ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.22(2016) No.2,pp.109-112

MHD Free Convection Flow through a Porous Medium Bounded by an Infinite Vertical Plate with Constant Heat Flux

Anuj Jhankal *
Department of Mathematics, Army Cadet College, Indian Military Academy, Dehradun UTTARAKHAND - 248 007 (India)

(Received 1 February 2013, accepted 16 May 2016)

Abstract: This paper considers the problem of free convective flow of a viscous incopressible fluid through a porous medium bounded by an infinite vertical porous plate in the presence of transverse magnetic field. The plate is subjected to a normal suction velocity and the heat flux at the plate is constant. The solution of coupled non linear differential equations are solved by using maple software. The effect of various parameters are presented and discussed.

Keywords: MHD; free convection flow; constant heat flux; numerical study

1 Introduction

The study of flows through porous media has been motivated by its immense importance and continuing interest in many engineering and technological field, for example, soil mechanics, petroleum engineering, transpiration cooling, food p-reservation, cosmetic industry blood flow and artificial dialysis etc. Free convective phenomenon has been the object of extensive research, because it is often encountered in cooling of nuclear reactor or in the study of the structure of stars and planets etc, from the technological point of view, MHD free convection flows have also great significance for the application in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics,(Cramer and Pai (1975)).
The theory, of laminar flow through, homogeneous porous media is based on an experiment originally conducted by Darcy (1856). The free convection flow past a vertical plate studied by Kolar et al (1988) and Ramanajah et al. (1992) with different boundary conditions. Problem of natural convective cooling of a vertical plate solved numerically by Camargo et at.(1996). Raptis (1986) studied mathematically the case of time varying two dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Helmy (1998) analyzed MHD unsteady free convection flow past a vertical porous plate embedded in a porous medium. Elabashbeshy (1997) studied heat and mass transfer along a vertical plate in the presence of magnetic field. Chemkha and Khaled (2001) investigated the problem of coupled heat and mass transfer by magnetohydrodynamic free convection from an inclined plate in the presence of internal heat generation or absorption. Dursunkaya and Worek (1992) studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafonssian and Willams (1995) presented the same effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Alam and Rahmam (2005) studied the Dufour and Soret effect on study MHD free convective heat and mass transfer flow past a semi infinite vertical porous plate embedded in a porous medium. Anavda et al. (2009) studied thermal diffusion and chemical effects with simultaneous thermal and mass diffusion in MHD mixed convection flow. Chen (2004) has studied MHD free convection from an inclined surface with suction effects. Anwar Beg and Ghosh (2010) studied the steady and unsteady MHD free and forced convective flow of electrically conducting, new tonium fluid in the presence of appreciable thermal radiation heat transfer and surface temperature oscillation. It is proposed here to study the effects of a magnetic field on the free convective flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux.

* **Corresponding author**. *E-mail address*: anujjhankal@yahoo.com

IJNS.2016.10.15/930

2 Mathematical formulation

Consider the axis of $\overline{x}$ be taken along the vertical plate in the upward direction and $\overline{y}$ axis perpendicular to it, the applied magnetic filed is of uniform strength B_0 and is applied transversely to temperature differences except that the density in the body force term. The momentum with temperature is negligible. The porous material containing the fluid is, in fact, a non-homogeneous medium, but it is possible to replace it with a homogeneous fluid that has dynamical properties equal to the local averages of the original non-homogeneous continuum. Thus, the complicated problem of the motion of a viscous fluid in a porous solid reduces to the motion of the homogeneous fluid with some reduces to the motion of homogeneous fluid with some additional resistance. We consider the case of short circuit problem in which the applied electric field E=0, and also assure that the induced magnetic field is small compared to external magnetic field B_0 , this implies a small magnetic Reynolds number. Under these assumption the equation, which govern the motion are (Raptis (1983)):

$$\frac{\partial \overline{v}}{\partial \overline{y}} = 0, \tag{1}$$

$$\overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} = g\beta(\overline{T} - \overline{T}_\infty) + \nu\frac{\partial^2 \overline{u}}{\partial, \overline{y}^2} - \frac{\nu}{\overline{K}} - \sigma\frac{B_0^2}{\rho}\overline{u}. \tag{2}$$

$$\overline{v}\frac{\partial \overline{T}}{\partial \overline{y}} = \frac{k}{\rho c_p}\frac{\partial^2 \overline{T}}{\partial \overline{y}^2} + \frac{\nu}{c_p}\left(\frac{\partial \overline{u}}{\partial \overline{y}}\right)^2 \tag{3}$$

where all the symbols have their usual meanings. The last two terms on the right hand side of equation (2) signify the additional resistance due to the porous medium, with permeability $\overline{K}$, and the electromagnetic body force term which acts on the fluid elements, respectively. Also, the Joule heating terms in the energy equation (3) is assumed to be negligible. The boundary conditions are:

$$\overline{y} = 0 : \overline{u} = 0, \frac{\partial \overline{T}}{\partial \overline{y}} = -\frac{\overline{q}}{k}$$

.

$$\overline{y} \to \infty : \overline{u} \to 0, \overline{T} \to \overline{T}_\infty. \tag{4}$$

The continuity equation (1) gives

$$\overline{v} \to -v_0 \tag{5}$$

where $v_0 > 0$ is the constant suction velocity at the plate and the negative sign indicates that the suction velocity is directed towards the plate. Introduce the following non-dimensional quantities:

$$u = \frac{\overline{u}}{v_0}\text{(Velocity)}, y = \frac{v_0\overline{y}}{\nu}, \theta = \frac{T - T_\infty}{\overline{q}\nu/kv_0}\text{(Temprature)}$$

$$Pr = \frac{\rho\nu c_p}{k}\text{(Prandtl Number)}, Ec = \frac{kv_0^3}{\overline{q}\nu c_p}\text{(Eckert Number)}$$

$$Gr = \frac{g\beta\nu^2\overline{q}}{kv_0^4}\text{(Grashof number)}, K = \frac{v_0^2\overline{k}}{\nu^2}\text{(Permeability parameter)}$$

$$M = \frac{\sigma B_0^2\nu}{\rho v_0^2}\text{(Magnetic field parameter)}. \tag{6}$$

On putting these values into equations (2) and (3), we get

$$u^{''}(y) + u^{'}(y) = -\text{Gr}\theta(y) + \frac{u(y)}{K} + \text{M}u(y). \tag{7}$$

$$\theta^{''}(y) + \text{Pr}\theta^{'}(y) = -\text{Pr Ec}u^{'2}(y). \tag{8}$$

Along, with the boundary conditions:

$$y = 0 : u = 0, \theta^{'} = -1$$

$$y \to \infty : u \to 0, \theta \to 0. \tag{9}$$

3 Numerical solutions

The non- liner coupled differential equations (7) and (8) subject to boundary condition (9) are solve numerically using Runge-Kutta-Fehlberg forth-fifth order method. To solve these equations we adopted symbolic algebra software Maple. Maple uses the well known Runge-Kutta-Fehlberg Fourth-fifth order (RFK45) method to generate the numerical solution of a boundary value problem. The boundary condition $y \to \infty$ were replaced by those at y=7 in accordance with standard practice in the boundary layer analysis. The effect of various parameters on velocity distribution and rate of heat transfer in terms of Nusselt number are shown in figure 1 and 2 respectively.

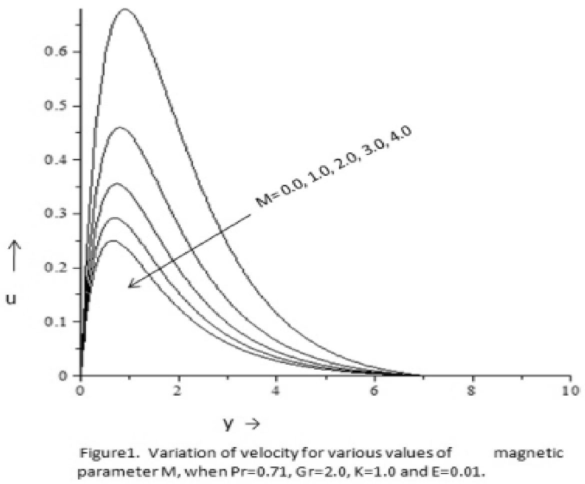

Figure 1: Variation of velocity for various values of magnetic parameter M, when $Pr = 0.71, Gr = 2.0, K = 1.0, andE = 0.01$.

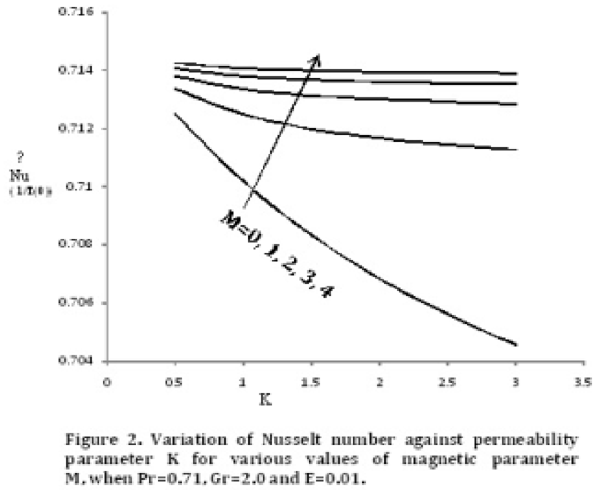

Figure 2: Variation of Nusselt number against permeability parameter K for various values of magnetic parameter M, when $Pr = 0.71, Gr = 2.0, andE = 0.01$.

4 Conclusions

A mathematical model has been presented for the effects of a magnetic field on the free convective flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux. From the study, following conclusions can be drawn:

1. Figure 1 shows the Maple generated numerical solutions for a Prandtl number 0.71, Grashof number 2.0, Eckert number 0.01, permeability parameter 1.0 and for a range of values of the magnetic parameter (M). We notice that the velocity decreases as the magnetic parameter (M) increases. Thus we conclude that we can control the velocity field by introducing magnetic field.

2. Figure 2 shows the rate of heat transfer in terms of Nusselt number with the permeability parameter K, for a Prandtl number 0.71, Grashof number 2.0, Eckert number 0.01 and for a range of values of the magnetic parameter (M). We notice that the rate of heat transfer is increases as the magnetic parameter (M) increases.

References

[1] M. S. Alam and M. M. Rahman. Dufour and Soret effects on MHD free convective heat and mass transfer flow past a vertical flat plate embedded in porous medium. *Journal Naval Architecture and Marine Engineering*. 2(2005)(1): 55-65.

[2] R. N. Ananda et al.. Thermal diffusion and chemical effects with simultaneous thermal and mass diffusion in MHD mixed convection flow with ohmic heating. *Journal of Naval Architecture and Marine Engineering.* 6(2009): 84-93.

[3] B. O. Anwar and S. K. Ghosh. Analytical study of magnetohydrodynamic radiation convection with surface temperature oscillation and secondary flow effects. *International Journal of Applied Mathematics and Mechanics.* 6(2010)(6): 1-22.

[4] R. Camargo et al.. Numerical study of the natural convective cooling of a vertical plate. *Heat and Mass Transfer*. 32(1996): 89-95.

[5] A. J. Chamkha and A. R. A. Khaled. Similarity solutions for hydromagnetic simultaneous heat and mass transfer by natural convection from an inclined plate with internal heat generation or absorption. *Heat Mass Transfer*. 37(2001): 117-123.

[6] C. H. Chen. Heat and Mass transfer in MHD flow by natural convection from a permeable, inclined surface with variable wall temperature and convection. *Acta Mechanica*. 22(2004): 219-235.

[7] K. R. Cramer and S. L. Pai. Magnetofluiddynamics for engineers and applied Physicist. *McGraw-Hill, New York.* (1975).

[8] H. P. G. Darcy. Les fountains publiques dela ville de Dijam Dalmont. *Pariser.*(1856).

[9] Z. Dursunkaya and W. M. Worek. Diffusion-thermo and thermal diffusion effects in transient and steady natural convection from a vertical surface. *Int. J. Heat MassTransfer.* 35(1992): 2060-2065.

[10] E. M. A. Elabashbeshy. Heat and mass transfer along a vertical plate with variable temperature and concentration in the presence of magnetic field. *Int. J. Eng. Sci.* 34(1997): 515-522.

[11] K. A. Helmy. MHD unsteady free convection flow past a vertical porous plate. *ZAMM.* 78(1998): 255-270.

[12] N. G. Kafoussias and E. M. Williams. Thermal-diffusion and Diffusionthermo effects on free convective and mass transfer boundary layer flow with temperature dependent viscosity. *Int. J. Eng. Science*. 33(1995): 1369-1376.

[13] A. K. Kolar and V. M. Sastri. Free convective transpiration over a vertical plate: a numerical study. *Heat and Mass Transfer.* 23(1988)(6): 327-336.

[14] G. Ramanaiah and G. Malarvizhi. Unified treatment of free convection adjacent to a vertical plate with three thermal boundary conditions. *Heat and Mass Transfer.* 27(1992)(6): 393-396.

[15] A. Raptis. Effects of a Magnetic Field on the Free Convective Flow Through a Porous Medium Bounded by an Infinite Vertical Porous Plate with Constant Heat Flux. *J. of Franklin Institute.* 316(1983)(6): 445-449.

[16] A. Raptis. Flow through a porous medium in the presence of magnetic field. *Int. J. Energy Res.* 10(1986): 97-101.