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Target Control Based on Propagation Immunization

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Abstract: Control of complex network reflects humans comprehension of complex system and the ability to reform it. Up-to-date research establishes the controllability theory of target control of complex network. The theory could find that one node can control a set of target nodes if the path length to each target node is unique. However, we find that this theory does not include the immune node or failure node which blocks the control signal. Following the advantages of the theory, we analyze the controllability of ER and RR networks. Finally, we find that the network with immune nodes is more difficult to control.

Keywords: complex network; controllability theory; target control; propagation immunization.

1 Introduction

In recent years, network science is widely used in physics, ecology and evolutionary biology, engineering and diverse fields. It explores the evolution model of complex network which is helpful for us to explain different phenomenons of complex system in various fields, in some degree, the purpose of explaining phenomenon is to forecast what can be controlled finally. The meaning of network controllability is to reform the network. Recently, following the problem of network controllability, the controllability and control method of complex system become one of top issues. People develop analytical tools to study the controllability of an arbitrary complex directed network, identifying the set of driver nodes with time-dependent control that can guide the systems entire dynamics. Established the LB model is important for us to analyze structure controllability [1]. After that, researchers made more in-depth work, while to drive a large, complex, networked dynamical system toward some desired state using as few external signals as possible is a fundamental issue in the emerging field of controlling complex networks. Optimal control is referred to the situation where such a network can be fully controlled using only one driving signal. We propose a preferential matching algorithm to find MDSs that have a specific degree property. Then, we show that the MDSs obtained by preferential matching can be composed of high- and medium-degree nodes [2-3]. Also, a significant body of work has emerged focusing on how to organize such networks to facilitate their control and make them amenable to human interactions, we summarize these activities by connecting the network topology, that is, the layout of the interconnections in the network, to the classic notion of controllability [4].

Recently, network analysis is widely used in diverse fields and can be a powerful framework for studying the structure of biological systems. We examine the controllability of systems for which the timescale of the dynamics we control and the timescale of changes in the network are comparable. We provide analytical and computational tools to study controllability based on temporal network characteristics and provide a worked example analysis and highlight new research opportunities in ecology and evolutionary biology [5-6]. We may face the problem of control range which propose a novel structural index, the control range, motivated by recent studies on the structural controllability of large-scale directed networks. It is observed that the control range of the nodes is mainly influenced by the networks degree distribution and that nodes with a low degree may have a high control range [7]. Although the integration of control theory and network analysis is important, we argue that the application of the structural controllability framework to most if not all real-world networks leads to the conclusion that a single control input, applied to the power dominating set, is all that is needed for

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structural controllability [8]. But, structural controllability theory is aim at linear time-invariant directed network, and to control the spread of the signal is too simple. Propagation control signal is an important basis for network control. The idealized propagation of control signal is accept may transpond, while in real network blocking the spread of the network nodes is common. Immunization algorithms originated from the complex system theory are feasible for large-scale systems based on a scale-free network model, the immunization strategy for complex systems which includes random and targeted immunizations to solve energy consumption issues and uses traffic to judge the energy savings from the node immunization [9]. Also It shows that although current immunization strategies can effective restrain virus propagation in homogeneous networks, the efficiency of strategies should be improved in interdependent networks [10-12].

For us, the introduction of immunization policy for the control of complex network is of great significance [13-16]. In particular, when used in medicine and epidemiology is very valuable [17-20]. While in this paper, we focus on the immunization strategy of structural controllability. We highlight the structural framing, and we want to learn applying immune signal for what type of nodes can increase the control difficulty of complex network. It also has deep implications for the feasibility and effectiveness of network control.

2 Model and method

This paper based on immune phenomenon, and introduce the concept of immune nodes in the complex network control, we will further improve the target control theory. This paper we adopt the method about wiping out the specific nodes outgoing edges, and then interdict signal transmission in the network. Physical significance of the immune signal is embodied in that interrupt or offset control signal through the influence from this node to other nodes'state in the network. After the network an addition to immune signal, we may find significant changes in the topology. At the moment, before the source node by the greedy algorithm which we are looking for may change, so we need to look for the original node of new network. Then randomly selecting a fraction a of nodes from the new network as target nodes, and constituting a target set of nodes. We use GA algorithm to calculate the original node, proportion of part nodes we called f.

2.1 Target controllability

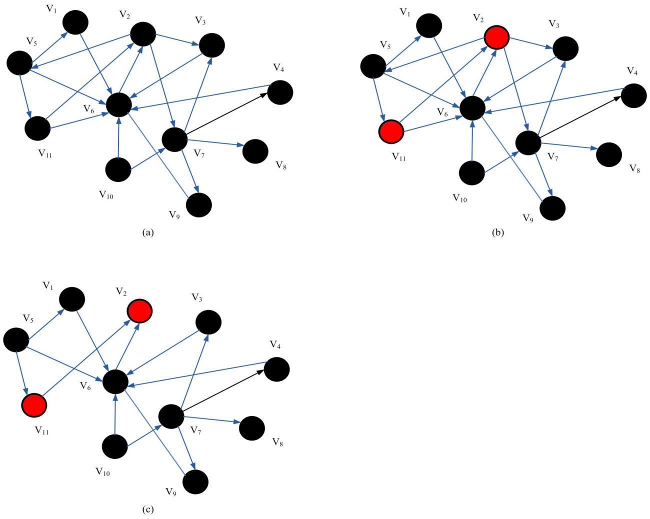

Figure 1: Isolation in network, where $v_1, v_2, v_3, \ldots, v_{11}$ represent eleven nodes of network. v_2 and v_{11} are immune nodes, (a) represent the original state of network, (b) represent the network applying immune signal, and (c) represent the changed network.

For a canonical linear, time-invariant dynamics:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

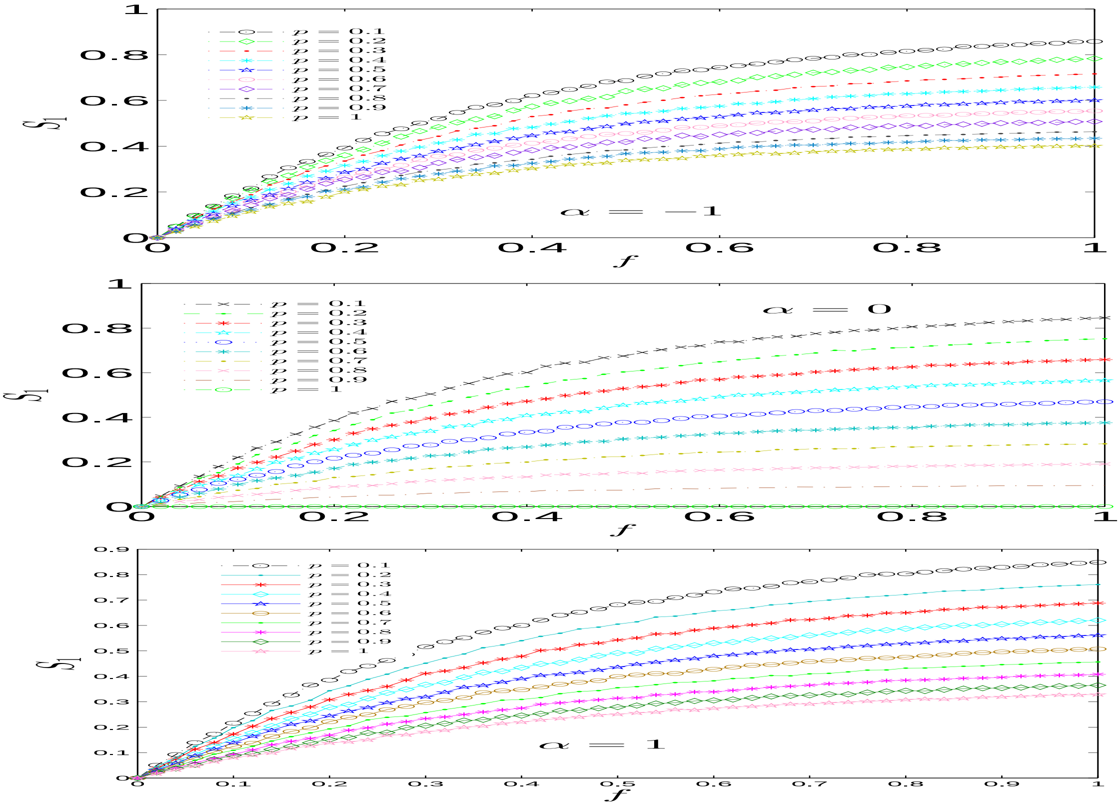

Figure 2: Simulation of ER networks (a-c) For ER networks with average degree $< k >= 5.6$, the nodes $N = 10000$. And p represent the p part of nodes removed from networks, f represent the f part of nodes chosen from new networks, S_1 represent the part after removing p part of nodes, f represent the originating nodes of part nodes proportion. (a) represent when parameter $\alpha = -1$. (b) represent when parameter $\alpha = 0$. (c) represent when parameter $\alpha = 1$.

where $x \in R^N$, $u \in R^M$ and $y \in R^S$ represent the systems state, input and output vector, respectively. $A \in R^{N\times N}$, $B \in R^{N\times M}$ and $C \in R^{S\times N}$ denote the state, input and output matrices, respectively. The system (A, B, C) is target controllable if and only if the dimension of the output controllable subspace $d(A, B, C)$ satisfies

$$d(A, B, C) \equiv rank[CB, CAB, CA^2B, \ldots, CA^{N-1}B] = S, \tag{2}$$

In allusion to the controllable system, we develop a greedy algorithm that offers a good approximation to the minimum set of inputs sufficient for target control. In some ways the target control is more difficult than the full control problem. Full control has a graphical condition, which can be easily checked by exactly mapping the controllability problem to the maximum matching problem. Target control lacks such an exact mapping. Therefore, to solve the target control problem in the single-input case, we develop a new algorithm named greedy algorithm.

2.2 Propagation immunization

We use segregating strategy to interdict signal transmission in the network. In fact, it means wiping out the specific nodes outgoing edges. In this paper we called isolation. Firstly, we want to determine the immune nodes for a given network, by just random immune or target immune, these are two tactics for propagation immunization. After then, we apply immune signal on the immune nodes. For example, if we put immune signal to the nodes in the disease transmission network, those nodes will never be infection source; in the power network, applying immune signal means breaking off electric signal from the higher node to the lower node; in the spread of behavior, the propagation will quit by immune signal. Following a small network as an example, to describe basic procedure of the algorithm above. It should be noted that our purpose

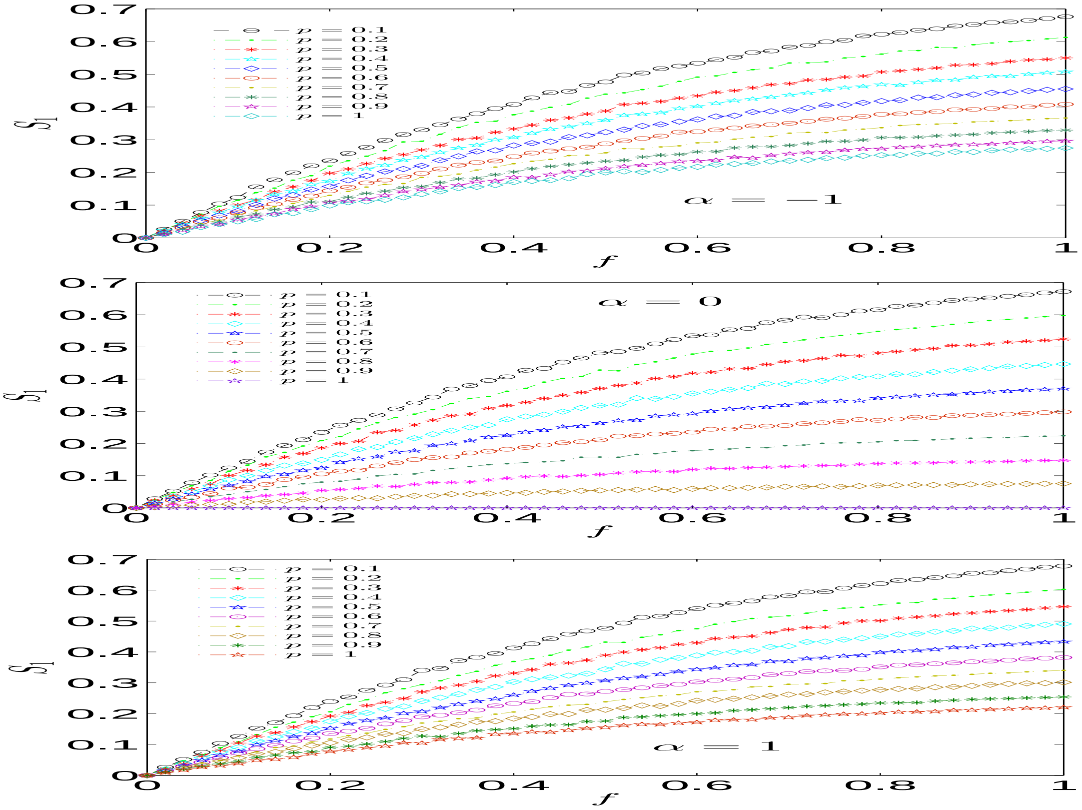

Figure 3: Simulation of RR networks (d-f) For ER networks with average degree $< k >= 3$, the nodes $N = 10000$. And p represent the p part of nodes removed from networks, f represent the f part of nodes chosen from new networks, S_1 represent the part after removing p part of nodes, f represent the originating nodes of part nodes proportion. (d) represent when parameter $\alpha = -1$. (e) represent when parameter $\alpha = 0$. (f) represent when parameter $\alpha = 1$.

is different from the propagation immunization of disease. Our goal is to control the state of nodes, but the propagation immunization of disease wants to reduce the propagation velocity of disease.

3 Simulation

We want to know if the network apply immune signal, the originating nodes of target control what will be the corresponding change. First, we set up a network in random. Then probability for $\frac{k^\alpha}{\Sigma_k k^\alpha}$, remove the p part of nodes outgoing edges, it may come into being a new network. Finally, we can use greedy algorithm to find the originating nodes of the new network. Here we do some simulation for ER network and RR network, as following figure 2 and figure 3

We find when $\alpha = -1$, with the removal part of p values different, the proportion of the original node share the nodes of new network show variation width is not large; when $\alpha = 0$, with the removal part of p values different, the proportion of the original node share the nodes of new network show popular impact; when $\alpha = 1$, just like the situation when $\alpha = -1$.

We find in the new network, f and S_1 similar to proportional relationship, this is just common sense like all know. Following the increase of S_1, f appear increase gradually. We also find on this premise of selecting the same number of immune nodes, targeted immunization is more effective than random immunization in affecting the control of network.

4 Conclusion

In the actual network, not all nodes are accept may transpond, we introduce immune nodes to the target control , find the network is more difficulty to control, also targeted immunization is more effective than random immunization in affecting the control of network. In the future, we have more work to research for the immunization strategy what can be suitable for network control, and we want to do more simulation of the real network.

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