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Analysis of the Optimal Growth Path Based on Environment and Energy Resources

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Abstract: We consider a economic model of endogenous growth with creative destruction in which we introduce environmental quality, pollution intensity and energy resources, then discusses the effects of the three factors above on the long-term equilibrium growth of economy with the method of the static analysis. The conclusion demonstrates that the rate of economic growth is subject to a variety of factors. First, the stronger the environmental self-purification capacity and the rate of resources regeneration are, the greater the rate of economic growth is. Second, pollution intensity is determined by the degree of pollution index and the difference between the discount rate and the environmental self-purification capacity. Pollution intensity changes with the change of the difference between discount rate and environmental self-purification capacity.

Keywords: environmental quality; environmental policies; energy; economic equilibrium growth;

1 Introduction

In recent years, a discussion concerning economic growth, energy use, and environmental protection causes a hot argument. The economic growth at the expense of environment for a long time devastated the environment. Greenhouse effect and energy shortage not only affect economic growth but also directly threaten the survival and development of humankind. How to achieve the vision of sustainable energy use and the sustainable economic growth on the protection of green environment, in response to this question scholars at home and abroad conduct a lot of researches. Since the 1980s, the endogenous growth theory emerged in order to overcome the defects of neoclassical growth theory whose representative model is the endogenous growth theory built by Romer [1,2], Scholz , Ziemes[3] and so on. Then Brock and Taylor[4] among others, brought energy and environment into the economic growth model, and they reckoned that the continuous development of economy and the high levels of emission reduction were mutually incompatible when energy stock reached a certain lower limit. By constructing the endogenous economic growth model, Tahvonen and Salo[5] analyzed the mutual conversion relationship between non-renewable energy and renewable energy in different stages of economic development,the study shows that the production evolution of energy can be divided into two forms in the context of a certain history: One is the evolution from using renewable energy to using non-renewable energy. The other is the evolution from using non-renewable energy to using renewable energy. In general, the two types of energy production evolution are applied simultaneously. Moreover, it indicates that on the equilibrium path, the consumption of non-renewable energy may increase, which leads to a fall in energy price. In the absence of environmental policy constraints, carbon emissions and income levels follow the inverted u-shaped relationship. Bovenberg and Smulders[6] built a two-sector endogenous growth model by introducing pollution and environment simultaneously into the production function and introducing environmental quality into utility function. The result shows that, on the optimal growth path, the optimal scale of government budget tends to increase when the total amount of the pollution tax revenue exceeds the public expenditure on the research and development of technology so as to reduce pollution. Antle and Heidebrink[7] demonstrate that there is no positive correlation between income and environmental quality and thus there is no inevitable correlation between environmental deterioration and economic growth. Lucas Bretschger[8] thoroughly studies how the non-renewable energy and the technological innovation influence economy in the economic growth model, and finds that economy will be affected negatively while the replacement level is insufficient.

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In China, most of scientists using empirical analysis method for theoretical model, but less research on incorporating environmental quality, pollution intensity, and energy into economic growth model that is mainly represented by Wang Chenwei, Peng Shuijun and so on. Chenwei Wang and Gang Shi[9] deem that customers urgency for current consumption contributes to an increase of the growth rate of both technological progress and economy, but the enjoyment timely is achieved at the expense of the exploitation rate of energy, the environmental destruction, and the increase of pollution rate. Shuijun Peng [10] proposes that government can achieve the sustainable development in five ways: ensuring the accumulation of human capital; promoting the technological innovation; boosting the progress of clean technology; setting the strict standard; raising the nations awareness of environmental protection. Fengliang Liu and others[11] introduces non-renewable energy into the endogenous growth model and he points out that the continuous economic growth is likely to be achieved when non-renewable energy and capital carry out an incomplete replacement. Based on studies at home and abroad, this paper incorporates energy, environment, and pollution intensity into the economic growth model simultaneously, and analyzes the effects of environmental quality, pollution intensity, and energy on economic growth.

2 The Model

2.1 The environmental quality

Since the economic activity is carried out in the whole ecosystem, it is bound to be influenced by the environmental quality. Grimaud and Roug[12-14]among others, consider the pollution problem caused by economic development while introducing energy constraints and environmental constraints. This paper follows the basic idea above. Generally,the depiction of environment can be divided into two different ways. One deems that the long-run continuous growth of economy relies on nature and regards nature as a source of economic growth represented by Brock and Taylor[4]. Assuming that environmental quality is a negative value, which is used to measure the quantity of pollutants storage in the environment. The other one depicts the environmental quality at time t with a positive value E_t represented by Chenwei Wang and Gang Shi[9],it denotes that the bigger the environmental quality value is, the better the environmental quality is. This paper employs the depiction way of the latter environmental quality. Then

$$E_t = E_{t-1} + \eta E_t - P_t, \tag{1}$$

where $E \geq E_{min}$, E_{t-1} is the last phrase of environmental quality, $\eta(\eta > 0)$is the environmental self-purification capacity, and P_t is the pollution emission at time t. E_{min} is the minimum of environment capacity. Environment is subject to the irreversible damage and the normal production and living activity will not be able to continue while E_t is below the minimum. Therefore, the prerequisite for economic equilibrium growth is that E_t must be no lower than E_{min} .
The motion equation of environmental quality is as follows:

$$\dot{E} = dE/dt = \eta E_t - P_t. \tag{2}$$

The Copenhagen conference required to control greenhouse gas emissions, and pollution intensity has an effect on the amount of pollution emissions and energy use directly. Here, it is assumed that $z_t(0 < z_t < 1)$ denotes pollution intensity at time t and $\tau(\tau > 0)$ denotes pollution index. Hence, pollution P_t relates to pollution intensity and economic output Y_t. The formula is:

$$P_t = {z_t}^{-t} Y_t. \tag{3}$$

τ bigger means a bigger ${z_t}^{-t}$ and heavier pollution; τ smaller means a smaller ${z_t}^{-t}$ and lighter pollution.

2.2 The energy

Energy saving and emission reduction is an important way to reduce environment pollution. In the context of the dwindling original fossil resources, an increase of energy regeneration plays a significant role in the sustainable economic growth. This practice resembles that the study of D'Arge[15] to bring renewable energy and non-renewable energy into the model at the same time. In this thesis, the energy constraint denotes non-renewable energy(which mainly consists of oil, coal and gas) and renewable energy(like wind energy, solar energy, geothermic energy and so on). Since the reform and opening up, economic growth rate has showed a trend of increase year by year, but it also caused non-renewable energy increasingly exhausted and severe environment pollution. The use of alternative and renewable energy will be an inevitable tendency in the future. This paper represents the usage levels of alternative and renewable energy with energy regeneration rate, and fossil energy and renewable energy are not distinguished in the model any more. Assuming that,energy storage is S_t and energy regeneration rate is ϕ at time t. Then,the dynamic equation of energy storage at time t is:

$$\dot{S}_t = \phi S_t - R_t \tag{4}$$

2.3 The equation of capital accumulation

If irrespective of capital depreciation, the standard equation of capital accumulation can be presented as follows:

$$\dot{K}_t = Y_t - C_t \tag{5}$$

where C_t is the consumption of the representative household at time t, and K_t is the amount of capital at time t.

2.4 The level of production technology

The technological innovation can enhance the average level of productivity, so increasing the efficiency of fossil energy utilization and conversion of renewable energy is one of feasible ways to implement energy saving and emission reduction. Assuming that one unit of labor is used for research and development department, then technology progress is subject to the Poisson process whose parameter is $\lambda(\lambda > 0)$ and each innovative technology replaces the old one[16].

$$A_t = \mu A_{t-1}, (\mu > 1), \tag{6}$$

where A_t indicates the technology level of the final sector at time t , and μ indicates how degree technology level at time t replaces that technology level at time $t-1$. For the convenience of discussion, the "one for one" production technology is adopt proposed by Aghion P and Howitt P[17]. Assuming that, at time t , the labor invested to $RandD$ sector is L_{A_t} , the labor invested to the final sector is L_{Y_t} , and $L = L_{A_t} + L_{Y_t}$. Proposed that people are fully employed and standardize the total flow of labor to one, then . The dynamic equation of technological progress is

$$\dot{A}_t = (\mu - 1)\lambda L_{A_t} A_t. \tag{7}$$

2.5 The production function and the utility function

Based on Cobb-Douglas production function, it is assumed that socioeconomic system is composed of final product sector and $RandD$ sector, then production function of the final product sector is:

$$Y_t = A_t {L_{Y_t}}^{\alpha 1} {K_t}^{\alpha 2} {R_t}^{\alpha 3} z_t, \tag{8}$$

where A_t is production technology, L_{Y_t} is labor number of the final production sector, K_t is capital amount, and R_t is energy inputs at time t . $\alpha 1$,$\alpha 2$, and $\alpha 3$ is labor of the final product sector, capital, and output elasticity of energy respectively, $0 < \alpha_i < 1$, $i = 1, 2, 3$, and $\alpha 1 + \alpha 2 + \alpha 3 = 1$. z_t denotes pollution intensity. When $z_t = 1$, it shows that pollution does not affect the output of economy; When $z_t < 1$, the output of economy is less than its potential output. This paper studies the situation of $0 < z_t < 1$. The utility function is the level of customers satisfaction from consuming the given commodity combination. If represents consumption of the representative household at time , then the effectiveness of the representative household is as follows:

$$U(C_t) = \frac{{C_t}^{1-\theta}}{1-\theta}, \tag{9}$$

where $\theta(\theta > 0)$ is the elasticity coefficient of marginal utility (the inverse of the intertemporal elasticity of substitution), which reflects the representative households desire to change consumption. If assuming the utility discount rate at different time is constant $\rho(\rho > 0)$, thus, in the infinite horizon, the total utility of discount rate is:

$$\int_0^{+\infty} U(C_t) e^{-\rho t} dt. \tag{10}$$

2.6 The optimal growth path

According to $(1)-(10)$, the social planner is confronted with the following problems,

$$\max \int_0^\infty \frac{C_t^{1-\theta}}{1-\theta} e^{-\rho t} dt$$

$$s.t. Y_t = A_t {L_{Y_t}}^{\alpha 1} {K_t}^{\alpha 2} {R_t}^{\alpha 3} z_t$$

$$\dot{K}_t = Y_t - C_t$$

$$\dot{A}_t = (\mu - 1)\lambda L_{A_t} A_t$$

$$\dot{S}_t = \phi S_t - R_t$$

$$\dot{E}_t = dE/dt = \eta E_t - {z_t}^{-\tau} Y_t.$$

where λ_1,λ_2,λ_3, and λ_4 is shadow price of state variables A_t,K_t,S_t, and E_t respectively, and C_t,L_{A_t} ,R_t ,z_t is control variable. In order to simplify the formula, the subscript time t is omitted below.

The optimal model above is Hamilton function and it is shown as follows:

$$H = \frac{{C_t}^{1-\theta}}{1-\theta} + \lambda_1[A(1-L_A)^{\alpha_1} K^{\alpha_2} R^{\alpha_3} z - C] + \lambda_2(\mu-1)\lambda L_A A + \lambda_3(\phi S - R) + \lambda_4(\eta E - {z_t}^{-\tau} Y_t). \tag{11}$$

According to the optimal theory, the first-order condition of optimization problem follows: see $(12)-(15)$

$$\frac{\partial H}{\partial C} = C^{-\theta} - \lambda_1 = 0 \rightrightarrows \lambda_1 = C^{-\theta}. \tag{12}$$

$$\frac{\partial H}{\partial L_A} = \frac{-\lambda_1 \alpha_1 Y}{1-L_A} + \lambda_2(\mu-1)\lambda A = 0 \Rightarrow \lambda_2 = \frac{\lambda_1 \alpha_1 Y}{(\mu-1)(1-L_A)\lambda A} \tag{13}$$

$$\frac{\partial H}{\partial R} = \frac{\lambda_1 \alpha_3 Y}{R} - \lambda_3 = 0 \Rightarrow \lambda_3 = \frac{\lambda_1 \alpha_3 Y}{R} \tag{14}$$

$$\frac{\partial H}{\partial z} = \frac{\lambda_1 Y}{z} + \lambda_4(\tau-1) z^{-(\tau+1)} Y = 0 \Rightarrow \lambda_4 = \frac{\lambda_1 z^\tau}{(1-\tau)} \tag{15}$$

Euler equation is shown in$(16)-(19)$:

$$\frac{\partial H}{\partial K} = \rho\lambda_1 - \dot{\lambda}_1 \Rightarrow \dot{\lambda}_1 = \rho\lambda_1 - \frac{\lambda_1 \alpha_2 Y}{K} \tag{16}$$

$$\frac{\partial H}{\partial A} = \rho\lambda_2 - \dot{\lambda}_2 \Rightarrow \dot{\lambda}_2 = \rho\lambda_2 - \frac{\lambda_1 Y}{A} - \lambda_2(\mu-1)\lambda L_A \tag{17}$$

$$\frac{\partial H}{\partial S} = \rho\lambda_3 - \dot{\lambda}_3 \Rightarrow \dot{\lambda}_3 = \rho\lambda_3 - \phi\lambda_3 \tag{18}$$

$$\frac{\partial H}{\partial E} = \rho\lambda_4 - \dot{\lambda}_4 \Rightarrow \dot{\lambda}_4 = \rho\lambda_4 - \eta\lambda_4. \tag{19}$$

The transversality condition is as follows:

$$\lim_{t\to\infty} \lambda_1 K e^{\rho t} = 0, \lim_{t\to\infty} \lambda_2 A e^{\rho t} = 0, \lim_{t\to\infty} \lambda_3 S e^{\rho t} = 0, \lim_{t\to\infty} \lambda_4 E e^{\rho t} = 0.$$

From(12)-(19), the result can be obtained

$$G_{\lambda_1} = -\theta G_Y = \frac{\dot{\lambda}_1}{\lambda_1} = \rho - \frac{\alpha_2 Y}{K}. \tag{20}$$

$$G_{\lambda_2} = G_{\lambda_1} + G_Y - G_A = \frac{\dot{\lambda}_2}{\lambda_2} = \rho - \frac{\lambda}{\alpha_1}(\mu - 1) + (\frac{1}{\alpha_1} - 1)G_A. \tag{21}$$

$$G_{\lambda_3} = G_{\lambda_1} + G_Y - G_R = \frac{\dot{\lambda}_3}{\lambda_3} = \rho - \phi. \tag{22}$$

$$G_{\lambda_4} = G_{\lambda_1} + \tau G_z = \rho - \eta. \tag{23}$$

Using(8),(20) $-$ (23), the results can be obtained

$$(1-\theta)G_Y = G_R + \rho - \phi \tag{24}$$

$$(1-\theta)G_Y = \rho - \frac{\lambda}{\alpha_1}(\mu - 1) + \frac{1}{\alpha_1}G_A \tag{25}$$

$$G_{\lambda_1} + \tau G_Z = \rho - \eta \tag{26}$$

In the equilibrium growth path, the growth rate G of per capital variable is constant[18]. Observing from the relationship among production, consumption and investment, and thus $G_Y = G_K = G_C$ is given; since(7) yields $G_{L_A} = G_A$; since(4) yields $G_S = G_R$.

After taking two sides logarithm from (8) and then taking derivatives with respect to time t, the result follows

$$(1-\alpha_2)G_Y = (1+\alpha_1)G_A + \alpha_3 G_R + G_z \tag{27}$$

Using (24) $-$ (27), the result follows

$$G_R = (1-\theta)G_Y - \rho + \phi \tag{28}$$

$$G_A = \alpha_1[(1-\theta)G_Y - \rho + \frac{\lambda}{\alpha_1}(\mu - 1)]. \tag{29}$$

$$G_z = frac\theta G_Y + \rho - \eta\tau \tag{30}$$

$$G_Y = \frac{\tau[\lambda(\mu-1) - \rho\alpha_1](1+\alpha_1) + \rho - \eta - \tau\alpha_3(\rho - \phi)}{\tau\theta(1+{\alpha_1}^2 - \alpha_2) - \tau({\alpha_1}^2 + \alpha_3) - \theta}. \tag{31}$$

From (31), it is obtained that when $\alpha_1 + \alpha_2 + \alpha_3 = 1$ and $\alpha_1 + {\alpha_1}^2 + \alpha_3 \leq 1$, the denominator in (31) is

$$\tau\theta(1+{\alpha_1}^2 - \alpha_2) - \tau({\alpha_1}^2 + \alpha_3) - \theta \leq (\theta - 1)\tau(\alpha_1 + {\alpha_1}^2 + \alpha_3) + \tau\alpha_1 - \theta \leq 0. \tag{32}$$

$G_Y > 0$,if$\rho < \frac{-\tau\lambda(\mu-1)(1+\alpha_1)+\eta-\tau\alpha_3\phi}{1-\tau(\alpha_1+{\alpha_1}^2+\alpha_3)}$,which implies that along the steady-state optimal growth path, output per head and consumption is positive growth; $G_Y < 0$,if$\rho > \frac{-\tau\lambda(\mu-1)(1+\alpha_1)+\eta-\tau\alpha_3\phi}{1-\tau(\alpha_1+{\alpha_1}^2+\alpha_3)}$,which implies that along the steady-state optimal growth path, output per head and consumption is negative growth. Of course, it is expected that output per head and consumption is positive growth while economy develops along the optimal growth path. Therefore, the following discuss only the situation $G_Y > 0$.

3 An static analysis and conclusion of economic growth

G_Y,G_R,G_A,and G_z obtained from the preceding section include some parameters, for instance,μ , λ , η , ϕ , ρ , τ , θ and so on. Then the growth rate of each variable is used to take derivatives of each parameter. The results are presented in Table 1

From the computation results, since the effects of parameters ρ , μ , and θ on each variables growth resemble the study result obtained by Wang Chenwei and Shi Gang[9], and are alike the study result obtained by Honglin Yang , Weixing Tian [16] and others, so we won't cover those again. Here, discussing only the reasons why μ has uncertainty effects on technological progress if $\theta > 1$. In effect

$$\frac{\partial G_A}{\partial \mu} = \frac{\tau(\lambda - 1)(\alpha_1 + {\alpha_1}^2) + \theta(\tau\lambda\alpha_3 - 1)}{\tau\theta(\alpha_1 + {\alpha_1}^2 + \alpha_3) + \tau({\alpha_1}^2 + \alpha_3) - \theta} \tag{33}$$

See (32) and (33), the technological progress can not be simply determined by μ due to the fact that technological growth rate is subject to both μ and λ.
After analyzing emphatically the effects of η , ϕ , and τ on each variables growth rate, summaries are given.

Proposition 1: The environmental self-purification η has positive effects on economic growth and energy production. If other parameters remain unchanged, an increase of η means an increase of G_Y and G_R accordingly. The research and development of new energy technology is not so urgent(the conclusion illustration: when environmental self-purification capacity is enhanced within short time and environmental pollution reduces, in general, the government will lower its strict control of energy production and by consequence energy production will increase in order to increase G_Y).

Proposition 2: ϕ is used to depict energy regeneration rate. If other parameters keep unchanged, an increase of energy regeneration rate means an increase of G_Y,G_z,G_R, and G_A. (the conclusion illustration: The energy regeneration rate keeps a positive increase within a reasonable range, which will definitely results in an increase of G_Y and G_R , and the severer environmental pollution. In order to tackle the pollution caused by energy production, the government needs to strengthen the study and development of new energy technology.)

Proposition 3: As shown in Table 1, when τ changes, G_Y,G_R,G_A will increase accordingly as τ increases. Yet, the effects of τ on G_z are related to the difference of $\rho - \eta$. Assuming that

$$G_Y = \frac{\tau[\lambda(\mu-1) - \rho\alpha_1](1+\alpha_1) + \rho - \eta - \tau\alpha_3(\rho - \phi)}{\tau\theta(1+{\alpha_1}^2 - \alpha_2) - \tau({\alpha_1}^2 + \alpha_3) - \theta} = \frac{M}{N}$$

$$\frac{\partial G_z}{\partial \tau} = -\frac{\theta}{\tau^2}\frac{(\rho-\eta)N + (\theta - \tau^2 N)M}{\tau^2} \tag{34}$$

See (34), the results are obtained that, on the premise of economic growth, pollution intensity is not only influenced by τ but also by the difference of $\rho - \eta$. If $\rho - \eta \geq 0$ then $\frac{\partial G_z}{\partial \tau} \geq 0$, and pollution intensity increases; if $\rho - \eta < 0$ then $\frac{\partial G_z}{\partial \tau} < 0$, and pollution intensity decreases.

4 Conclusion

This paper analyzes the influence factors of economic growth mainly in aspects of pollution intensity and energy. By means of analyzing models optimal growth path, the rate of economic growth and the steady-state features are given.

The results demonstrate that,at the steady-state, the rate of economic growth will increase accordingly as μ , η , ϕ , and τ increase when economy keeps the positive growth rate; however, the economic growth rate will decrease as discount rate ρ and elasticity coefficient of marginal utility θ increase. it's worth noting that the degree μ , η , ϕ , and τ impact on energy use and levels of technology related to θ . If $\theta < 1$, then the rate of energy consumption and levels of technology will increase. Otherwise, energy consumption and growth rate of technology levels will reduce. When ρ , η , and ϕ increase, the study and development of low pollution technology will be strengthened so as to control the increase of energy consumption or pollution intensity generated by economic growth. Nevertheless, how τ influences pollution intensity is linked to the difference of $\rho - \eta$ and pollution intensity changes with the change of difference of $\rho - \eta$. In view of the results derived from the theoretical analysis, the government is able to take approriate measures to ensure sustained economic growth.

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Table 1: An static analysis

	$x=\rho$	$x=\mu$	$x=\eta$	$x=\phi$	$x=\theta$	$x=\tau$
$\frac{G_Y}{x}$	<0	>0	>0	>0	<0	>0
$\frac{G_z}{x}$	>0	>0	<0	>0	<0	Subject to both τ and $\rho-\eta$
$\frac{G_R}{x}$	<0	>0,if $\theta<1$; <0,if $\theta>1$	>0,if $\theta<1$; <0,if $\theta>1$	>0 ,if $\theta<1$; <0 ,if $\theta>1$	>0	>0 ,if $\theta<1$; <0 ,if $\theta>1$
$\frac{G_A}{x}$	<0	>0 , if $\theta<1$, Whether $\theta>1$ or not is uncertain	>0 , if $\theta<1$; <0 , if $\theta>1$	>0 , if $\theta<1$; <0 ,if $\theta>1$	>0	>0 , if $\theta<1$; <0 , if $\theta>1$

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