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A New Method to the Rumor Spreading

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Abstract: According to the method of infectious disease model, a rumor spreading model was built in heterogeneous network.The spreading threshold , R_0 ,is an important tool in measuring the dynamic characteristics of the whole system. By the calculation of the next generation matrix, we got a new method to solve the R_0 of rumor spreading model, and gave an explicit expression. During the simulation,The crowd was divided into two categories: minors and adult. based on this classification , a network model with two kinds of people and two nodes was established. Through limiting and changing different parameters, the simulation results of a certain regularity were obtained, we also made the results analyzed and compared. To summarize, our study made a statement about the influence factors of the spreading threshold, and put forward related control strategy about the rumor spreading.

Keywords: rumor spreading; infectious disease model; the next generation matrix; heterogeneous network

1 Introduction

Early rumors spread model studies began in 1960. The process of rumors spreading is similar to the spread of the virus, which makes the establishment of the model mostly associated with infectious disease model .In this models, People is in generally divided into several categories, For example, in the earliest classical rumors spread DK model[1], the crowd was divided into three categories With the corresponding probability: S(represents rumor spreaders, who know and spread rumor, similar to the infected person in the infectious disease model), Istands for ignorants ,who do not know and obviously cannot spread the rumor, Similar to the susceptible in the infectious disease model), R(shows people who accept the rumor but do not spread it,similar to the removed in the infectious disease model). Then, Maki[2]Thomson and Murray[3]have applied mathematical model to the rumors spread model, and carried on the corresponding theoretical research. However, the introduction of the complex network makes rumors spread research entered a new period[4], Zanette[5]firstly introduced the complex network theory to the rumor spreading model,and established a rumor spreading model in the small world network, having proven the existence of the threshold[6]. MorenoNekovee and Pacheo built a rumors spread model on the scale-free network[7], In another Nekovees article[8],the forgetting mechanism was joined in the research, he also discuss the change of threshold .In the later research, scholars considered various factors in the rumors spreading, and conducted a similar study on its characteristics.

In addition, In view of the classification of the crowd in the classical model, Huang and Jin put forward a classification method of R[9]. They said, when individuals infected by the virus, one cannot choose to accept or refuse to infection, but the spread of rumors process can change according to individual choice, therefore, R can be further divided into two categories: Ra represents people who know the rumor choose to accept rather than spreadand Rustands for those who refuse to accept rumor). However, they only studied transmission under heterogeneous network, and dont consider the forgetting mechanism.

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2 Construction of the model

Consider N interconnected individuals in the network, Vertex represents individual, edge reflects the relationship between individuals, we can get the corresponding undirected graph. Assuming that gossip spreads directly by communicators and other population, we get the diagram.

In the process, individuals are divided into four categories, namely: ignorant, spreaders, people who know the rumor choose to accept rather than spread and those who refuse to accept rumor. Expressed in I, S, R and U , respectively.

When a person comes into contact with the rumor, Three possible changes will be appeared: (1)turn to the rumor spreader with probability λ;(2)turn to people who know the rumor choose to accept rather than spread with probability γ;(3)turn to those who refuse to accept rumor with probability β. In the second case the rumor continues to be spread .For example, when a rumor spreader comes into contact with other spreaders (include S,R and U), he will be change into R in probability α. In addition, by forgetting mechanism, we can learn that, due to their own forgotten or lose interest in such rumors during the spreading process, different spreaders may spontaneously to stop focusing on the rumors, f represents the probability.

According to the above analysis, we established the basic rumors spread model:

$$\begin{cases} \frac{dI(t)}{dt} = b - \lambda I(t) S(t) - (\gamma + \beta) I(t) - f I(t) \\ \frac{dS(t)}{dt} = \lambda I(t) S(t) - \alpha S(t) - f S(t) \\ \frac{dR(t)}{dt} = \alpha S(t) + \gamma I(t) - f R(t) \\ \frac{dU(t)}{dt} = \beta I(t) - f U(t) \end{cases} \tag{1}$$

In the model, $I(t)$, $S(t)$, $R(t)$ and $U(t)$ represent the proportion of four groups in the total number, obviously, $I(t) + S(t) + R(t) + U(t) = 1$.

In real life, the process of rumors spreading is influenced by many factors, such as the age of the communicator, the sex of the communicators, and so on. In order to construct a more practical model, we made further research on the above model.

Considering people in different age stages may have different cognition of things, the corresponding judgment is also different .For example, when rumors spread to a person, because of lack of judgment for new knowledge, minors may accept and spread it easily. While adults can judge the credibility of the message according to their own experience and knowledge , and then make the corresponding choice. Therefore, we divided individuals into two age groups, namely the minors and adults. Thus we constructed the rumors spread system under a simple network contains two nodes :

$$\begin{cases} \frac{dI_{ki}}{dt} = b_{ki} - \lambda_{1ki} I_{ki} S_{1i} / N_{1i} - \lambda_{2ki} I_{ki} S_{2i} / N_{2i} - (\gamma_{ki} + \beta_{ki}) I_{ki} - f_{ki} I_{ki} + \sum\limits_{j=1, j\neq i}^{2} \omega_{kji} I_{kj} - \sum\limits_{j=1, j\neq i}^{2} \omega_{kij} I_{ki} \\ \frac{dS_{ki}}{dt} = \lambda_{1ki} I_{ki} S_{1i} / N_{1i} + \lambda_{2ki} I_{ki} S_{2i} / N_{2i} - \alpha_{ki} S_{ki} - f_{ki} S_{ki} + \sum\limits_{j=1, j\neq i}^{2} \omega_{kji} S_{kj} - \sum\limits_{j=1, j\neq i}^{2} \omega_{kij} S_{ki} \\ \frac{dR_{ki}}{dt} = \alpha_{ki} S_{ki} + \gamma_{ki} I_{ki} - f_{ki} R_{ki} + \sum\limits_{j=1, j\neq i}^{2} \omega_{kji} R_{kj} - \sum\limits_{j=1, j\neq i}^{2} \omega_{kij} R_{ki} \\ \frac{dU_{ki}}{dt} = \beta_{ki} I_{ki} - f_{ki} U_{ki} + \sum\limits_{j=1, j\neq i}^{2} \omega_{kji} U_{kj} - \sum\limits_{j=1, j\neq i}^{2} \omega_{kij} U_{ki} \end{cases} \tag{2}$$

In the above model, b_{ki} represents individuals of type k increased in node i everyday, and N_{ki} reflects the total number of type k in node k, obviously, $N_{ki} = I_{ki} + S_{ki} + R_{ki} + U_{ki}$. Besides, J_{ki} shows individuals of type k in node i within the area J. $\lambda_{1ki} I_{ki} S_{1i} / N_{ki}$ and $\lambda_{2ki} I_{ki} S_{2i} / N_{ki}$ express the number changed into spreaders of age 1 and age 2 in node i everyday, respectively. Write the individuals who forget the rumor within the area J as $f_{ki} J_{ki}$. After rumors spreaders in contact with the other people that knew the rumor, they tend to accept but not spread gossip, the number of changed individuals in this process everyday can be denoted by $\alpha_{ki} S_{ki}$. Besides, When the ignorant in contact with gossip may also produce other two cases, that is, $\gamma_{ki} I_{ki}$ individuals turn into the area where people accept the rumor but do not spread it and $\beta_{ki} I_{ki}$ individuals become those who refuse the rumor. $\sum\limits_{j=1, j\neq i}^{2} \omega_{kji} J_{kj}$ and $\sum\limits_{j=1, j\neq i}^{2} \omega_{kij} J_{ki}$ stand for the individual speed of moving and remove of type k in node i within the area J.

3 The solution of the problem

Similar to infectious disease model, when considering a rumor, the most important thing is the ability to invade to the crowd. Many existed system have a disease-free equilibrium, where individual is not infected by virus. These models usually exist a threshold variable, which is the so-called basic reproductive number R_0, for example, in the $SIRS$ model[10], if $R_0 < 1$, the system is locally asymptotically stable in the disease-free equilibrium, virus do not invade individuals; if $R_0 > 1$, then the system is unstable in the disease-free equilibrium. In a word, on the analysis of the differential equation system, the traditional way is to make a definition to R_0, and analysis the stability of the equilibrium under different conditions($R_0 > 1$, $R_0 = 1$ and $R_0 < 1$).

In this paper, we introduce a new definition method łThe spectral radius of the next generation matrix. Easy to operation, we only consider the area of I and S.

Firstly, The original model can be divided into the difference between two columns, that is,$F = (F_i)$, $V = (V_i)$, where F_i and V_i represent the i row of F and V, respectively. More specifically, F_i represents the speed of the emerging rumor spreader in the region I; $V_i = V_i^- - V_i^+$, V_i^- and V_i^+ represent the speed of individual removing and moving the region i, respectively.

Next, we will solve the Jacobi matrix of the above-mentioned vector. Obviously, the Jacobi matrix F and the Jacobi matrix V represent transmission and transformation, respectively. Their specific expressions are as follows:

$$F = \left[\frac{\partial F_i\left(x^o\right)}{\partial x_j}\right], V = \left[\frac{\partial V_i\left(x^o\right)}{\partial x_j}\right] \tag{3}$$

where x^0 represents the disease-free equilibrium of system, x_j represents the number or proportion of rumor spreader in the region j. Assume the region of uninitiated and rumor spreader are m, we have $j = 1, 2..., m$.

In the end, we will solve the spectral radius of the next generation matrix, namely, the transmission threshold R_0 in the rumor spreader model. Here, the next generation matrix is $\mathbf{FV^{-1}}$, then we obtain the spectral radius of the matrix, that is, $R_0 = \rho\left(\mathbf{FV^{-1}}\right)$.

Before calculate R_0 by utilizing the method of the next generation matrix, we prove the existence and uniqueness of the disease-free equilibrium. In the disease-free equilibrium DFE, because $I_{1i}^0 = N_{1i}^0$, $I_{2i}^0 = N_{2i}^0$ and $S_{1i}^0 = R_{1i}^0 = U_{1i}^0 = S_{2i}^0 = R_{2i}^0 = U_{2i}^0$,so there is an unique solution[11], denoted by $\left[N_{1i}^0 \;\; N_{2i}^0\right]^T$.

For the region of uninitiated and rumor disseminator, we have the following solving process:

$\frac{d}{dt}[I_{11}, I_{12}\ , I_{21},\ I_{22}\ , S_{11}\ , S_{12}\ , S_{21}\ , S_{22}]^T = F_H - V_H$,

where, $$F_H = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \lambda_{111}I_{11}S_{11}/N_{11} + \lambda_{211}I_{11}S_{21}/N_{21} \\ \lambda_{112}I_{12}S_{12}/N_{12} + \lambda_{212}I_{12}S_{22}/N_{22} \\ \lambda_{121}I_{21}S_{11}/N_{11} + \lambda_{221}I_{21}S_{21}/N_{21} \\ \lambda_{122}I_{22}S_{12}/N_{12} + \lambda_{222}I_{22}S_{22}/N_{22} \end{bmatrix}$$

$$\mathbf{V_H} = \begin{bmatrix} -b_{11} + \lambda_{111}I_{11}S_{11}/N_{11} + \lambda_{211}I_{11}S_{21}/N_{21} + (\gamma_{11}+\beta_{11})I_{11} + f_{11}I_{11} - \omega_{121}I_{12} + \omega_{112}I_{11} \\ -b_{12} + \lambda_{112}I_{12}S_{12}/N_{12} + \lambda_{212}I_{12}S_{22}/N_{22} + (\gamma_{12}+\beta_{12})I_{12} + f_{12}I_{12} - \omega_{112}I_{11} + \omega_{121}I_{12} \\ -b_{21} + \lambda_{121}I_{21}S_{11}/N_{11} + \lambda_{221}I_{21}S_{21}/N_{21} + (\gamma_{21}+\beta_{21})I_{21} + f_{21}I_{21} - \omega_{221}I_{22} + \omega_{212}I_{21} \\ -b_{22} + \lambda_{122}I_{22}S_{12}/N_{12} + \lambda_{222}I_{22}S_{22}/N_{22} + (\gamma_{22}+\beta_{22})I_{22} + f_{22}I_{22} - \omega_{212}I_{21} + \omega_{221}I_{22} \\ \alpha_{11}S_{11} + f_{11}S_{11} - \omega_{121}S_{12} + \omega_{112}S_{11} \\ \alpha_{12}S_{12} + f_{12}S_{12} - \omega_{112}S_{11} + \omega_{121}S_{12} \\ \alpha_{21}S_{21} + f_{21}S_{21} - \omega_{221}S_{22} + \omega_{212}S_{21} \\ \alpha_{22}S_{22} + f_{22}S_{22} - \omega_{212}S_{21} + \omega_{221}S_{22} \end{bmatrix}$$

From (3), we can get the Jacobi matrix of the model as:

$$F_H = \begin{bmatrix} 0 & 0 \\ L & \Lambda \end{bmatrix}, \quad V_H = \begin{bmatrix} Y & \Lambda \\ 0 & \oplus_{k=1}^2 X_k \end{bmatrix}.$$

Here we introduce a kind of calculation method of matrix, it can be denoted by $\oplus$, specifically, $A \oplus B = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$.

In the above Jacobi matrix, L, Λ, M_k, X_k, Y are as follows

$$L=\oplus_{k=1}^{2}(\oplus_{i=1}^{2}(\frac{\lambda_{1ki}S_{1i}^0}{N_{1i}^0}+\frac{\lambda_{1ki}S_{1i}^0}{N_{1i}^0})),\ \Lambda=\begin{bmatrix}\frac{\lambda_{111}I_{11}^0}{N_{11}^0} & 0 & \frac{\lambda_{211}I_{11}^0}{N_{21}^0} & 0\\ 0 & \frac{\lambda_{112}I_{12}^0}{N_{12}^0} & 0 & \frac{\lambda_{212}I_{12}^0}{N_{22}^0}\\ \frac{\lambda_{121}I_{21}^0}{N_{11}^0} & 0 & \frac{\lambda_{221}I_{21}^0}{N_{21}^0} & 0\\ 0 & \frac{\lambda_{222}I_{22}^0}{N_{12}^0} & 0 & \frac{\lambda_{222}I_{22}^0}{N_{22}^0}\end{bmatrix},$$

$$M_1=\begin{bmatrix}\gamma_{11}+\beta_{11}+f_{11}+\omega_{112} & -\omega_{121}\\ -\omega_{112} & \gamma_{12}+\beta_{12}+f_{12}+\omega_{121}\end{bmatrix},$$

$$M_2=\begin{bmatrix}\gamma_{21}+\beta_{21}+f_{21}+\omega_{212} & -\omega_{221}\\ -\omega_{212} & \gamma_{22}+\beta_{22}+f_{22}+\omega_{221}\end{bmatrix},$$

$$X_1=\begin{bmatrix}\alpha_{11}+f_{11}+\omega_{112} & -\omega_{121}\\ -\omega_{112} & \alpha_{12}+f_{12}+\omega_{121}\end{bmatrix},$$

$$X_2=\begin{bmatrix}\alpha_{21}+f_{21}+\omega_{212} & -\omega_{221}\\ -\omega_{212} & \alpha_{22}+f_{22}+\omega_{221}\end{bmatrix},$$

$$Y=L+\oplus_{k=1}^{2}M_k\ .$$

Since M_1, M_2, X_1, X_2 are all invertible matrix, we can obtain: $V_H^{-1}=\begin{bmatrix}Y^{-1} & -Y^{-1}\Lambda\oplus_{k=1}^{2}X_k\\ 0 & \oplus_{k=1}^{2}X_k^{-1}\end{bmatrix}$.

Hence , the spectral radius of the next generation matrix can be expressed as

$$\rho\left(F_H V_H^{-1}\right)=\begin{bmatrix}0 & 0\\ LY^{-1} & -L\,Y^{-1}\Lambda\oplus_{k=1}^{2}X_k+\Lambda\ \oplus_{k=1}^{2}X_k{}^{-1}\end{bmatrix}.$$

Furthermore, we can obtain $R_0=\rho\left(F_H V_H^{-1}\right)=\rho(-L\,Y^{-1}\Lambda\oplus_{k=1}^{2}X_k+\Lambda\ \oplus_{k=1}^{2}X_k{}^{-1})$.

4 Parameter analysis

We build a heterogeneous network with one hundred nodes to further study the relationship between R_0 in rumors spread process and each relevant parameter. Suppose that the parameter $f_{2i}, \alpha_{2i}, \mu_{2i}$ are heterogeneous, and for different i, j, at least one in the following inequality is established: $f_{2i}\neq f_{2j}, \alpha_{2i}\neq\alpha_{2j}, \mu_{2i}\neq\mu_{2j}$. Then we can find that $a_1=f_{21}+\gamma_{21}+\beta_{21}$ in the Jacobi matrix. In consideration of the diagonal dominance of X_2, M_2, M_2^{-1}, X_2^{-1} are non-negative matrixs[12].

By calculating, we find that if $\omega_{212}\left(a_2-a_1\right)>\left(a_1c_1-a_2c_2\right)-\left(a_2-a_1\right)\omega_{221}$ (where $a_2=f_{22}+\gamma_{22}+\beta_{22}$ and $c_1=f_{21}+\alpha_{21}+\mu_{21}$ and $c_2=f_{22}+\alpha_{22}+\mu_{22}$), then $\rho(\ -L\,Y^{-1}\Lambda\oplus_{k=1}^{2}X_k+\Lambda\ \oplus_{k=1}^{2}X_k{}^{-1})$ decreases progressively at ω_{212}, otherwise progressive increases.

When $a_1=a_2$, if $c_2>c_1$, then $\rho(\ -L\,Y^{-1}\Lambda\oplus_{k=1}^{2}X_k+\Lambda\ \oplus_{k=1}^{2}X_k{}^{-1})$ decreases progressively at ω_{212}, otherwise progressive increases.

When $a_1\neq a_2$, $\omega_{212}^*:=\frac{a_1c_1-a_2c_2}{a_2-a_1}-\omega_{212}$ is a critical point of $\rho(\ -L\,Y^{-1}\Lambda\oplus_{k=1}^{2}X_k+\Lambda\ \oplus_{k=1}^{2}X_k{}^{-1})$.

When $a_1>a_2$, $\rho(\ -L\,Y^{-1}\Lambda\oplus_{k=1}^{2}X_k+\Lambda\ \oplus_{k=1}^{2}X_k{}^{-1})$ achieves the maximum value at ω_{212}^*.

When $a_1<a_2$, $\rho(\ -L\,Y^{-1}\Lambda\oplus_{k=1}^{2}X_k+\Lambda\ \oplus_{k=1}^{2}X_k{}^{-1})$ achieves the minimum value at ω_{212}^*.

In the process of simulation, we can clearly see that the limit of R_0 in the network with two nodes, three nodes and even one hundred nodes are always unchangeable.

In order to study the effect of rumors spread speed on R_0, We had the program run 100 times, each time keeping the spread speed increasing, while keeping other parameters constant.

Circumstance 1: Make $\beta_{12i}>\beta_{12j}, \beta_{21i}>\beta_{21j}$, when ω_{2ji} is increased, R_0 increases. when ω_{2ij} is increased, R_0 declines.(Figure 1(a) and (b))

Circumstance2: Make , when ω_{2ji} is increased, R_0 decline. when ω_{2ij} is declined, R_0 increase. (Figure 2(a) and (b))

Circumstance3: Make , when ω_{2ji} is increased, R_0 decline. when ω_{2ij} is increased, R_0 increase. (Figure 3(a) and (b))

Circumstance4: Make , when ω_{2ji} is increased, R_0 decline. when ω_{2ij} is increased, R_0 increase. (Figure 4(a) and (b))

Make the the corresponding adjustment on parameters which is less than 1 and above 1. To sum up, the population movement speed under different condition, has a very important role in the spread of rumors and stop.

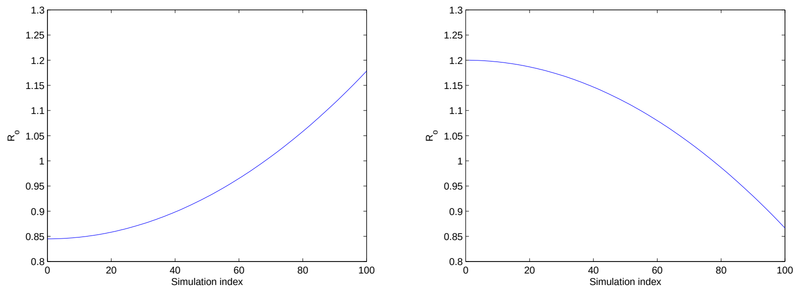

Figure 1: When contact speed is different, the 100 run results of R_0.

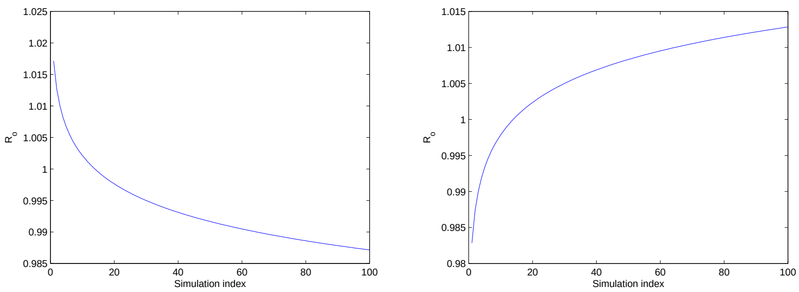

Figure 2: When forgotten speed is different, the 100 run results of R_0.

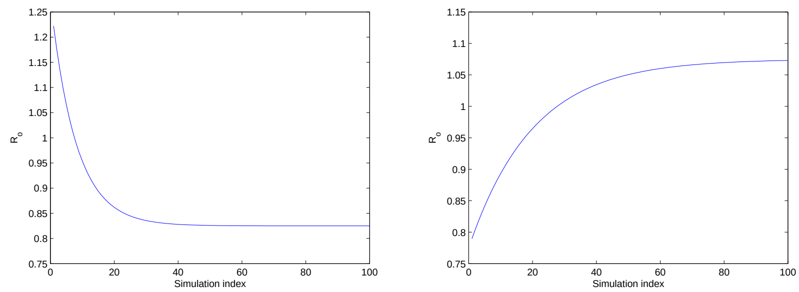

Figure 3: When spread speed is different, the 100 run results of R_0.

5 Conclusions

This paper gives a kind of new method for solving the transmission threshold R_0 and the concrete expression. We simplify the solving process of R_0 by means of the spectral radius of the next generation, and study the related characteristics of R_0 via the analysis of the small matrix. This method is also applicable to similar infectious disease model. We study the different parameters with respect to R_0 through simulation. The network in real life is uniform; the education level of individual, the size of the age, and many external factors will affect the spread of rumors. Considering the cognitive situation of minors to the outside world, we have a more meaningful parameter setting: $\beta_{12i} > \beta_{12j}, \beta_{21i} > \beta_{21j}$. We find that, individuals movement speed only affect the value of R_0 and lead to the value of R_0 floating up and down under the heterogeneous network.

Although the model consider some influencing factors, but it has certain gap with the actual situation in life. In the next study, in order to make the model to be more perfect and realistic, we will try to consider the influence of emotional contagion in such problems, and adjust the scale of parameters reasonably.

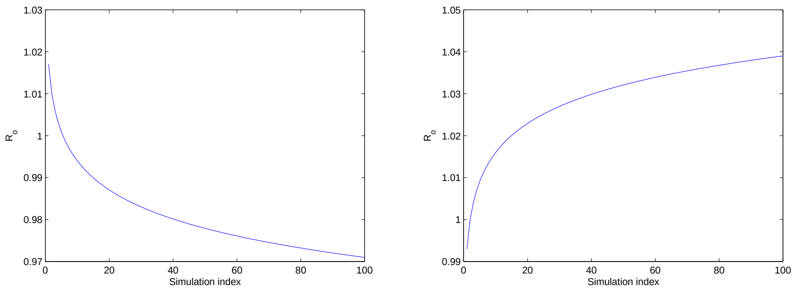

Figure 4: When forgotten speed is different, the 100 run results of R_0.

Acknowledgments

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