-384.
[8] Scott,R.H. and S.Pressman. A Double Squeeze on the Middle Class. *Journal of Economics Issues*.(2)2011:333-342.
[9] Galor,O.and J.Zeira. Income distribution and macroeconomics. *Mpra Paper*. (1)1989:35-52.
[10] Ceroni, C. B. Poverty traps and human capital accumulation. *Economica*.(270)2010: 203-219.
[11] Banerjee,A.V. and A,F.Newman. Occupational choice and the process of development. *Journal of Political Economy*, (2)1993: 274-298.
[12] Dasgupta,P. and D. Ray. Inequality as a determinant of malnutrition and unemployment: theory. *Economic Journal*, (384)1986:1011-1034.