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Risk Propagation Model of Inter-enterprise on Scale-free Networks

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Abstract: Based on the network structure of the financial system and combining with the thought of infectious disease modeling, this paper build a model of inter-enterprise spreading risk on the scale-free network. Through the consequence of the risk-spreading basic reproduction number (R_0) and the analysis of the stability of the equilibrium point ,this paper draw the results that if $R_0 < 1$, the risk on the network will disappear; but if $R_0 > 1$,the risk will remain. By the numerical simulations, this paper analyses the influencing factors of the inter-enterprise risk transmission and then talks about the impact of these factors on the R_0, thus find that the ways of reducing the rate of risk transmission, increasing the risk elimination rate, shorting the cycle of risk appeared and reducing the degree of heterogeneity of the network can help eliminate the inter-enterprise risk.

Keywords: scale-free networks; inter-enterprise risk transmission; basic reproduction number; global stability

1 Introduction

In recent years, scholars have found that the risk spreading in the financial system is similar to the diseases spreading in the crowd, and they have begun try to study the risk spreading in the financial system based on the epidemiology, biology and mathematics. For instance, Xie et al.[1] established the mathematical model that affects the enterprises in the financial crisis and tried to give an economic explanation as well as a solution to the model. Based on the traditional SIR model, Mi et al.[2] built the SEIRS distress diffusion differential equation for enterprise group, studied on the distress diffusion mechanism inside enterprise group, and provided theoretic base for the detecting, prevention and control of distress in enterprise group. Li et al.[3] also proposed the stochastic models of contagion of banking risk based on the SI model, and the results show that the transmission of banking risk can be controlled by decreasing the spread rate of banking risk, reducing the inter-bank correlation degree or increasing the treat rate. However, the application of infection model in the research of risk transmission of financial system is still in its infancy stage, few factors considered by the models, and the network structure was not taken into account.

With the maturing of complex network theory, epidemic model based on complex network has made great progress. On one hand, Zhu et al.[4] proposed a generalized epidemic model on complex heterogeneous networks, and presented the mathematical analysis of the epidemic dynamic. And a modified SIS model with an infective vector on complex networks is proposed and analyzed by Wang et al.[5], which incorporates some infections diseases that are not only transmitted by a vector, but also spread by direct contacts between human beings. On the other hand, the time delayed has also been taken into account, considering the time needed in the recovery of the disease. For example, Techenche et al.[6] showed that time delayed can reduce the transmission threshold for the SIR model. Then, Xu et al.[7] presented a time delayed SIS model on complex network, and found that for small-word network, the epidemic threshold and the delay time have a power-law relation. In this paper, we present a risk propagation model of inter-enterprise with time delayed. Then we analyse the global dynamics and the influencing factors of the risk transmission.

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The rest of this paper is organized as follows. Section 2 describes our model in detail. Then the global dynamic of the model are studied in sections 3. In section 4, we perform some numerical simulations and have some discussions on sensitivity of the risk-spreading basic reproduction number. We finally conclude the paper in section 5.

2 Model formulation

According to the theory of inter-enterprise risk transmission, epidemiology and the topological properties of scale-free networks, we make the following assumptions:

(1) Considering the majority of real networks is scale free, this paper will study the inter-enterprise risk transmission in the scale-free network. The nodes of network represent enterprises and the links between enterprises means the possible correlation of fund technology, property rights and so on. And the situation of enterprise removed due to insolvency or new adding will not be included here. That is to say, the total enterprise N keeps constant.

(2) In the network, enterprises are divided into four types according the states they are in: susceptible(risk uninfected), exposed(latent, having infected risk but not showing up, and having the ability to spread it), infected(having infected and is showing the risk, also having the ability to spread it), recovered(immune, risk eliminated and having some resistance), and we let S, E, I and R to denote the four types, respectively. Taking into account the heterogeneity induced by the enterprises with different degree k, all the enterprises are divided into four groups with densities at time step t being $S_k(t)$,$E_k(t)$,$I_k(t)$ and $R_k(t)$, and the enterprises in each groups have the same degree k. Also the total enterprise number is constant, i.e.$S_k(t) + E_k(t) + I_k(t) + R_k(t) = 1$, then we can easily obtain $0 \leq S_k(t), E_k(t), I_k(t), R_k(t) \leq 1$.

(3) Transmission among the four types in the network is governed by the following rules: First, if a susceptible enterprise(S) has a correlation of fund, technology or property rights with a latent enterprise(E) or an infected enterprise(I), it can acquire risk from them with the probability of ρ_1 and ρ_2 respectively. Here the parameter $\rho_1(\rho_2)$ means the risk transmission rate of latent (infected) enterprise. Because of enterprise differences, the probability of showing the risk will be α, while the probability of not showing will be $1 - \alpha$. Second, the latent enterprise (E) will no longer stay in the risk latency with probability α, and if the managers can eliminate the risk timely, it will turn into the state susceptible with probability β, otherwise, it will turn into the state infected with probability $1 - \beta$. Third, the infected enterprise(I) should go through a certain period from showing risk to eliminating it, i.e., if an enterprise is infected and show the risk at any time step t, it will be in the state of infection until the time step $t + T + 1$ and then the risk will be eliminated with the probability $b(I_{k,0}, \cdots, I_{k,T}$ denote the infective enterprises at different stages and $I_k(t) = \sum_{f=0}^{T} I_{k,f}(t))$. And after the enterprise eliminate the risk, it will turn into susceptible with probability γ or get some risk resisting ability with probability $1-\gamma$. Finally, due to the weakening of risk prevention awareness, the recession of enterprises own strength and other reasons, the immune enterprise will lose the ability to resist risk with the probability c. From a practical perspective, $\rho_1, \rho_2, a, b, c, \alpha, \beta$ and γ are constants with $0 \leq \rho_1, \rho_2, a, b, c, \alpha, \beta, \gamma \leq 1$ and $T \in N$.(The risk transmission sketch is described in Figure 1) To study the spreading of the inter-enterprise risk, we write the dynamic mean-field reaction rate

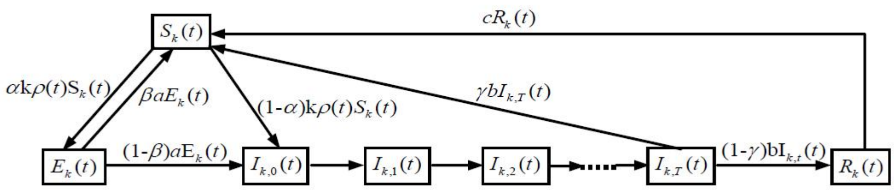

Figure 1: Flow chart of the inter-enterprise risk transmission

equations on scale-free networks as follows:

$$\begin{cases} \dfrac{dS_k(t)}{dt} = -kS_k(t)(\rho_1\Theta_1(t)+\rho_2\Theta_2(t)) + \beta aE_k(t) + \gamma bI_{k,T}(t) + cR_k(t), \\ \dfrac{dE_k(t)}{dt} = \alpha kS_k(t)(\rho_1\Theta_1(t)+\rho_2\Theta_2(t)) - aE_k(t), \\ \dfrac{dI_{k,0}(t)}{dt} = (1-\alpha)kS_k(t)(\rho_1\Theta_1(t)+\rho_2\Theta_2(t)) + (1-\beta)aE_k(t) - I_{k,0}(t), \\ \dfrac{dI_{k,1}(t)}{dt} = -I_{k,1}(t) + I_{k,0}(t), \\ \dfrac{dI_{k,2}(t)}{dt} = -I_{k,2}(t) + I_{k,1}(t), \\ \qquad \vdots \\ \dfrac{dI_{k,T}(t)}{dt} = -bI_{k,T}(t) + I_{k,T-1}(t), \\ \dfrac{dR_k(t)}{dt} = (1-\gamma)bI_{k,T}(t) - cR_k(t). \end{cases} \tag{1}$$

where $k = 1, 2, \cdots, n$, $\Theta_1(t)(\Theta_2(t))$is the probability of a link points to an latent(infected) enterprise. Here, $P(k)$ is the probability that an enterprise has k contacts and $\langle k\rangle$ is the average degree of the network, i.e.$\langle k\rangle = \Sigma_{i=1}^n iP(i)$, so

$$\begin{aligned} \Theta_1(t) &= \Sigma_{i=1}^n P(i|k)E_i(t) = \frac{\Sigma_{i=1}^n iP(i)E_i(t)}{\langle k\rangle}, \\ \Theta_2(t) &= \Sigma_{i=1}^n P(i|k)\sum_{f=0}^{T} I_{i,f}(t) = \frac{\Sigma_{i=1}^n P(i|k)\sum_{f=0}^{T} I_{i,f}(t)}{\langle k\rangle}. \end{aligned}$$

Let $\rho(t) = \rho_1\Theta_1(t) + \rho_2\Theta_2(t)$ for short. From a practical perspective, the initial conditions for (1) satisfy

$$\begin{cases} \rho(0) > 0, \\ 0 \le S_k(0), E_k(0), I_{k,0}(0), \cdots I_{k,T}(0), R_k(0) \le 1, \\ S_k(0) + E_k(0) + \Sigma_{f=0}^T I_{k,f}(0) + R_k(0) = 1. \end{cases} \tag{2}$$

Since $S_k(t) + E_k(t) + I_k(t) + R_k(t) = 1$, we obtain $0 \le S_k(t), E_k(t), I_{k,0}(t), \cdots I_{k,T}(t), R_k(t) \le 1, 0 \le \Sigma_{f=0}^T I_{k,f}(t) \le 1$ and $\rho(t) \le \rho_1 + \rho_2$. Substituting $R_k(t) = 1 - S_k(t) - E_k(t) - \Sigma_{f=0}^T I_{k,f}(t)$ into (1), then (1) is equivalent to the following model:

$$\begin{cases} \dfrac{dS_k(t)}{dt} = -kS_k(t)\rho(t) + \beta aE_k(t) + \gamma bI_{k,T}(t) + c1 - S_k(t) - E_k(t) - \Sigma_{f=0}^T I_{k,f}(t), \\ \dfrac{dE_k(t)}{dt} = \alpha kS_k(t)\rho(t) - aE_k(t), \\ \dfrac{dI_{k,0}(t)}{dt} = (1-\alpha)kS_k(t)\rho(t) + (1-\beta)aE_k(t) - I_{k,0}(t), \\ \dfrac{dI_{k,1}(t)}{dt} = -I_{k,1}(t) + I_{k,0}(t), \\ \dfrac{dI_{k,2}(t)}{dt} = -I_{k,2}(t) + I_{k,1}(t), \\ \qquad \vdots \\ \dfrac{dI_{k,T}(t)}{dt} = -bI_{k,T}(t) + I_{k,T-1}(t), \end{cases} \tag{3}$$

3 Global dynamics analysis of risk propagation model of inter-enterprise

Based on the model above, we will analyse the global dynamics in order to know weather the risk will disappear or remain and how to control the spreading of the risk in this part.

Now, from $\frac{dS_k(t)}{dt} = \frac{dE_k(t)}{dt} = \frac{dI_{k,0}(t)}{dt} = \cdots = \frac{dI_{k,T}(t)}{dt} = \frac{dR_k(t)}{dt} = 0$, a direct calculation yields

$$\begin{cases} S_k(t) = \dfrac{abc}{A_k}, \\ E_k(t) = \dfrac{bc\alpha k\rho(t)}{A_k}, \\ I_{k,f}(t) = \dfrac{ackb(1-\alpha\beta)\rho(t)}{A_k}, (f = 0, 1, 2, \cdots, T-1) \\ I_{k,T}(t) = \dfrac{ack(1-\alpha\beta)\rho(t)}{A_k}, \\ R_k(t) = \dfrac{abk(1-\alpha\beta)(1-\gamma)\rho(t)}{A_k}. \end{cases} \tag{4}$$

where

$$A_k = bc(a + \alpha k\rho) + ak\rho(1-\alpha\beta)[b(1-\gamma) + (Tb+1)c].$$

Obviously, $\rho(t) = 0$ satisfies (4). Hence, $S_k(t) = 1, E_k(t) = I_{k,0}(t) = \cdots = I_{k,T}(t) = R_k(t) = 0$ is an equilibrium of (1),which is called the risk-free equilibrium. Then, substitute $E_k(t), I_{k,0}(t), \cdots, I_{k,T}(t)$ of (4) into $\rho(t)$, we have $\rho(t)f(\rho(t)) = 0$, where

$$f(\rho(t)) = 1 - \frac{bc\alpha\rho_1 + ac(1-\alpha\beta)(Tb+1)\rho_2}{\langle k\rangle}\sum_{i=1}^{n}\frac{i^2P(i)}{A_i}.$$

Since $f'(\rho(t)) < 0$ and $f(\rho_1 + \rho_2) > 0$, the equation $f(\rho(t)) = 0$ has a unique nonzero solution if and only if $f(0) < 0$,i.e.,$\frac{[b\alpha\rho_1 + a(1-\alpha\beta)(Tb+1)\rho_2]\langle k^2\rangle}{ab\langle k\rangle} > 1$, where $\langle k^2\rangle = \sum_{i=1}^{n} i^2P(i)$.

The analysis above yields the following theorem 1.

Theorem 1 *Define*

$$R_0 = \frac{[b\alpha\rho_1 + a(1-\alpha\beta)(Tb+1)\rho_2]\langle k^2\rangle}{ab\langle k\rangle},$$

there always exists a risk-free equilibrium $E_0 = (1, 0, \underbrace{0, \cdots, 0}_{T+1}, 0)$ *for (1), and when* $R_0 > 1$*, (1) has a unique positive equilibrium* $E_1 = (S_k^*, E_k^*, I_{k,0}^*, \cdots, I_{k,T}^*, R_k^*)$ *satisfying (4), where* $k = 1, 2, \cdots, n$.

R_0 in theorem 1 is called the risk-spreading basic reproduction number.

Theorem 2 *(I) when* $R_0 < 1$*, the risk-free equilibrium is globally asymptotically stable;*
(II) When $R_0 > 1$*, the risk of inter-enterprise on the network is permanent, i.e., there exists an* $\varepsilon > 0$*, such that* $\liminf_{t\to\infty}\{S_k(t), E_k(t), I_{k,0}(t), \cdots, I_{k,T}(t)\} \geq \varepsilon$ *for* $k = 1, 2, \cdots, n$*, where* $(S_k(t), E_k(t), I_{k,0}(t), \cdots, I_{k,T}(t))$ *is the any solution of (3), satisfying (2) and at least one of* $E_k(0), I_{k,0}(0), \cdots, I_{k,T}(0)$ *is greater than* 0.

Then, we are in a position to present the main result.

Theorem 3 *Suppose thatis* $(S_k(t), E_k(t), I_{k,0}(t), \cdots, I_{k,T}(t))$ *the solution of (3), satisfying (2) and at least one of* $E_k(0), I_{k,0}(0), \cdots, I_{k,T}(0)$*is greater than* 0*.If* $R_0 > 1$*,then* $\lim_{t\to\infty}\{S_k(t), E_k(t), I_{k,0}(t), \cdots, I_{k,T}(t)\} = (S_k^*, E_k^*, I_{k,0}^*, \cdots, I_{k,T}^*)$*,where* $(S_k^*, E_k^*, I_{k,0}^*, \cdots, I_{k,T}^*)$*is the unique positive equilibrium of (3) and satisfying (4),*$k = 1, 2, \cdots, n$.

We can use the theorem 3.2 and theorem 3.3 in Ref.[4] to certify theorem 2 and theorem 3 above,so we no longer prove here.

4 Sensitivity analysis of risk-spreading basic reproduction number

According to the model and the analysis of the global dynamical, we know that the risk-spreading basic reproduction number determines whether the risk will remain in the enterprise network. If $R_0 < 1$, the risk in the enterprise will finally disappear, otherwise if $R_0 > 1$, the risk will remain and the proportions of susceptible, latent and infected enterprises will reach the unique stationary positive levels eventually. Therefore, in order to control the spread of risk, the sensitivity analysis of R_0 is especially important.

By $R_0 = \frac{[b\alpha\rho_1 + a(1-\alpha\beta)(Tb+1)\rho_2]\langle k^2\rangle}{ab\langle k\rangle}$, we obtain

$$\frac{\partial R_0}{\partial \rho_1} = \frac{\alpha\langle k^2\rangle}{a\langle k\rangle}, \frac{\partial R_0}{\partial \rho_2} = \frac{(1-\alpha\beta)(Tb+1)\langle k^2\rangle}{b\langle k\rangle}, \frac{\partial R_0}{\partial a} = -\frac{\alpha\rho_1\langle k^2\rangle}{a^2\langle k\rangle}, \frac{\partial R_0}{\partial b} = -\frac{(1-\alpha\beta)\rho_2\langle k^2\rangle}{b^2\langle k\rangle},$$

$$\frac{\partial R_0}{\partial \alpha} = \frac{[b\rho_1 - a\beta\rho_2(Tb+1)]\langle k^2\rangle}{ab\langle k\rangle}, \frac{\partial R_0}{\partial \beta} = -\frac{\alpha(Tb+1)\rho_2\langle k^2\rangle}{b\langle k\rangle}, \frac{\partial R_0}{\partial T} = \frac{(1-\alpha\beta)\rho_2\langle k^2\rangle}{\langle k\rangle}.$$

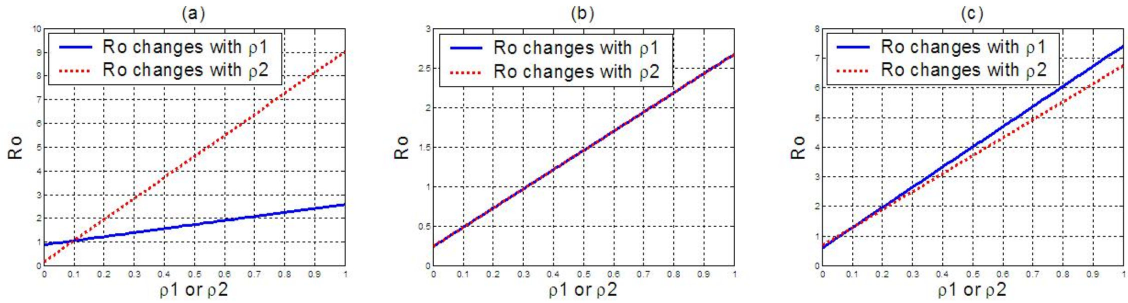

Figure 2: The impact of ρ_1 or ρ_2 on R_0 with different value of $b\alpha - a(1-\alpha\beta)(Tb+1)$. (a) when $a = 0.8, b = 0.7, \alpha = 0.4, \beta = 0.6, \langle k^2\rangle/\langle k\rangle = 3.4, T = 2$ and $\rho_1 = 0.1$ or $\rho_2 = 0.1$. (b) $b\alpha - a(1-\alpha\beta)(Tb+1) = 0$ when $a = 0.8, b = 0.5, \alpha = 0.8, \beta = 0.9375, \langle k^2\rangle/\langle k\rangle = 3.4, T = 2$ and $\rho_1 = 0.1$ or $\rho_2 = 0.1$. (c) $b\alpha - a(1-\alpha\beta)(Tb+1) > 0$ when $a = 0.8, b = 0.7, \alpha = 0.4, \beta = 0.6, \langle k^2\rangle/\langle k\rangle = 3.4, T = 2$ and $\rho_1 = 0.1$ or $\rho_2 = 0.1$.

(1) ρ_1 is the risk transmission rate of latent enterprise, and ρ_2 is the risk transmission rate of infected enterprise. R_0 increases as ρ_1 or ρ_2 increases. In order to control the spread of the risk and make it eliminate eventually, the enterprise managers should make effective prevention strategies to avoid infection risk when a susceptible enterprise has a contact with a latent or infected enterprise, i.e. reduce ρ_1 or ρ_2, so that make $R_0 < 1$. From Figure 2(a), it can be seen that when $b\alpha - a(1-\alpha\beta)(Tb+1) < 0$, ρ_2 works more effectively than ρ_1 when reducing R_0, hence, at this time the susceptible enterprise should be more careful about infected enterprise. In Figure 2(b), we can see when $a(1-\alpha\beta)(Tb+1) = 0$, the influence of ρ_1 and ρ_2 is the same, so we also need to pay more attention to latent and infected enterprises at the same time. However, when $b\alpha - a(1-\alpha\beta)(Tb+1) > 0$, the ρ_2 works less efficiently to reduce R_0 than ρ_1,which is displayed in Figure 2(c). Hence, we should be more careful about latent enterprise at this time.

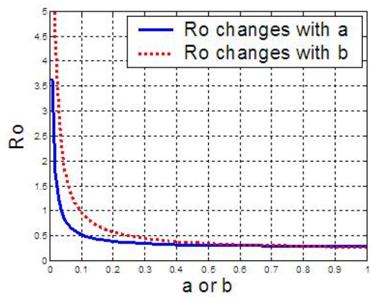

Figure 3: The impact of a or b on R_0 here $\alpha = 0.4, \beta = 0.6, \rho_1 = 0.02, \rho_2 = 0.03, \langle k^2\rangle/\langle k\rangle = 3.4, T = 2$ and $a = 0.8$ or $b = 0.8$.

(2) a is the probability of that latent enterprise no longer stay in this period and b is the probability of that infected enterprise eliminate the risk after risk appearing period. R_0 decreases as a or b decreases, and b works more effectively

than a (see Figure 3). That is to say, long lasting of the latent period especially infected period increase the chance of risk spreading. Hence, once managers find the enterprise infected, they should eliminate such risk as soon as possible, especially for the enterprises which is in the risk appearing period. And the more enterprises that experienced risk appearing period can eliminate risk, the more effective they can control the spread of risk.

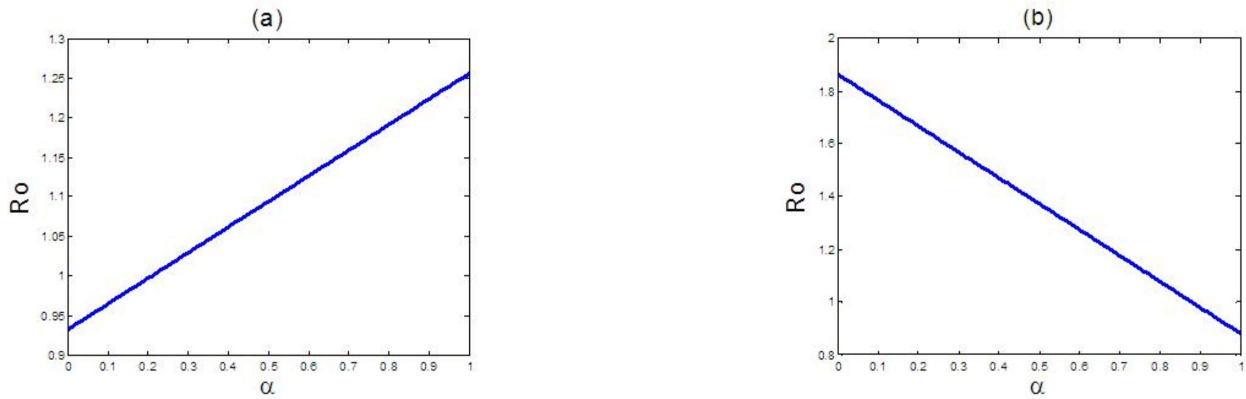

Figure 4: The impact of α on R_0 with different value of $b\rho_1 - \alpha\beta\rho_2$. (a) $b\rho_1 - \alpha\beta\rho_2 > 0$ when $a = 0.8, b = 0.7, \beta = 0.2, \rho_1 = 0.12, \rho_2 = 0.08, \langle k^2\rangle/\langle k\rangle = 3.4, T = 2$. (b) $b\rho_1 - \alpha\beta\rho_2 < 0$ when $a = 0.8, b = 0.7, \beta = 0.8, \rho_1 = 0.12, \rho_2 = 0.16, \langle k^2\rangle/\langle k\rangle = 3.4, T = 2$.

(3) α is the probability that enterprise will enter the latent period and not appear the risk after it infect risk, β is the probability that enterprise can eliminate risk before the risk appear. From Figure 4(a), it can be seen that when $b\rho_1 - \alpha\beta\rho_2 > 0$, R_0 increases as α increases. However, at this time reducing α is good for the risk disappearing in the network, but it makes more enterprises to appear the risk and they may eliminate the risk only after the risk appearing period. This will generate greater harm to enterprises, hence, we should make other strategies. On the other hand, when $b\rho_1 - \alpha\beta\rho_2 < 0$, R_0 decreases as α decreases, at this time the managers should minimize the impact of risk so that the risk does not appear. From Figure 5, R_0 decreases as β decreases, therefore the risk management should strengthened to help more latent enterprises(E) to eliminate the risk.

(4) T is the risk appearing time. Figure 6 manifests that R_0 increases as T increases, that is to say, after the enterprises infect risk and appear, the shorter the time, the more favorable to eliminate the risk in the network. So managers should enable infected enterprises(I) to eliminate the risk as soon as possible.

(5) $\langle k^2\rangle/\langle k\rangle$ is the level of heterogeneity of the enterprise network. From the expression of R_0, we know that decreasing the level of heterogeneity is good for the disappearing of the risk in the enterprise network. Therefore, managers need to pay more attention to the enterprise network when they make the prevention strategies.

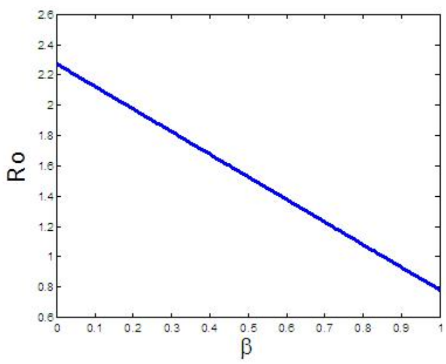

Figure 5: The impact of β on R_0, here $a = 0.8, b = 0.7, \alpha = 0.6, \rho_1 = 0.12, \rho_2 = 0.16, \langle k^2\rangle/\langle k\rangle = 3.4, T = 2$.

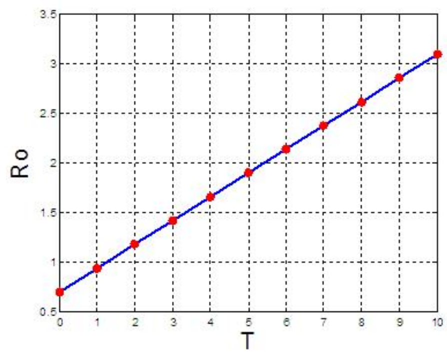

Figure 6: The impact of T on R_0, here $a = 0.8, b = 0.7, \alpha = 0.7, \beta = 0.8, \rho_1 = 0.12, \rho_2 = 0.16, \langle k^2\rangle/\langle k\rangle = 3.4$.

5 Conclusions

In summary, through the construction of the risk propagation model of inter-enterprise and the analysis of the global dynamics and the influencing factors of the risk transmission, we confined that if $R_0 < 1$, it is proven that the risk in the enterprise network will disappear; otherwise, if $R_0 > 1$, the risk will remain and the propotion of four states of enterprises will reach the unique stationary positive levels eventually. Hence, managers should make some comprehensive risk prevention, monitoring and control strategies, in particular, discover the existence of the risk timely and prevent the enterprises from being infected. However, our research of inter-enterprise risk spreading focused more on the theoretical analysis, hence the actual strategies and other related questions will be further discussed in the future.

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