ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.21(2016) No.3, pp.187-192

Self-similarity of Carbon Market from Complex Network Analysis

Li Wang, Xinghua Fan *, Shasha Li
Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu 212013, China
(Received 28 May 2015 , accepted 22 January 2016)

Abstract This article shows the European carbon market is self-similar by applying the complex network analysis. Carbon future prices are selected as the indicator of carbon market. Firstly, we translate carbon price time series into complex network by using the visibility graph algorithm. The nodes were the data of daily carbon prices. The edges were determined by visible property between two nodes. Then, topological properties of the constructed network are adopted to measure the main characteristics of the carbon future prices series. The complex network derived from Dec13 is found to have the most complicate behavior. Finally, fractal dimension of clustering coefficient of each complex network is used to prove the self-similarity of the carbon market. All clustering coefficients have fractal dimension which means the carbon market is self-similar.
Keywords carbon prices; self-similarity; complex network; visible graph; clustering coefficient.

1 Introduction

Carbon emission trading is a useful policy instrument for controlling CO_2 emissions through market mechanism[1]. Since 2005, the European Union Emissions Trading Scheme (EU ETS) has seen a rapid growth in trading volume activity[2]. Currently, the carbon futures market under EU ETS is the largest one in the world, whose transaction volume and price fluctuation both play a significant implication for the global carbon market.

In recently years, research about the carbon prices gradually concentrates on the carbon prices distribution and behavior and the carbon prices analysis adopting the measurement model. Hintermann[3] researched the relationship between the variation of weather factors and carbon prices, which indicated that the variation of weather factors would effected the fluctuation of carbon prices and there existed nonlinear relationship between the extreme cold weather and carbon prices. Alberola[4] found that carbon prices occurred structural mutation and the policy and system effected the variation of carbon prices from the breakpoint of carbon prices mutations. Convery and Redmond [5] verified the carbon prices of EUs first stage, which showed that the impactor influencing the carbon prices fluctuation is energy prices. Feng et al.[6] identified that the carbon prices was a biased random walk distribution and had some irregular market characteristics by adopting random walk model and R/S analysis. Besides, the influence for carbons history prices to future development was short. Seifert et al. [7] researched the carbon spot prices for the carbon emissions market of the EU. By establishing the stochastic general equilibrium model to carry out the quantitative analysis, the results showed that the spot price was related to the cost of carbon emissions quota and EU penalty cost. Besides, carbon spot prices have not shown some seasonal characteristics. Chevallier [8] investigated the EUs carbon futures and option price volatility by constructing GARCH model and BCS model, which found that the carbon prices was influenced by economic mechanism, quota and energy price and the unstable environment.

The theory of complex network has been introduced into the study of time series[9]. By transforming time series into complex network, many scholars proposed various approaches to explore the dynamics in time series[10]. There are several methods to translate time series into complex network, such as the cycle network, the correlation network, the visibility graph, the k-nearest-neighbor network and recurrence networks[11]. Many researchers have also analyzed the self-similarity of complex networks. Zhang et al. proposed different algorithms to calculate the fractal dimension of complex networks[12].

* **Corresponding author**. *E-mail address*: 245082251@qq.com

IJNS.2016.06.15/915

The structure of the paper is as follows. In Sec. 2, we present the methodology, including the method of visible graph, measurements of network structure as well as fractal dimension. In Sec.3, we give the data and experimental results. Five topological characteristics of the mapped complex networks: average path length, average, diameter, density and clustering coefficient are compared and analyzed. The conclusions are presented in Sec.4.

2 Methodology

2.1 Complex network

In general, the complex networks can be represented by an undirected and unweighed graph $G = (V, E)$, which consists of a set of $V = \{v_1, v_2, \cdots, v_N\}$ nodes or vertices and a set of $E = \{e_1, e_2, \cdots, e_E\}$ edges or links. We constructed complex networks by visible graph. The nodes were determined by the daily carbon prices, and the edges were determined by the visibility algorithm.

The visibility graph is a natural bridge between complex network theory and time series analysis. We use the visibility graph to translate carbon prices time series into complex networks[17]. A node in the visibility graph is a point or location in Euclidean plane; an edge connected two inter-visible locations, which means there is no obstacle between the two locations. The visibility graph algorithm is very simple and ingenious. Giving two data points of a time series are connected, if they can see each other without any obstacle. Any two points $(t_a, y_a) and (t_b, y_b)$ (assume time $t_a < t_b$ and $y_a and y_b$ are the values of the two data points, respectively) in the time series are mutually visible, and become two connected nodes in the visibility graph, if and only if for any $t_c(t_a < t_c < t_b)$[16].

$$y_c < y_a + (y_b - y_a)\frac{t_c - t_a}{t_b - t_a}$$

The graph thus generated from a time series always displays the following characteristics:

1. Connectivity: each node can see at least its left and right nearest neighbors;
2. Undirectedness: the links of a visibility graph generated in this way have no directions;
3. Invariance under affine transformations of the series data: the visibility criterion is invariant under rescaling of both horizontal and vertical axes and under horizontal and vertical translations.

2.2 Measurements of network structure

Distinct features of a time series can be mapped onto networks with distinct topological characteristics of the complex networks.

The degree and the degree distribution are the common measurements to analyze complex networks. The degree of a node is the number of edges incident with it, and degree distribution $p(k)$ is defined as the probability of a node with degree k.

The diameter D of a network is the largest of the shortest path distances between any pair of nodes, and the average path length $< d_{ij} >$ is the average distance of a network connecting any pair of nodes i and j.

$$D = \max_{ij} d_{ij}, \quad < d_{ij} >= \frac{1}{N(N-1)}\sum_{i,j} d_{ij}. \tag{1}$$

For the overall network structure, the average path length and distance is an important measured characteristic [18].

The clustering coefficient of a node i is a measure of network transitivity, expressing the extent to which neighbors of a node are neighbors of each other, and is defined:

$$c_i = \frac{2e_i}{k_i(k_i - 1)}, \tag{2}$$

where k_i is the number of neighbors of node i and e_i is the number of connected pairs between all neighbors of node i. The average clustering coefficient $< c_i >$ of a network is the average c_i as follows

$$< c_i >= \frac{1}{N}\sum_{i=1} c_i. \tag{3}$$

The density is defined as the number of edges divided by the largest number of edges possible.

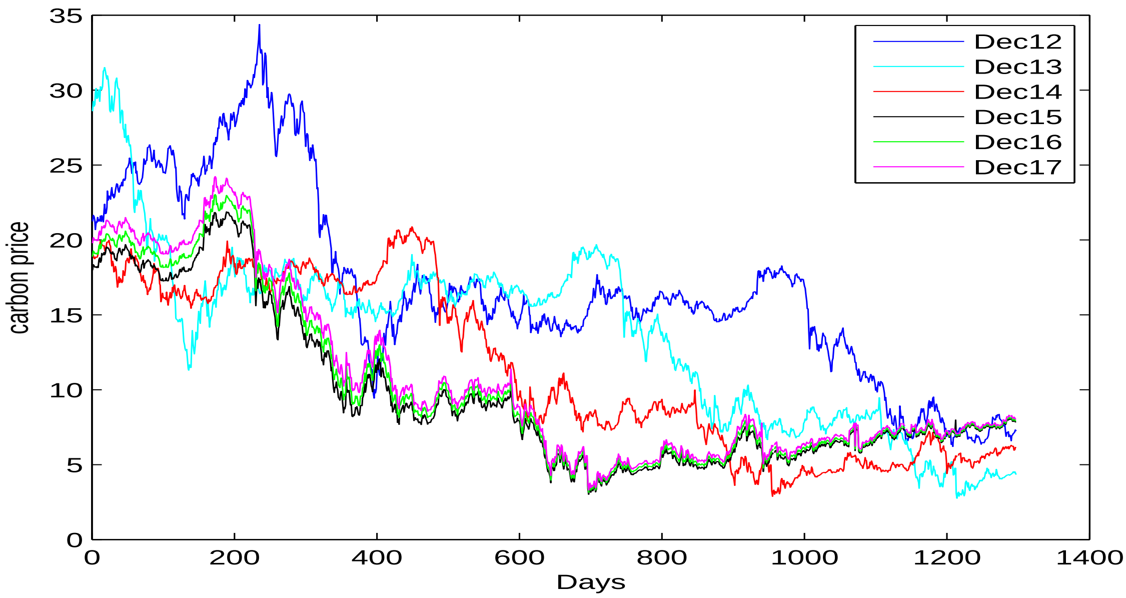


Figure 1: Carbon prices of Dec12 to Dec17 (Color on line).

2.3 Self-similarity of complex network

The self-similarity property is the typical characteristic of the fractal object. The self-similarity in the distribution of the clustering coefficient can be served as a general characteristic of complex networks. Researchers found that fractal dimension is one of the fundamental properties of complex networks characterizing not only its topological properties but also dynamic characteristics [19, 20]. To unfold the self-similarity property of complex network we calculate the fractal dimension using Box covering algorithm[21].

Box covering algorithm is one of the most typical ways to calculate the fractal dimension. A power-law relation between the number of boxes needed to cover the network and the size of the box [22]. For a given network G and box size l_B, a box is a set of nodes where all distances l_{ij} between any two nodes i and j in the box are smaller than l_B. The minimum number of boxes required to cover the entire network is denoted by N_B. The fractal dimension or box dimension d_B calculated with the box covering algorithm is given as follows[23]:

$$N_B \approx l_B^{-d_B}.$$

3 Data and experimental results

3.1 Data

We consider carbon future prices of the EU allowance (EUA) from the European Climate Exchange (ECX). The ECX is the largest carbon market under the EU ETS, in which there are spot, futures, options of the EU allowance (EUA) and Certified Emission Reduction(CER) and the trading volume of EUA is the largest. We selected carbon future prices, matured in December from 2012 to 2017 as sample data. The corresponding carbon future prices are denoted by Dec12-Dec17. Each carbon price time series covers a period of five years, exclusive of public holidays. Dec12 has selected sample points from 8 August 2007 to 8 August 2012. Dec13 and Dec14 are just one year and two years shifting to Dec12. All Dec15, Dec16 and Dec17 have elected points from 8 August 8 2010 to 8 August 2015. All data come from the EUA website. Fig. 1 shows the plots of the six carbon price time series. We can see that carbon price presets high uncertainty and nonlinearity.

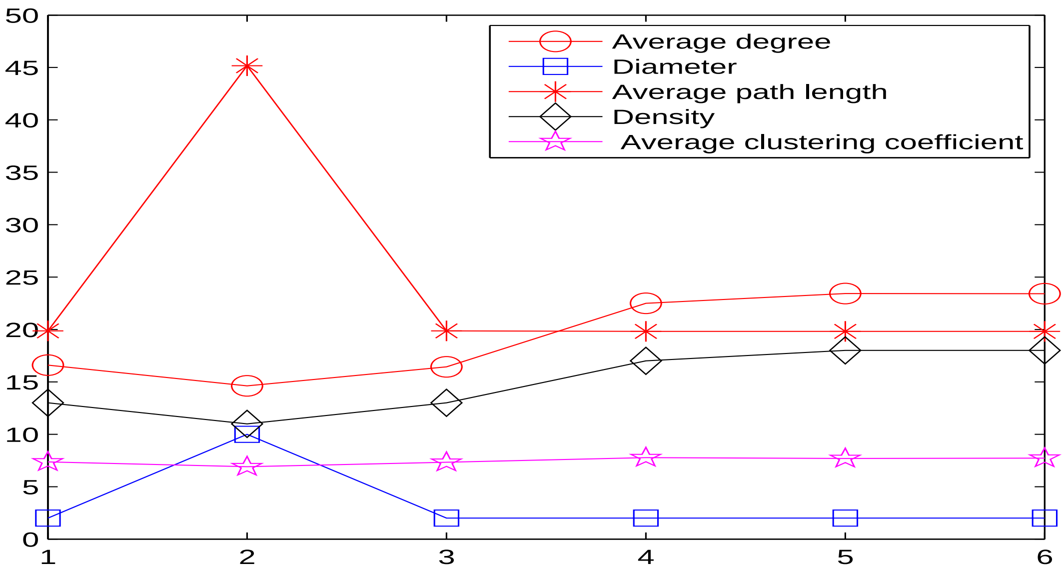


Figure 2: Topological characteristics of each complex network. Horizontal axis refers to Dec12 to Dec17. Different vertical scale are taken to ensure a same range: Average path length *10, Average clustering coefficient*10, density*1000.

Table 1: Fractal dimension of clustering coefficients for different complex networks.

Box size	1	2	4	8	16	32
Dec12	0.9989	0.9983	0.9967	0.9934	0.9956	0.9826
Dec13	0.9978	0.9989	1	1	0.9911	0.9738
Dec14	0.9989	0.9983	0.9967	0.9934	0.9956	0.9826
Dec15	0.9989	0.9983	0.9967	0.9934	0.9956	0.9826
Dec16	0.9989	0.9983	0.9967	0.9934	0.9956	0.9826
Dec17	0.9989	0.9983	0.9967	0.9934	0.9956	0.9826

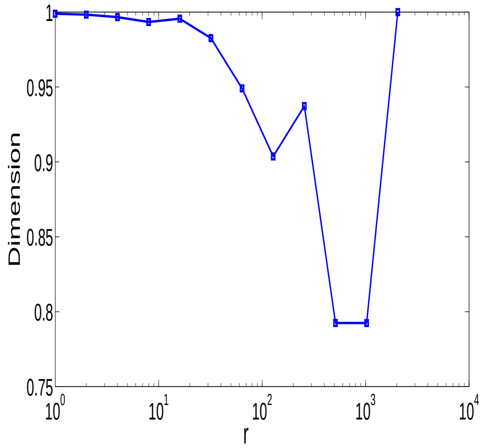
3.2 Experimental results

We translated carbon future prices into complex networks by visible graph algorithm. The nodes were determined by the daily carbon prices, and the edges were determined by the visibility propertity. Then we calculated the topological properties of those complex networks. Fig. 2 shows the results of topological properties. One can see that the average path length and diameter properties increase first and reach their maximum at Dec13, then they decline sharply to the initial value of Dec12, and keep the value steady from Dec14 to Dec17. In contrast, the other three properties, density, average degree and average clustering coefficient, reduce smoothly to the minimal value at Dec13, then they increase and go up to their stable value at Dec15. This means that the network derived from Dec13 is the most complex.

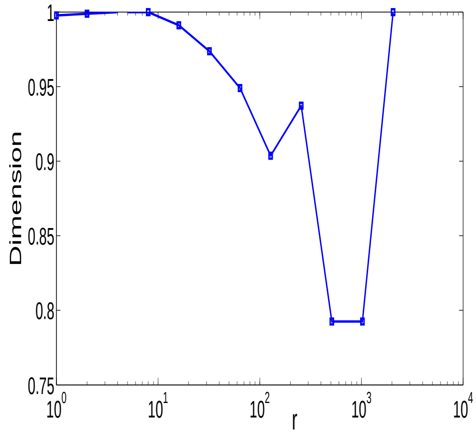
To gain further understanding of carbon prices, we study features of complex network from the perspective of self-similarity.We consider the clustering coefficients as quantitative measures of the local properties of the complex networks. Fractal dimension has been confirmed to be able to determine the self-similarity of complex network directly. Using Box Covering algorithm, we calculated the fractal dimension of clustering coefficients for different complex networks.

Table 2: Fractal dimension of clustering coefficients for different complex networks.

Box size	64	128	256	512	1024	2048
Dec12	0.9491	0.9037	0.9372	0.7925	0.7925	1
Dec13	0.9491	0.9037	0.9372	0.7925	0.7925	1
Dec14	0.9491	0.9037	0.9372	0.7925	0.7925	1
Dec15	0.9491	0.9037	0.9372	0.7925	0.7925	1
Dec16	0.9491	0.9037	0.9372	0.7925	0.7925	1
Dec17	0.9491	0.9037	0.9372	0.7925	0.7925	1



(a) $Dec12$



(b) $Dec13$

Tab. 1 and 2 shows that all the complex networks have same behavior in their fractal dimension. For given box size, all the complex networks have almost the same fractal dimension. For given complex networks, the fractal dimension almost increase by the same value. Thus we only draw the picture of Dec12 and Dec13 in Fig. 3.2 as the represent of those complex networks. And all of the clustering coefficients of complex networks have the properties of self-similarity. Based on the analysis of topological property and fractal dimension, we obtained that the complex network derived from the Dec13 carbon prices, were more complex than others. This means the Dec13 carbon prices were more complexity. Therefore, we also obtain that the carbon market have the self-similarity property and is a fractal object.

4 Conclusions

In this study, we studied the self-similarity of carbon market by translating carbon price time series into complex networks. The visibility graph algorithm was applied to map the time series into a graph with nodes and edges. We analyzed the topological properties of these networks with respect to degree distributions, clustering coefficient or average path length. we adopted the clustering coefficient as the measure of the complex network. The seif-similarity in clustering coefficient was determined by the fractal dimension. We found that the carbon market is self-similar in the meaning of fractal dimension. We have provided a new way to analyze the carbon market from the perspective of complex network . In the future, we may study the relationship between carbon prices and other economic factors by using complex network theory.

Acknowledgements

Research is supported by the National Natural Science Foundation of China (No. 51276081).

References

[1] P. Zhou et al. Modeling economic performance of interprovincial {CO2} emission reduction quota trading in china. *Applied Energy*. 112(2013): 1518–1528.

[2] J. Crossl et al. Is the european union emissions trading scheme (eu ets) informationally efficient evidence from momentum-based trading strategies. *Applied Energy*. 109(2013): 10–23.

[3] Hintermann. Allowence price drivers in the first phase of the EU ETS. *Environ. Econ. Manag.* 59(2010): 43C56.

[4] A. Emilie et al. Price drivers and stuctural breaks in European carbon prices 2005C2007.*Energy Policy* . 36(2008): 787–797.

[5] F. J. Convery and R. Luke. Market and price developments in the European Union Emissions Trading Scheme. *Rev. Environ. Econ. Policy.* 1(2007): 88–111.

[6] Z. H. Feng et al. Carbon price volatility: evidence from EU ETS. *Appl. Energy.* 88(2011): 590–598.

[7] S. Jan et al. Dynamic behavior of CO2 spot prices. *J. Environ. Econ. Manag.* 56(2008): 180–194.

[8] J. Chevallier et al. Risk aversion and institutional information disclosure on the European carbon market: A case-study of the 2006 compliance event. *Energy Policy.* 37(2009): 15–28.

[9] X. Lan et al. Fast transformation from time series to visibility graphs. *Chaos*. 25(2015): 0831051–0831057.

[10] Y. Moreno et al. Complex networks: Structure and dynamics. *Phys. Rep*. 424(2006): 1705308.

[11] P. Caraiani. Characterizing emerging european stock markets through complex networks: From local properties to self-similar characteristics. *Physica A*. 391(2012): 3629–3637.

[12] X. Zhang et al. Identifying influential nodes in weighted networks based on evidence theory. *Phys A: Stat Mech Appl*. 392(2013): 2564–2575.

[13] Y. Deng. Generalized evidence theory. *Appl Intell*. 43(2015): 530–543.

[14] X. Qin et al. An improved method to rank generalized fuzzy numbers with different left heights and right heights. *J Intell Fuzzy Syst*. 28(2015): 2343–2355.

[15] X. Lan et al. Fuzzy fractal dimension of complex networks. *Appl Soft Comput*. 25(2014): 514–518.

[16] X. Li et al. A complex network of well log based on visibility graph. *nternational Journal of Signal Processing, Image Processing and Pattern Recognition*. 7(2014): 365–376.

[17] Z. K. Gao et al. from time series based on phase space reconstruction. *Chaos*. 19(2009): 033137.

[18] D. J. Watts and S. H. Strogatz. Collective dynamics of 'small-world' networks. *Nature*. 393(1998): 440–442.

[19] P. Xu. Dependence assessment in human reliability analysis using evidence theory and ahp. *Risk Anal*. 35(2015): 296–316.

[20] Y. Luo et al. Determining basic probability assignment based on the improved similarity measures of generalized fuzzy numbers. *Comput Commun Control*. 10(2015): 333–347.

[21] M. Newman. Analysis of weighted networks. *Phys Rev E*. 70(2004): 056131.

[22] A. Jamakovic and S. Uhlig. On the relationships between topological measures in real-world networks. *Netw Heterog Med*. 3(2008): 345–356.

[23] A. Bunde et al. Dimension of spatially embedded networks. *Nat Phys*. 7(2011): 481–484.