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A Two-mode Combined Adaptive Fuzzy Control

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Abstract: This paper proposes a two-mode combined adaptive fuzzy controller, which comprises learning mode and operating mode. The two modes make that calculation is simple and accuracy is higher. In the learning mode, the fuzzy parameters are tuned using simple adaptive law. And at the same time the approximation error estimator uses an adaptive law to ensure the upper bound of the approximation error. In the operating system, the fuzzy parameters are fixed. Only the approximation is updated. Finally, simulation study of a duffin forced oscillating system demonstrates the proposed controller.

Keywords: two-mode; adaptive fuzzy control; direct adaptive control; adaptive law

1 Introduction

In recent years, fuzzy controllers have been successfully applied to many nonlinear problems [1-4]. However, we encounter a lot of problems when we solve the real problem of fuzzy mathematics. Such as the calculations are too complex and the priori knowledge of the plant is unknown [5].

To solve these problems, many experts have proposes a variety of other methods. Stable direct adaptive fuzzy control of plants modeled was first proposed by Wang [1], and it has been widely adopted. The proposed controller comprises an ideal controller and a supervisory control signal. However, the controller cannot guarantee that the tracking error will remain within the predefined region. To overcome these problems, Lee and Zak proposed a high-gain feedback controller to generate the supervisory signal [6]. Despite of these developments, the supervisory signal is also unable to guarantee the smallest possible tracking error.

Then, many scholars used a two-mode method in adaptive fuzzy system. The method can make calculation is simple and accuracy is higher. In [7], a two-mode fuzzy controller is designed, it can achieve the desired control performance by adjusting the controller parameters. A two-mode controller of PI and PD is proposed in [8]. In the paper, state variable is replaced by feedback error signal of designing the input of the system. This method can reduce the reliance on the priori knowledge about the plant, and it can be better to solve the uncertainty of the nonlinear system. In [9], the two-mode method is applied in photonic Crystal Fiber. The two-mode method is also applied in other fields [10-12].

In this paper, the two-mode method is first used for combined adaptive fuzzy system. Then we design a combined adaptive fuzzy controller, the fuzzy controller would be better since it has two modes: learning mode and operating mode. The use of the two modes makes our fuzzy controller has a stronger leaning ability, while the computational efficiency is higher. In the learning mode, the fuzzy parameters are tuned using simple adaptive law. The main difference from [13-15] is that our proposed adaptive law also allows us to set explicit bounds on the estimated bounds. And at the same time the approximation error estimator uses an adaptive law to ensure the upper bound of the approximation error. In the operating system, the fuzzy parameters are fixed. Only the approximation is updated.

In this paper, section 2 describes the structure of the used fuzzy system. The proposed adaptive fuzzy system is given in section 3. Section 4 performs the simulation example. Finally, some conclusions are given in section 5.

2 Stable adaptive fuzzy control

2.1 Plant model and the ideal control law

In this paper, we will consider a class of SISO nonlinear system, which can be represented in the controllable form

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$$\dot{x_1} = x_2, \quad \dot{x_2} = x_3, \quad \cdots, \quad \dot{x_n} = f(x) + bu, \quad y = x_1 \tag{1}$$

where u is the control input, y is the output, $f(x)$ is unknown continuous function, b is positive constant. $x = (x_1, x_2, \cdots, x_n)^T$ is the state vector of the system, which is assumed available for measurement.

The objective is to design a control law that:

(1) derives the output to track the reference signal and guarantees that the tracking error is ultimately bounded;

(2) guarantees that state variables and control signals are bounded.

To achieve these goals, two assumptions are made:

(1)The tracking signals r and its derivative $r^{(i)}$ are bounded ,$i = 1, 2, \cdots, n$. This condition guarantees that the ideal control law is bounded.

(2)The upper and lower bounds of $f(x)$, $\underline{f} \leq f(x) \leq \overline{f}$ is known.. This assumption ensures that the fuzzy logic systems used to approximate $f(x)$ is bounded in corresponding regions.

Suppose $f(x)$ is known. Then, based on the theory of feedback linearization, the ideal control law can be expressed as

$$u^* = \frac{1}{b}[-f(x) - Ke - r^n], \tag{2}$$

where $E = (e, \dot{e}, \cdots, e^{n-1})$,$K = (k_1, k_2, \cdots, k_n$, and r^n is the n th derivation of reference error model is

$$\dot{e} = (A - bK)e, \tag{3}$$

where The stability of the closed systemEq.(3)can be guaranteed, if given any symmetric positive definite matrix Q , there exists a symmetric positive definite P that solves the Lyapunov equation [16].

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & & & \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & b \end{bmatrix}, \tag{4}$$

$$\Lambda^T P + P\Lambda = -Q, \tag{5}$$

where

$$\Lambda = A - bK.$$

2.2 Adaptive fuzzy controller

Here, the control signal of a stable combined adaptive fuzzy controller is

$$u = u_c + u_s, \tag{6}$$

$$u_c = \alpha u_I + (1 - \alpha)u_d, \tag{7}$$

$$u_I = -(\hat{f}(x) + \tilde{f}(x|\theta_I)) + K^T + r^n, \tag{8}$$

$$u_D = \theta_D^T \xi(x), \tag{9}$$

where u_c is the approximation of the ideal control law and u_s is the supervisory control signal, $\hat{f}(x) + \tilde{f}(x|\theta_I)$ is the optimal estimation of $f(x)$. In the paper, the final output value is

$$y(x) = \frac{\sum_{l=1}^{M} y^l (\prod_{i=1}^{n} \mu_{F_i^l}(x_i))}{\sum_{l=1}^{M} (\prod_{i=1}^{n} \mu_{F_i^l}(x_i))}, \tag{10}$$

where $y(x)$ is the output of the fuzzy system, x is the input vector, $\mu_{F_i^l(x_i)}$ is the membership functions for $i = 1, 2, \cdots, n$ and$l = 1, 2, \cdots, M$.

When the antecedent membership functions $\mu_{F_i^l(x_i)}$ are fixed and y^l is adjustable, Eq.(10) can be written as a linear-in-the-parameter equation:

$$y(x) = \theta^T \xi(x), \tag{11}$$

where $\theta = (y^1, y^2, \cdots, y^M)^T$, $\xi(x) = (\xi^1, \xi^2, \cdots, \xi^M)^T$

The fuzzy system defined in Eq.(10) can be used to approximate any continuous function ,because it is universal approximator [17]. As fuzzy systems are universal approximates.

Hencethe approximation error w is given by

$$w = \alpha[\hat{f}(x) + \tilde{f}(x|\theta_I) - f(x)] + (1-\alpha)[u^* - u_D(x|\theta_D)]. \tag{12}$$

In practice, the approximation error given by Eq.(11) will never zero. Consequently, a supervisory control signal is needed to guarantee the tracking error is uniformly ultimately bounded. The supervisory control u_s is constructed as the following type

$$u_s = Lsgn(e^T Pb_c)[|u_c| + \frac{1}{b_L}(f^U + |y_m^{(n)}| + |k^T e|)], \tag{13}$$

where f^U is the upper bound of f, $I = 1$ when $V_e \geq \bar{V}$, $I = 0$ otherwise and E_0 is a predefined small error. Thus, the supervisory control u_s can be implemented, and it can ensure that the Lyapunov stability condition is satisfied even in the worst-case scenario. Because of the above issues with currently existing adaptive fuzzy controllers, this paper proposes a stable combined adaptive fuzzy controller that is simple to design and achieve better tracking performances.

3 Main results

3.1 Design of the adaptive fuzzy systems

To present the method, our control law includes an approximation of the ideal control signal u_c and a supervisory control signalu_s, which can be proposed as

$$u = u_c + u_s = \alpha[-(\hat{f}(x) + \tilde{f}(x|\theta_I)) + K^T e + r^n] + (1-\alpha)\theta_D^T \xi(x)) - \hat{\omega}. \tag{14}$$

Then, the approximation error ω can be defined

$$w = \alpha[\hat{f}(x) + \tilde{f}(x|\theta_I) - f(x] + (1-\alpha)[u^* - u_D(x|\theta_D)]. \tag{15}$$

Theorem 1 *Consider the system(1), if it satisfies*

$< 1 >$ the controller in Eq.(2) ;

$< 2 >$ the rules in $R^{(l)}$;

$< 3 >$ the adaptive laws

$$\dot{\theta}_I = -r_1 e P b \xi(x) \tag{16}$$

$$\dot{\theta}_D = -r_2 e P b \zeta(x) \tag{17}$$

$$\dot{\hat{\omega}} = -r_\omega e P b \tag{18}$$

then:

(1) The tracking error $e = r - y$ is uniformly ultimately bounded. $\hat{\omega}\theta_I$,θ_D are bounded.

(2) The tracking error e and the parameter error vectors φ_I , φ_D, φ_ω will converge to

$$||e(t)|| \leq \sqrt{\frac{2V(0)e^{-\mu t}}{\lambda_{Q_{\min}}}} \tag{19}$$

$$||\varphi_I|| \leq \sqrt{2r_1 V(0)e^{-\mu t}} \tag{20}$$

$$||\varphi_D|| \leq \sqrt{2r_2 V(0)e^{-\mu t}} \tag{21}$$

$$||\phi_\omega|| \leq \sqrt{2r_\omega V(0)e^{-\mu t}} \tag{22}$$

where

$$V(0) = \frac{1}{2}e(0)^T Pe(0) + \frac{\alpha}{2r_1}\varphi_I(0)^T\varphi_I(0) + \frac{1-\alpha}{2r_2}\varphi_D(0)^T\varphi_D(0) + \frac{1}{2r_\omega}\phi_\omega(0)^2 \tag{23}$$

$$\mu = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} \tag{24}$$

(3) The control signal $\phi_w = -\int_{t=0}^{0} r_w e^T Pbdt$ *is also bounded .*

$$u = \frac{1}{b}(|f| + ||K||\sqrt{\frac{2V(0)e^{-\mu t}}{\lambda_{Q_{\min}}}} + r^{(n)} + \bar{\phi}_w) \tag{25}$$

Proof. As the fuzzy systems are universal approximators, there exist vector such that

$$\theta_I^* = \arg\min_{\prod_{i=1}^n}[\sup_{x\in R^n}|\hat{f}(x) + \tilde{f}(x|\theta_I) - f(x)|] \tag{26}$$

$$\theta_D^* = \arg\min_{\prod_{i=1}^n}[\sup_{x\in R^n}|u^* - u_D(x|\theta_D)|] \tag{27}$$

The Lyapunov design approach will be used to prove the stability of the system. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}e^T Pe + \frac{\alpha}{2r_1}\varphi_I^T\varphi_I + \frac{1-\alpha}{2r_2}\varphi_D^T\varphi_D + \frac{1}{2r_\omega}\phi_\omega^2 \tag{28}$$

In which $\varphi_I = {\theta_I}^* - \theta_I$, $\varphi_D = {\theta_D}^* - \theta_D$,$\phi_\omega = \hat{\omega} - \omega$.

Its derivative along the solution is

$$\begin{aligned}
\dot{V} &= \frac{\partial}{\partial t}(\frac{1}{2}e^T Pe) + \frac{\partial}{\partial t}(\frac{\alpha}{2r_1}\varphi_I^T\varphi_I) + \frac{\partial}{\partial t}(\frac{1-\alpha}{2r_2}\varphi_D^T\varphi_D) + \frac{\partial}{\partial t}(\frac{1}{2r_\omega}\phi_\omega^T\phi_\omega) \\
&= \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T P\dot{e} + \frac{\alpha}{r_1}\dot{\varphi}_I^T\varphi_I + \frac{1-\alpha}{r_2}\dot{\varphi}_D^T\varphi_D + \frac{1}{r_\omega}\dot{\phi}_\omega^T\phi_\omega \\
&= \frac{1}{2}[\Lambda e + b\alpha\varphi_I^T\xi(x) + b(1-\alpha)\varphi_D^T\zeta(x) - b_c u_s + b_c\omega]^T Pe + \frac{1}{2}e^T P[\Lambda e + b\alpha\varphi_I^T\xi(x) + \\
&\quad b\alpha(1-\alpha)\varphi_D^T\zeta(x) - b_c u_s + b_c\omega] + \frac{\alpha}{r_1}\dot{\varphi}_I^T\varphi_I + \frac{1-\alpha}{r_2}\dot{\varphi}_D^T\varphi_D + \frac{1}{r_\omega}\dot{\phi}_\omega^T\phi_\omega \\
&= \frac{1}{2}e^T\Lambda Pe + \frac{1}{2}b_c^T[\alpha\varphi_I^T\xi(x) + (1-\alpha)\varphi_D^T\zeta(x) - u_s + \omega]Pe + \frac{1}{2}e^T P\Lambda e \\
&\quad + \frac{1}{2}e^T Pb_c[\alpha\varphi_I^T\xi(x) + (1-\alpha)\varphi_D^T\zeta(x) - u_s + \omega] + \frac{\alpha}{r_1}\dot{\varphi}_I^T\varphi_I + \frac{1-\alpha}{r_2}\dot{\varphi}_D^T\varphi_D + \frac{1}{r_\omega}\dot{\phi}_\omega^T\phi_\omega
\end{aligned}$$

where r_1, r_2, r_ω are positive scalars and Q is an $n \times n$ positive definite matrix.

Using the facts that $(\wedge e)^T = e^T\wedge^T$, $e^T Pb = b^T Pe$, and $\wedge^T P + P\wedge^T = -Q$, we have:

$$\begin{aligned}
\dot{V} &= e^T Pb_c[\Lambda e + \alpha\varphi_I^T\xi(x) + (1-\alpha)\varphi_D^T\zeta(x) - u_s + \omega] + \frac{\alpha}{r_1}\dot{\varphi}_I^T\varphi_I + \frac{1-\alpha}{r_2}\dot{\varphi}_D^T\varphi_D + \frac{1}{r_\omega}\dot{\phi}_\omega^T\phi_\omega \\
&= -\frac{1}{2}e^T Qe + \alpha e^T Pb_c\varphi_I^T\xi(x) + (1-\alpha)e^T Pb_c\varphi_D^T\zeta(x) + \frac{\alpha}{r_1}\dot{\varphi}_I^T\varphi_I + \frac{1-\alpha}{r_2}\dot{\varphi}_D^T\varphi_D + \frac{1}{r_\omega}\dot{\phi}_\omega^T\phi_\omega - e^T Pb_c u_s \\
&\quad - e^T Pb_c\omega \\
&= -\frac{1}{2}E^T Qe + \alpha\varphi_I^T[e^T Pb\xi(x) + \frac{\dot{\varphi}_I}{r_1}] + (1-\alpha)[e^T Pb\zeta(x) + \frac{\dot{\varphi}_D}{r_2}] + \phi_\omega[e^T Pb + \frac{\dot{\phi}_\omega}{r_\omega}] \\
&\quad + (1-\alpha)[e^T Pb\xi(x) + \frac{\dot{\varphi}_D}{r_2}] + \phi_\omega[e^T Pb + \frac{\dot{\phi}_\omega}{r_\omega}]
\end{aligned}$$

Eq.(24) reduces to $\dot{V} = -\frac{1}{2}e^T Qe$ when

$$\dot{\varphi}_I = -r_1 e^T Pb\xi(x) \tag{29}$$

$$\dot{\varphi}_D = -r_2 e^T Pb\xi(x) \tag{30}$$

$$\dot{\phi}_\omega = -r_\omega e^T Pb \tag{31}$$

Which are equivalent to the adaptive control laws defined

$$\dot{\theta}_I = r_1 e^T Pb\xi(x) \tag{32}$$

$$\dot{\theta}_D = r_2 e^T Pb\zeta(x) \tag{33}$$

As ϕ_ω=$\hat{\omega} - \omega$, $\dot{\phi}_\omega = \dot{\hat{\omega}} - \dot{\omega}$. Suppose the value of $\phi_\omega = -r_\omega e^T Pb$ in (31) is made large enough that $\dot{\phi}_\omega \gg \dot{\omega}$ by considering a large r_ω. Then , $\dot{\omega}$ can be neglected, and the following can be derived

$$\dot{\hat{\omega}} = -r_\omega e^T Pb \tag{34}$$

Hence, the supervisory controller provides an integral action that drive the bounded tracking error zero. Meanwhile, $\dot{V}$ is negative definite, so the adaptive controller can ensure that e is bounded.

For the formula $\dot{V} = -\frac{1}{2}e^T QE$, we have

$$\dot{V} \leq -\lambda_{\min}(Q)||e||^2 \leq -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}e^T PE = -\mu V \quad . \tag{35}$$

where $\mu = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$

Multiplying Eq.(28) and integrating over[0, t] leads to

$$0 \leq V(t) \leq V(0)e^{-\mu t}.$$

where $V(0) = \frac{1}{2}e(0)^T Pe(0) + \frac{1}{2r_\theta}\varphi_\theta(0)^T\varphi_\theta(0) + \frac{1}{2r_\omega}\phi_\omega(0)^2$

Using the fact that

$$\frac{1}{2}\lambda_{Q_{\min}}||e(t)||^2 \leq \frac{1}{2}e(t)^T Qe(t) \leq V(t)$$

$$\frac{1}{r_1}||\varphi_I||^2 \leq V(t)$$

$$\frac{1}{r_2}||\varphi_D||^2 \leq V(t)$$

$$\frac{1}{r_\omega}||\phi_\omega||^2 \leq V(t)$$

Then, we can have

$$||e(t)|| \leq \sqrt{\frac{2V(0)e^{-\mu t}}{\lambda_{Q_{\min}}}}$$

$$||\varphi_I|| \leq \sqrt{2r_1 V(0)e^{-\mu t}}$$

$$||\varphi_D|| \leq \sqrt{2r_2 V(0)e^{-\mu t}}$$

$$||\phi_\omega|| \leq \sqrt{2r_\omega V(0)e^{-\mu t}}$$

As the tracking error is bounded, the upper bound of the control signal can be found by replacing the parameters in its denominator with their lower bounds and replacing the terms its numerator with their upper bounds:

$$|u| < \frac{1}{b}(|f| + ||K||\sqrt{\frac{2V(0)e^{-\mu t}}{\lambda_{\min}}} + r^{(n)})$$

■

3.2 Extension to two-mode operating system

Here, two-mode operating system is applied to adaptive fuzzy control. The learning mode is designed to produce better performance, and the operating mode aims to reduce the computational cost. Then, the two-mode operating system combines the advantages of each modes. Thereby the producing systems are more applicable to practical problems.

The two modes can be defined as follows:

The learning mode : θ_I^T, θ_D^T, which is uses to approximate θ_I, θ_D, is continuously updated to reduce the approximation error. The supervisory control signal u_s uses to ensure the stability of the whole system. The main effect of this mode is to reduce the affect of the unknown priori knowledge and thus minimize the tracking error.

The operating mode : In the operating mode, the parameters of the θ_I^T, θ_D^T is fixed, and supervisory signal u_s is kept running. This mode is designed to reduce the computational cost.

In this paper, it is sufficient to establish that the supervisory signal u_s to guarantee stability of the proposed two-mode adaptive fuzzy system.

4 Case study

Consider control of the duffin forced oscillating system. It can be represented by the following state equations:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos(t) + u(t)$$

Without control, i.e. $u(t) \equiv 0$, the system is a chaos system. Our task is to control x_1 to track reference trajectory $y_m = \sin(t)$. The proposed scheme is compared against a two-mode adaptive fuzzy controller that can be characterized as

$$u(t) = u_c + u_s == \alpha[-(\hat{f}(x) + \tilde{f}(x|\theta_I)) + K^T e + r^n] + (1-\alpha)\theta_D^T \zeta \text{x})\text{-}\hat{\omega}$$

with adaptive law

$$\dot{\varphi}_I = -r_1 e^T P b \xi(x)$$
$$\dot{\varphi}_D = -r_2 e^T P b \zeta(x)$$
$$\dot{\hat{\omega}} = -r_\omega e^T P b$$

The design parameters are chosen as $r_1 = 2$, $r_2 = 2$ $r_w = 200$, α=0.5.

The values of the remaining controller parameters are the same as those of the proposed adaptive fuzzy system, i.e.

$$K = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$$

where $x_1(0) = x_2(0) = 2$.

The membership functions are chosen as

$$\mu_1 = 1/(1 + (\exp(5(x+2))) \quad \mu_2 = \exp(-(x+0.5)^2)$$
$$\mu_3 = \exp(-(x-0.5)^2) \quad \mu_4 = 1/(1 + (\exp(5(x-2)))$$

Then, we can get the following pictures.From the Fig 1 and Fig.2, we know that the state variable $x_1(t)$ approach tracking signal $y = \sin(t)$ and $x_2(t)$ gets close to $\cos(t)$. Then, we achieve that the system is stable at last.

5 Conclusion

In this paper, a two-mode combined fuzzy controller is constructed with approximation error estimator. The main advantages of the controller are the learning and its computational efficiency. Mathematically, we prove the stability of the closed-loop system in both learning and operating modes. Moreover, we present the methods to tune the controller's parameters to improve the controller performance.

Finally, application is presented to demonstrate the controller. The applications shows that the controller successfully control the system to track desired reference signals with good tracking error in both learning and operating modes.

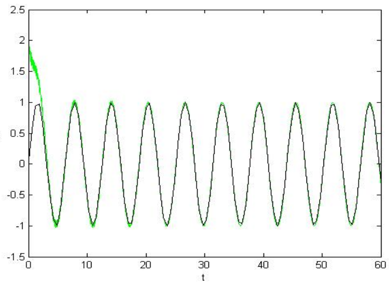

Figure 1: The tracking signal $y = \sin(t)$and state variable $x_1(t)$.

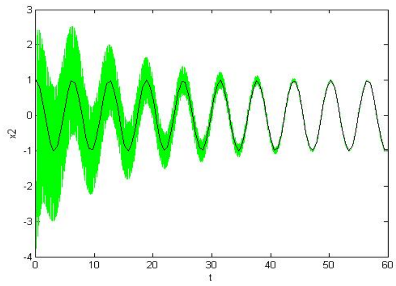

Figure 2: The tracking signal $\cos(t)$and state variable $x_2(t)$.

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