ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.22(2016),No.2,pp.113-117

Innovation Risk Propagation Model in Industrial Cluster with Time Lag

Jingjing Yao[1] *, Liang Jiang[2] , Yusheng Kong[1]
[1] School of Finance & Economic, Jiangsu University, Zhenjiang 212013, China1
[2] Jingjiang College of Jiangsu University , Zhenjiang 212013, China
(Received 18 May 2016, accepted 10 August 2016)

Abstract: In this p aper, we propose a model based on the characteristics of industrial cluster and the idea of infectious disease. We calculate the basic reproduction number of risk propagation and analyze the global stability of the equilibrium point. According to our study, we find that there is innovation risk in cluster network when $R_0 > 1$. When $R_0 < 1$, the innovation risk disappears in the cluster network finally. We also study the stability of the system with time delay of the immune. The results indicate that some factors can influence the innovation risk propagation in the cluster.

Keywords: Industry cluster; Infectious disease model; R_0; Global stability;

1 Introduction

Many complex systems in the real world can be described by complex networks. As a tool, the complex network is widely used to model a variety of complex issues [1, 2]. Some scholars found that the spread of risk in industrial clusters is similar to the spread of diseases, so they tried to apply system dynamics modeling to analyze the innovation risk of communication industry cluster system [3, 4]. Yuan [5] constructed the innovation single process model based on the risk contagion without considering the rate of birth and death. Xiao [6] found out the SIR model can analyze the dynamics of industrial clusters innovation system with the theory of industry cluster. Su [7] analyzed the problem of spreading of risk in the innovation system, the results suggest that it was similar to the spreading of rumor and virus. It provides the theoretical basis for the enterprises and government departments to formulate policies. However, the research about epidemic model in industry system of risk communication application is still in the early stage, and most of the proposed models fail to consider the network structure of the system.

At present, the SIS epidemic model and the SIR epidemic model are the most widely investigated and employed models in infectious diseases. Zhu et al.[8] constructed a general model of non-scale network, and this model used some other relevant epidemic models, such as SI, SIR, SEIR models. Xu[9] studied SEIRI epidemiological model with disease relapse, and took the nonlinear incidence rate and time delay into consideration. Techenche et al.[10] used SIR model to study propagation delay, and reviewed the recent discussion about the diseases model and the reduction of transmission threshold. Chen [11] established the rumor spreading model on the scale-free network, and compared the results obtained by the computer simulation with stochastic analysis methods.

In this paper, by applying the above classification methods, such as infectious disease models and the knowledge of complex network, we propose a new model which takes the time delay of industry cluster innovation risk into consideration. The results indicate that the influencing factors of the innovation risk propagate in the cluster and they also provide clues to the selection of risk control strategy.

The remainder of the paper is organized as follows. The proposed model is presented in Section 2. The simulation results are provided in Section 3. In the last section, we conclude the paper.

*Corresponding author. *E-mail address*: jjyao1989@sina.com

IJNS.2016.10.15/931

2 The industrial cluster innovation risk propagation model

2.1 Assumptions

Based the theory of industrial clusters, the model of SIR and the topological structure of the scale-free network, we build a new industrial cluster innovation risk propagation model. Firstly, we give the following assumptions:

(1)This paper considers the networks of innovation risk propagation as scale-free networks just like most of the real complex networks. Each of enterprises in the industrial cluster is regarded as a node. And the capital, technology and property are regarded as a connection between nodes. We consider the collapse of enterprise or annexation of new businesses in the scale-free networks and we assume that the increasing rate is equal to the rate of collapse and annexation.

(2)The system consists of three types of nodes, the ignorant, the spreaders and the stiflers. We denote the mean degree of the network by $< k >$ and describe the number of spreaders, ignorant and stiflers in the system by $S_k(t)$, $I_k(t)$, $R_k(t)$, $0 \leq S_k(t), I_k(t), R_k(t) \leq 1$. N is the number of nodes in the system and$N_k(t) = S_k(t) + I_k(t) + R_k(t)$.

(3)The spreading rules on the complex networks can be summarized as follows: first of all, if an individual gets the risk, then it will become a spreader and at each time step transmits the risk to its neighbors with probabilityα,Secondly, a spreader will turn into a stifler with probability γ if it makes a contact with another spreader or stifler. This is the mechanism when a spreader voluntarily ceases to spread the risk. At last, because of the reduction in risk prevention consciousness or strength recession of the enterprise, these risk-resistant members have a certain period of time to lose their resistance with the probability τ.We let $\delta = \gamma$ for convenience. According to the practical significance of the members within the industry cluster innovation risk contagion, α, γ, μ are constant in $[0, 1]$. And $\tau \in N$. The risk transmission flow chart is shown in Fig. 1.

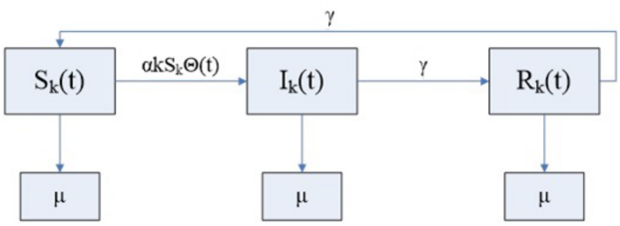

Figure 1: SIRS model for the spread of the industrial cluster innovation risk

2.2 The Model

According to the SIR model and risk spreading process elaborated above, the mean-field equations can be described as follows:

$$\begin{cases} \frac{dS_k(t)}{dt} = \mu - \alpha k S_k(t)\Theta(t) - \mu S_k(t) + \gamma I_k(t-\tau)e^{-\mu\tau} \\ \frac{dI_k(t)}{dt} = \alpha k S_k(t)\Theta(t) - \gamma I_k(t) - \mu I_k(t) \\ \frac{dR_k(t)}{dt} = \gamma I_k(t) - \mu R_k(t) - \gamma I_k(t-\tau)e^{-\mu\tau} \end{cases} \tag{1}$$

As $k = 1, 2, ..., n$, $\Theta(t)$ is susceptible probability between members $S_k(t)$ and members $I_k(t)$. Use $P(k)$ to represent the probability of node degree k, the average degree of the network $< k >$, and that is $< k >= \sum_{i=1}^{n} iP(i)$, so we obtain the following equation:

$$\Theta(t) = \sum_{i=1}^{n} P(i|k)I_i(t) = \frac{\sum_{i=1}^{n} iP(i)I_i(t)}{< k >} \tag{2}$$

The initial conditions for system(1) are of the form

$$S_k(\theta) = \phi_{k1}(\theta), I_k(\theta) = \phi_{k2}(\theta), R_k(\theta) = \phi_{k3}(\theta)$$

$$\phi_{ki}(\theta) \geq 0, \theta \in [-\tau, 0], phi_{ki}(0) > 0 (i = 1, 2, 3),$$

where $\phi_k = (\phi_{k1}, \phi_{k2}, \phi_{k3}) \in C$, such that $\phi_{ki} \geq 0 (i = 1, 2, 3)$ for all $\theta \in [-\tau, 0]$, C denotes the Banach space $C([-\tau, 0], R^{3n})$ and we designate the norm of an element ϕ_k in C by $\|\phi_k\| = sup_{-\tau \leq \theta \leq 0} |\phi_1(\theta)|, |\phi_2(\theta)|, |\phi_3(\theta)|$.

The system(1) always has a disease-free equilibrium $E_0 = (1, 0, 0)$. Further, we could have steady state as follows:

$$\begin{aligned} S_k &= \frac{\mu(\gamma + \mu)}{\alpha k \Theta(\gamma + \mu) + \mu(\gamma + \mu) - \gamma \alpha k \Theta e^{-\mu\tau}}, \\ I_k &= \frac{\mu \alpha k \Theta}{\alpha k \Theta(\gamma + \mu) + \mu(\gamma + \mu) - \gamma \alpha k \Theta e^{-\mu\tau}}, \\ R_k &= \frac{\alpha k \Theta(\gamma - \gamma e^{-\mu\tau})}{\alpha k \Theta(\gamma + \mu) + \mu(\gamma + \mu) - \gamma \alpha k \Theta e^{-\mu\tau}}, \end{aligned} \tag{3}$$

Put the second equation (3) into (2), we obtain

$$\Theta = \sum_{k'} \frac{k' P(k')}{< k >} \frac{\mu \alpha k \Theta}{\alpha k \Theta(\gamma + \mu) + \mu(\gamma + \mu) - \gamma \alpha k \Theta e^{-\mu\tau}}$$

We define a self-consistency equality that

$$\Theta = \frac{\mu \alpha}{< k >} \sum_{k'} \frac{k' P(k') k \Theta}{\alpha k \Theta(\gamma + \mu) + \mu(\gamma + \mu) - \gamma \alpha k \Theta e^{-\mu\tau}} = f(\Theta) \tag{4}$$

It is easy to see that $\Theta = 0$ is a solution of (4), and $f(1) = \sum_{k'} \frac{k' P(k')}{<k>} \frac{\mu \lambda k}{\alpha k(\gamma+\mu)+\mu(\gamma+\mu)-\gamma \alpha k e^{-\mu\tau}} < 1$. Following, we find the conditions under a nontrivial solution to Eq.(4) in the interval $0 < \Theta < 1$. Then, it corresponds to the inequality

$$\frac{df(\Theta)}{d\Theta}|_{\Theta=0} = \frac{d}{d\Theta}\left(\frac{\mu\alpha}{< k >} \sum_{k'} \frac{k' P(k') k' \Theta}{\alpha k' \Theta(\gamma + \mu) + \mu(\gamma + \mu) - \gamma \alpha k' \Theta e^{-\mu\tau}}\right)_{\Theta=0} > 1$$

Hence, the basic reproduction number R_0 is given by

$$R_0 = \frac{\alpha}{\gamma + \mu} \frac{< k^2 >}{< k >},$$

where

$$< k^2 > = \sum_k k^2 p(k).$$

Due to $\Theta = 0$, which means τ no effect on Θ. So R_0 is not dependent on the delay τ.

If $R_0 < 1$, the risk will gradually die out. Otherwise, the disease will spread on networks.

3 Numerical simulation

From the global stability analysis of the system (1), we know the basic reproductive number can determine whether the risk of industry cluster in the network transmission persists. We know the stability of the disease-free equilibrium and the endemic equilibrium. Here we present numerical simulations to support the results obtained in the previous sections.

In Fig.2, α is monotone increasing for R_0, in order to control the spread of the risk and make it fades out eventually, when susceptible S associate with infected I on capital, technology etc., managers should make some effective prevention strategies, avoiding to the risk of infection, that is reduced α, so $R_0 < 1$.

In Fig.3, when the more enterprise enter the immune period, the infection risk is decreased. If the enterprise does not take measures after it infected risk, it will increase the innovation risk communication risk. Therefore, the enterprise requires management once found the infection risk of the enterprise timely to govern and eliminate the risk.

In Fig.4, it demonstrates that they varies with time t. In a good business environment, the number of enterprises with risk infection will increase at the beginning, but it will gradually decrease over time.

In addition, the degree of heterogeneity on enterprise network $\frac{<k^2>}{<k>}$, known as the expression of R_0 know, reduce $\frac{<k^2>}{<k>}$ is also conductive to ultimately eliminate the risk of transmission of the crisis. So, enterprise managers should also pay attention to the enterprise network environment and set the corresponding risk prevention measures.

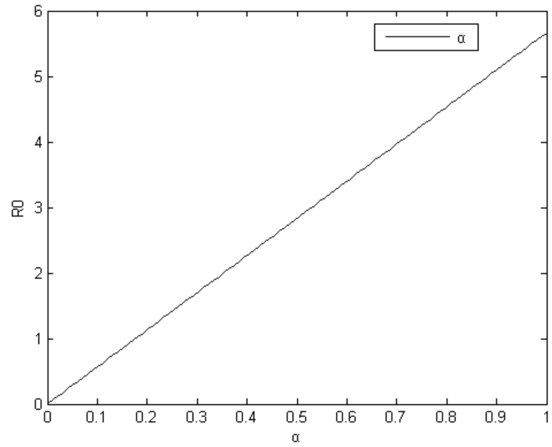

Figure 2: The relationship between the basic reproduction number R_0 parameters α When $\gamma = 0.2, \mu = 0.4, \frac{<k^2>}{<k>} = 3.4$

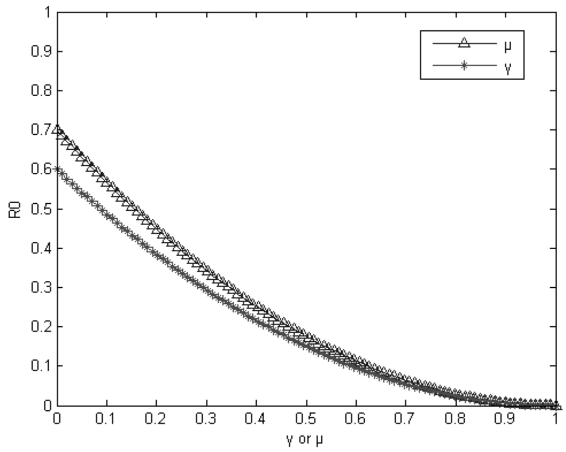

Figure 3: The influence of parameters γ or μ on R_0 when $\alpha = 0.6, \frac{<k^2>}{<k>} = 3.4$

4 Conclusions

In this paper, due to the similarity between the spread of innovation risk in the industrial cluster and the spread of infectious disease, we proposed a model to analyze the SIRS epidemic spread on complex networks with time delay. Then the global dynamics of system (1) was completely established. By means of a suitable Lyapunov function and LaSalles invariance principle, it shows that if $R_0 > 1$, the endemic equilibrium of system (1) is globally asymptotically stable and therefore the disease becomes endemic; if $R_0 < 1$, the disease-free equilibrium of system (1) is globally asymptotically stable and the disease fades out. Theoretical analysis and numerical simulation showed that the parameters μ, α, γ and τ have effects on R_0. The model provides a theoretical basis for the enterprise to make decisions. However, more studies need to be performed on open systems with complex networks for preventing and controlling the spread of innovation risk.

Acknowledgments

Supported by the National Natural Science Foundation of China(Grant No.71371087,71271103); Postgraduate research and innovation plan project in Jiangsu Province(Grant No.KYZZ_0294).

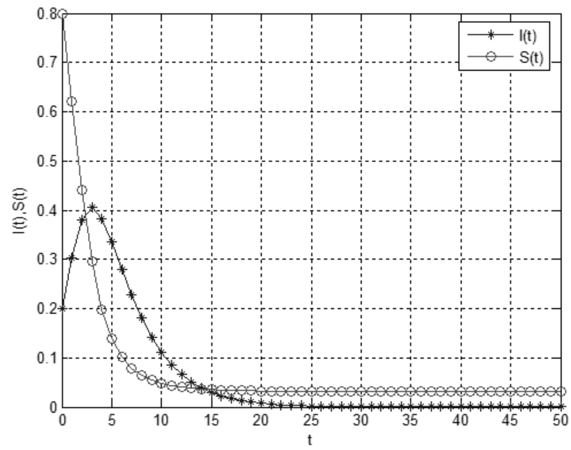

Figure 4: The density of S_k and I_k varies with t

References

[1] J. Li and Z. Ma. Stability switches in a class of characteristic equations with delay-dependent parameters[J]. *Nonlinear Anal.* (2004)(5): 389-408.

[2] R. G. Mcfarland et al. Supply Chain Contagion. *Journal of Marketing*. 72(2008)(2):63-79.

[3] L. Wang and Z. Wang. Research on Innovation Mechanism and Risk of Industrial Cluster. *Energy Procedia.* (2011)(13): 10061-10067.

[4] Q. Gan et al. Travelling waves in an infectious disease model with a fixed latent period and a spatioCtemporal delay[J]. *Mathematical and Computer Modelling*. 53(2011): 814-823.

[5] X. Yuan et al. Analysis of an epidemic model with awareness programs by media on complex networks[J]. *Chaos Solitons & Fractals*. 48(2013)(1): 1-11.

[6] Y. Xiao et al. Dynamical behavior for a stage-structured SIR infectious disease model [J]. *Nonlinear Analysis Real World Applications*. 3(2002)(2): 175-190.

[7] Y. Su et al. A Study on Diffusion of Technology Innovation Systems Based on Epidemic Model [J]. *Journal of Systems Science*. (2014)(2): 93-96.

[8] G. H. Zhu et al. Spreading dynamics and global stability of a generalized epidemic model on complex heterogeneous networks[J]. *Applied Mathematical Modeling*. 2012(36): 5808-5817.

[9] X. J. Xu and G. R. Chen. The SIS model with time delay on complex networks. *International Journal of Bifurcation and Chaos*. 2(2009)(12): 623-628.

[10] J. M. Tchuenche et al. Global behavior of an SIR epidemic model with time delay. *Mathematical Methods In The Applied Sciences*. (2007)(30): 733-749.

[11] G. R. Chen and X. J. Xu. The SIS model with time delay on complex networks[J]. *International Journal of Bifurcation and Chaos*. 2(2009)(12): 623-628.