ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.19(2015) No.1,pp.40-46

Travelling Wave Solutions to the Generalized Benjamin-Bona-Mahony (BBM) Equation Using the First Integral Method

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(Received 29 June 2013, accepted 27 February 2014)

Abstract: In this paper, we investigate the first integral method for solving the generalized Benjamin-Bona-Mahony (BBM) equation. This idea can obtain some exact solutions of this equations based on the theory of Commutative algebra.

Keywords: first integral method; generalized Benjamin-Bona-Mahony (BBM); travelling wave solutions

1 Introduction

In recent years, the investigation of exact solutions to nonlinear partial differential equations has played an important role in nonlinear phenomena. Nonlinear phenomena appear in a wide variety of scientific applications such as plasma physics, solid state physics, nonlinear optics, quantum field theory and fluid dynamics. In order to better understand these nonlinear phenomena, many mathematicians and physical scientists make efforts to seek more exact solutions to them. Several powerful methods have been proposed to obtain exact solutions of nonlinear partial differential equations, such as the Bäcklund transformation method [7, 11, 16], Hirotas direct method [15], tanh-sech method [6], extended tanh method [3, 36], the exp- function method [17, 18, 37], sine-cosine method [9, 12, 43], Jacobi elliptic function expansion method [19], F-expansion method [24] and so on .

The first integral method was first proposed in [4] in solving Burgers- KdV equation which is based on the ring theory of commutative algebra.The useful first integral method is widely used in many papers such as in Refs.[4, 38–40] and the reference therein. The present work is interested in generalized Benjamin-Bona-Mahony (BBM) equation [30]:

$$u_t + \alpha u_x + (\beta u^n + cu^{2n}) + ku_{xxx} = 0. \tag{1}$$

In the above equation, the first term of left side represents the evolution term while parameters β and c represent the coefficients of dual-power law nonlinearity, α and k are the coefficients of dispersion terms, n is the power law parameter, and variable u is the wave profile. In [31] , Biswas used the solitary wave ansatz and obtained an exact 1-soliton solution of (1). In order to find more exact solutions of some nonlinear evolutionary equations, Kuru [32, 33] discussed the BBM-like equation, and Estévez et al. [34] analyzed another type of generalized BBM equations. In this work, we use the first integral method to find the exact solutions of the generalized Benjamin-Bona-Mahony equation.
The rest of this paper is organized as follows : Section 2 is a brief introduction to the first integral method. In section 3, we apply the first integral method to find exact solutions of the generalized Benjamin-Bona-Mahony equation.

2 The First Integral Method.

Consider a general nonlinear partial differential equation (PDF) in the form

$$P(u, u_t, u_x, u_{xx}, u_{xt}, u_{tt}, ...) = 0, \tag{2}$$

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IJNS.2015.02.15/851

where $u(x,t)$ is the solution of nonlinear partial differential equation (2). By means of the transformation

$$u(x,t) = U(\xi), \quad \xi = (x - \lambda t), \tag{3}$$

where λ is arbitrary constants, we reduce eq (2)to an ordinary differential equation (ODE) of the form

$$P(u, u^{'}, u^{''}, u^{'''}, ...) = 0, \tag{4}$$

where $u(x,t) = u(\xi)$ and the primes denote ordinary derivatives with respect to ξ. Next, we introduce a new independent variable

$$V(\xi) = u(\xi), \;\; W(\xi) = u^{'}(\xi), \tag{5}$$

which leads to a system of ODEs of the form

$$\begin{cases} V^{'}(\xi) = & W(\xi), \\ W'(\xi) = & H(V(\xi), W(\xi)). \end{cases} \tag{6}$$

According to the qualitative theory of ordinary differential equations [2], if we can find two first integrabls to system (6) under the same conditions, then analytic solutions to (6) can be solved directly. However, in general, it is realty difficult to realize this even for a single first integral, because for a given plane autonomous system, there is no general theory telling us how ti find it's first integrals in a systematic way. A key idea of our approach here to find first integral is to utilize the division theorem . For convenience, first let us recall the Division theorem for two variables in the complex domain $\mathbf{C}$ [4].

Theorem 1 (Division theorem) *Suppose that $P(\omega, z)$ and $Q(\omega, z)$ are polynomials in $\mathbf{C}[\omega, z]$, and that $P(\omega, z)$ is irreductible $\mathbf{C}[\omega, z]$. If $Q(\omega, z)$ vanishes at any zero point of $P(\omega, z)$, then there exists a polynomial $G(\omega, z)$ in $\mathbf{C}[\omega, z]$ such that*

$$Q(\omega, z) = P(\omega, z).G(\omega, z). \tag{7}$$

3 The generalized Benjamin-Bona-Mahony (BBM) equation [30]

Assume that equation (1) has the solution of the form:

$$u(x,t) = u(\xi), \;\; \xi = (x - \nu t). \tag{8}$$

where ν is arbitrary constant. Substituting (8) into (1) we obtain

$$(\alpha - \nu)u^{'} + (\beta u^n + cu^{2n})u^{'} + ku^{'''} = 0, \tag{9}$$

where prime denotes derivative with respect to ξ. Integrating the equation of (9) with respect to ξ and taking the integration constants to zero yields:

$$(\alpha - \nu)u + \frac{\beta}{n+1}u^{n+1} + \frac{c}{2n+1}u^{2n+1} + ku^{''} = 0, \tag{10}$$

Making the following transformation:

$$v = u^n, \tag{11}$$

then (10) becomes

$$v^{''} - av + b\frac{(v^{'})^2}{v} + dv^2 + fv^3 = 0, \tag{12}$$

where

$$\begin{array}{c} a = \frac{n(\alpha-\nu)}{k}, \; b = \frac{1-n}{n}, \\ d = \frac{n\beta}{k(n+1)}, \; f = \frac{nc}{k(2n+1)}. \end{array} \tag{13}$$

and $v^{'}$ and $v^{''}$ denote $\frac{dv}{d\xi}$ and $\frac{d^2v}{d\xi^2}$, respectively. and the prime denotes derivative with respect to ξ. Next, we introduce new independent variables $u = z,\ u^{'} = \omega$. Then equation (12) can be rewritten as the two-dimensional autonomous system

$$\begin{cases} \frac{dz}{d\xi} = & \omega, \\ \frac{d\omega}{d\xi} = & az - b\frac{\omega^2}{z} - dz^2 - fz^3. \end{cases} \tag{14}$$

Assume that

$$\frac{d\xi}{z} = d\tau \tag{15}$$

thus system becomes

$$\begin{cases} \frac{dz}{d\tau} = & z\omega, \\ \frac{d\omega}{d\tau} = & az^2 - b\omega^2 - dz^3 - fz^4. \end{cases} \tag{16}$$

Now, we apply the Division Theorem to seek the first integral to (14). Suppose that $z = z(\tau)$ and $\omega = \omega(\tau)$ are the nontrivial solutions to (16), and $p(\omega, z) = \sum_{i=0}^{r} a_i(z)\omega^i$, is irreducible polynomial in $\mathbf{C}[\omega, z]$ such that

$$p(\omega(\tau), z(\xi)) = \sum_{i=0}^{r} a_i(z(\tau))\omega^i(\tau) = 0, \tag{17}$$

where $a_i(z)\ (i = 0, 1, ..., r)$ are polynomials of z and all relatively prime in $\mathbf{C}[\omega, z]$, $a_r(z) \neq 0$. Equation (17) is also called the first integral to (16). We start our study by assuming $r = 1$ in (17). Note that $\frac{dp}{d\tau}$ is polynomial in z and ω, and $p(\omega(\tau), z(\tau)) = 0$ implies $\frac{dp}{d\tau}\ |_{(16)}= 0$. By the Division Theorem, the exists a polynomial $H(z, \omega) = h(z) + g(z)\omega$ in $\mathbf{C}[\omega, z]$ such that

$$\begin{gathered} \frac{dp}{d\tau}\ |_{(16)}= (\frac{\partial p}{\partial z}\frac{\partial z}{\partial \tau} + \frac{\partial p}{\partial \omega}\frac{\partial \omega}{\partial \tau}\ |_{(16)} \\ = \sum_{i=0}^{1} a_i^{'}(z)\omega^{i+1}z + \sum_{i=0}^{1} ia_i(z)\omega^{i-1}(az^2 - b\omega^2 - dz^3 - fz^4) \\ = (h(z) + g(z)\omega)(\sum_{i=0}^{1} a_i(z)\omega^i), \end{gathered} \tag{18}$$

where prime denotes differentiating with respect to the variable z. On equating the coefficients of $\omega^i\ (i = 0,\ 1,\ 2)$ on both sides of (18), we have

$$za_1^{'}(z) - ba_1(z) = g(z)a_1(z), \tag{19}$$

$$za_0^{'}(z) = g(z)a_0(z) + h(z)a_1(z), \tag{20}$$

$$h(z)a_0(z) = a_1(z)[az^2 - dz^3 - fz^4], \tag{21}$$

Since, $a_1(z)$ and $g(z)$ are polynomials, from (19) we conclude that $a_1(z)$ is a constant and $g(z) = -b$. for simplicity, we take $a_1(z) = 1$, and balancing the degrees of $a_0(z)$, and $h(z)$, we conclude that $deg\ h(z) = 2$ and $deg\ a_0(z) = 2$, only. Now suppose that

$$h(z) = Az^2 + Bz + C,\ a_0(z) = Dz^2 + Ez + F\ (A \neq 0,\ D \neq 0), \tag{22}$$

where $A,\ B,\ C,\ D,\ E$ and F are all constants to be determined. Using (22) into (20) we obtain

$$h(z) = ((2 + b)D)z^2 + ((1 + b)E)z + bF, \tag{23}$$

Substituting $a_0(z)$, $a_1(z)$ and $h(z)$ in (21) and setting all the coefficients of powers z to be zero, we obtain a system of nonlinear algebraic equations, and by solving it, we obtain the following solutions:

$$\begin{gathered} F = 0, D = \frac{1}{2+b}\sqrt{(-(2+b)f)}, \\ E = \frac{1}{2+b}\sqrt{(-(2+b)f)}, d = \frac{f}{(2+b)}(3 + 2b) \end{gathered} \tag{24}$$

$$\begin{gathered} F=0, D=-\frac{1}{2+b}\sqrt{(-(2+b)f)}, \\ E=-\frac{1}{2+b}\sqrt{(-(2+b)f)}, d=\frac{f}{(2+b)}(3+2b) \end{gathered} \tag{25}$$

Using the conditions (24) in (17), we obtain

$$\omega=-\frac{1}{2+b}\sqrt{(-(2+b)f)}(z^2+z) \tag{26}$$

Combining this first integral with (17), the second-order differential equation (12) can be reduced to

$$\frac{dv}{d\xi}=-\frac{1}{2+b}\sqrt{(-(2+b)f)}(v^2+v). \tag{27}$$

Solving (27) directly and changing to the original variables, we obtain the complex exponential function solution to equation (1):

$$u(x,t)=\left(\frac{1}{-1+C_1\exp(\frac{-in}{n+1}\sqrt{(\frac{c(n+1)}{k(2k+1)})}(x-\nu t))}\right)^{\frac{1}{n}}, \tag{28}$$

where C_1 is arbitrary constants. Similarly, for the cases of (25), we have anther complex exponential function solutions:

$$u(x,t)=\left(\frac{1}{-1+C_2\exp(\frac{in}{n+1}\sqrt{(\frac{c(n+1)}{k(2k+1)})}(x-\nu t))}\right)^{\frac{1}{n}}, \tag{29}$$

where C_2 is arbitrary constants. These solutions are all new exact solutions.

Now we assume that $r=2$ in (17). by the Division Theorem, there exists a polynomial

$$\begin{gathered} \frac{dp}{d\tau}\mid_{(16)}=(\frac{\partial p}{\partial z}\frac{\partial z}{\partial \tau}+\frac{\partial p}{\partial \omega}\frac{\partial \omega}{\partial \tau}\mid_{(16)} \\ =\sum_{i=0}^{2}a_i^{'}(z)\omega^{i+1}z+\sum_{i=0}^{2}ia_i(z)\omega^{i-1}(az^2-b\omega^2-dz^3-fz^4) \\ =(h(z)+g(z)\omega)(\sum_{i=0}^{2}a_i(z)\omega^i) \end{gathered} \tag{30}$$

On equating the coefficients of ω^i $(i=0,\ 1,\ 2,\ 3)$ on both sides of (16), we have

$$za_2^{'}(z)-2ba_2(z)=g(z)a_2(z), \tag{31}$$

$$za_1^{'}(z)-ba_1(z)=g(z)a_1(z)+h(z)a_2(z), \tag{32}$$

$$g(z)a_0(z)+h(z)a_1(z)=2a_2(z)[az^2-dz^3-fz^4]+za_0^{'}(z), \tag{33}$$

$$h(z)a_0(z))=a_1(z)[az^2-dz^3-fz^4], \tag{34}$$

Since, $a_2(z)$ and $g(z)$ are polynomials, from (31) we conclude that $a_2(z)$ is a constant and $g(z)=-2b$. for simplicity, we take $a_2(z)=1$, and balancing the degrees of $a_0(z)$, $a_1(z)$ and $h(z)$, we conclude that $deg\ h(z)=2$ and $deg\ a_1(z)=2$, only. Now suppose that

$$h(z)=Az^2+Bz+C,\ a_1(z)=Dz^2+Ez+F\ (A\neq 0,\ D\neq 0), \tag{35}$$

where $A,\ B,\ C,\ D,\ E$ and F are all constants to be determined. Using (35) into (32) and (33) we obtain

$$\begin{gathered} h(z)=((2+b)D)z^2+((1+b)E)z+bF, \\ a_0(z)=(\frac{1}{2}\frac{2f+2D^2+D^2b}{2+b})z^4+(\frac{2d+3ED+2EDb}{3+2b})z^3+(\frac{1}{2}\frac{E^2-2a+2DF+2DFb+E^2b}{1+b})z^2+bE\frac{F}{1+2b}z+\frac{1}{2}F\frac{E+bE+bF}{b}. \end{gathered} \tag{36}$$

Substituting $a_0(z)$, $a_1(z)$ and $h(z)$ in (34) and setting all the coefficients of powers z to be zero, we obtain a system of nonlinear algebraic equations, and by solving it, we obtain the following solutions:

$$\begin{gathered} D=\frac{-2f}{\sqrt{(-f(2+b))}}, \\ E=\frac{2d\sqrt{(-f(2+b))}}{f(3+2a)},\ F=0,\ a=-((1+b)\,d^2\frac{2+b}{f(9+12b+4b^2)}). \end{gathered} \tag{37}$$

$$D = \frac{2f}{\sqrt{(-f(2+b))}},$$
$$E = -\frac{2d\sqrt{(-f(2+b))}}{f(3+2a)},\ F = 0,\ a = -((1+b)\,d^2\frac{2+b}{f(9+12b+4b^2)}). \tag{38}$$

Setting (37) in (17), we obtain that system (16) has two first integral

$$\omega = \frac{\sqrt[2]{f}(\sqrt[2]{2}-i)}{\sqrt[2]{2+b}}z^2 - \frac{i(\sqrt[2]{2+b})d}{\sqrt[2]{f}(3+2b)}z,\ (i^2=-1), \tag{39}$$

$$\omega = -\frac{\sqrt[2]{f}(\sqrt[2]{2}+i)}{\sqrt[2]{2+b}}z^2 - \frac{i(\sqrt[2]{2+b})d}{\sqrt[2]{f}(3+2b)}z,\ (i^2=-1). \tag{40}$$

Combining this first integral with (16), the second-order differential equation (12) can be reduced to

$$\frac{dv}{d\xi} = \frac{\sqrt[2]{f}(\sqrt[2]{2}-i)}{\sqrt[2]{2+b}}v^2 - \frac{i(\sqrt[2]{2+b})d}{\sqrt[2]{f}(3+2b)}v, \tag{41}$$

$$\frac{dv}{d\xi} = -\frac{\sqrt[2]{f}(\sqrt[2]{2}+i)}{\sqrt[2]{2+b}}v^2 - \frac{i(\sqrt[2]{2+b})d}{\sqrt[2]{f}(3+2b)}v. \tag{42}$$

Solving (41) and (42) directly and changing to the original variables, we obtain the exact solution to equation (1):

$$u(x,t) = \left(\frac{1}{(-1-i\sqrt{2})R + C_3\exp(\frac{ik(n+1)}{n(3k(n+1)+2(1-n))}\sqrt{\frac{k(n+1)(2n+1)}{c}})(x-\nu t)}\right)^{\frac{1}{n}} \tag{43}$$

$$u(x,t) = \left(\frac{1}{(-1+i\sqrt{2})R + C_4\exp(\frac{ik(n+1)}{n(3k(n+1)+2(1-n))}\sqrt{\frac{k(n+1)(2n+1)}{c}})(x-\nu t)}\right)^{\frac{1}{n}} \tag{44}$$

where,$R = \frac{n^2c(3k(n+1)+2(1-n))}{k^2(n+1)^2(2n+1)}$, C_3 and C_4 are arbitrary constants. Similarly, for the cases of (38), we have another complex exponential function solutions:

$$u(x,t) = \left(\frac{1}{(-1+i\sqrt{2})R + C_5\exp(\frac{-ik(n+1)}{n(3k(n+1)+2(1-n))}\sqrt{\frac{k(n+1)(2n+1)}{c}})(x-\upsilon t)}\right)^{\frac{1}{n}} \tag{45}$$

$$u(x,t) = \left(\frac{1}{(-1-i\sqrt{2})R + C_6\exp(\frac{-ik(n+1)}{n(3k(n+1)+2(1-n))}\sqrt{\frac{k(n+1)(2n+1)}{c}})(x-\upsilon t)}\right)^{\frac{1}{n}} \tag{46}$$

where,$R = \frac{n^2c(3k(n+1)+2(1-n))}{k^2(n+1)^2(2n+1)}$, C_5 and C_6 are arbitrary constants.

Notice that the results in this paper are based on the assumption of $r = 1,\ 2$ for The generalized Benjamin-Bona-Mahony (BBM) equation . For the cases of $r = 3,\ 4$ for these equations, the discussions become more complicated and involves the irregular singular point theory and the elliptic integrals of the second kind and the hyperelliptic integrals. Some solutions in the functional form cannot be expressed explicitly. One does not need to consider the cases $m \geq 5$ because it is well known that an algebraic equation with the degree greater than or equal to 5 is generally not solvable.

4 Conclusions

In this work, we are concerned with The generalized Benjamin-Bona-Mahony (BBM) equation for seeking their travelling wave solutions. We first transform each equation into an equivalent two-dimensional planar autonomous system then use the first integral method to find one first integral which enables us to reduce The generalized Benjamin-Bona-Mahony (BBM) equation to a first-order integrable ordinary differential equations. Finally, a class of travelling wave solutions for the considered equations are obtained . These solutions include complex exponential function solutions. We believe that this method can be applied widely to many other nonlinear evolution equations, and this will be done in a future work.

Acknowledgments

The authors would like to express their sincere thanks to referees for the valuable advice.

References

[1] M. J. Ablowitz and H. Segur. Solitons and inverse scattering transform. *Philadelphia, SIAM*, 1981.

[2] T. R. Ding, C.Z. Li. Ordinary Differential Equations. *Peking University Press, Peking*, 1996.

[3] E. Fan. Extended tanh-function method and its applications to nonlinear equations. *Phys. Lett. A.*, 277(2000):212-218.

[4] Z. S. Feng. On explicit exact solutions to the compound Burgers-KdV equation. *Phys. Lett. A.* 293(2002):57-66.

[5] J. H. He, X. H. Wu. Exp-function method for nonlinear wave equations. *Chaos Solitons Fractals*, 30(2006):700-708.

[6] W. Malfliet. *Solitary wave solutions of nonlinear wave equations,* Am. J. Phys. 60(7)(1992):650-654.

[7] T. C. Xia. Auto-Bäcklund transformation and exact analytical solutions for the Kupershimidt equation. *Appl. Math. E-Notes*, 3(2003):171-177.

[8] N. Bourbaki. Commutative Algebra. *Addison Wesley,Paris*, 1972.

[9] X. D. Zheng, T. C. Xia and H. Q. Zhang. New exact traveling wave solutions for compand KdV-Burgers equation in mathematical physics. *Appl. Math. E-Notes* 2(2002): 45-50.

[10] S. Zhang. The periodic wave solutions for the (2+1)-dimensional Konopelchenko Dubrovsky equations. *Chaos Solitons Fractals*, 30(2006):1213-1220.

[11] D. Lu, B. Hong, L. Tian. Bäcklund transformation and n-soliton-like solutions to the combined KdVBurgers equation with variable coefficients. *International Journal of Nonlinear Science* , 2(2O06):3-10.

[12] A. M. Wazwaz. A sine-cosine method for handling nonlinear wave equations. *Math. and Comput. Modelling*, 15(1982):197-211.

[13] B. P. Palka. An Introduction to Complex Function Theory. *Spring- Verlag, New York*, 1991.

[14] A. M. Wazwaz. The tanh method for travelling wave solutions of nonlinear equations. *Appl Math Comput*. 153(3)(2004):713-723.

[15] R. Hirota. The direct method in soliton theory. *Cambridge University Press, Cambridge*, 2004.

[16] M. R. Miura. Bäacklund Transformation. *Springer, Berlin*, 1978.

[17] J. H. He, H. X. Wu. Exp-function method for nonlinear wave equations. *Chaos, Solitons and Fractals*, 30(2006):700-708.

[18] J. H. He, M. A. Abdou. New periodic solutions for nonlinear evolution equations using Exp-function method. *Chaos, Solitons and Fractals*, 34(2007): 1421-1429.

[19] M. Inc, M. Ergut. Periodic wave solutions for the generalized shallow water wave equation by the improved Jacobi elliptic function method. *Appl Math E-Notes*, 5(2005):89-96.

[20] M. Wang, X. Li and J. Zhang. The $(G^{'}/G)$- expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics.*Phys. Lett. A*. 372(2008):417-423.

[21] Z. Feng and R. Knobel. Traveling waves to a Burgers-Korteweg-de Vries-type equation with higherorder nonlinearities. *Journal of Mathematical Analysis and Applications*, 328(2)(2007):1435-1450.

[22] M. J. Ablowitz and P. A. Clarkson. Nonlinear Evolution Equations and Inverse Scattering. *Cambridge University Press, Cambridge*, 1991.

[23] S. K. Paul and C. A. Roy. On the Analytical Approach to the N-Fold B$\ddot{a}$cklund Transformation of Davey-Stewartson Equation. *Journal of Nonlinear Mathematical Physics*, 5(4)(1998):349-356.

[24] H. T. Chen, H. Q. Zhang. New double periodic and multiple soliton solutions of the generalized (2 + 1)-dimensional Boussinesq equation. *Chaos Solitons Fractals*, 20(2004):765-769.

[25] A. Hendi. New Exact TravellingWave Solutions for Some Nonlinear Evolution Equations. *International Journal of Nonlinear Science* , 7(3)(2O09):259-267.

[26] A. A. Soliman. The Modified Extended Direct Algebraic Method for Solving Nonlinear Partial Differential Equations. *International Journal of Nonlinear Science* , 6(2)(2O08):136-144.

[27] Lin Ju. On Solution of the Dullin-Gottwald-Holm Equation. *International Journal of Nonlinear Science* , 1(1)(2O06):43-48.

[28] J. Biazar , M. Eslami. Differential Transform Method for Quadratic Riccati Differential Equation. *International Journal of Nonlinear Science* , 9(4)(2O10):444-447.

[29] Ya-zhou Li, Kai-ming Li, Chang Lin. Exp-function method for solving the generalized-Zakharov equations. *Applied Mathematics and Computation*, 205(1)(2008):197-201 .

[30] Xun Liu, Lixin Tian, and Yuhai Wu. Exact Solutions of the Generalized Benjamin-Bona-Mahony Equation *Mathematical Problems in Engineering*, Volume 2010, Article ID 796398,5 pages .

[31] A. Biswas. 1-soliton solution of Benjamin-Bona-Mahoney equation with dual-power law nonlinearity. *Communications in Nonlinear Science and Numerical Simulation*, 15(10)(2010):2744-2746.

[32] S. Kuru. Compactons and kink-like solutions of BBM-like equations bymeans of factorization. *Chaos, Solitons and Fractals*, 42(1)(2009):626-633.

[33] S. Kuru. Traveling wave solutions of the BBM-like equations. *Journal of Physics A*, 42(37)(2009):Article ID 375203, 12 pages.

[34] P. G. Estvez, S. Kuru, J. Negro, and L. M. Nieto. Travelling wave solutions of the generalized Benjamin-Bona-Mahony equation. *Chaos, Solitons and Fractals*, 40 (4):(2009), 2031-204 .

[35] E. T. Whittaker and G. N. Watson. A Course of Modern Analysis. An Introduction to the General Theory of Infinite Processes and of Analytic Functions. *Cambridge Mathematical Library, Cambridge University Press, Cambridge, UK*, 1996. 10(6)(2009):735-739.

[36] A. Bekir. Applications of the extended tanh method for coupled nonlinear evolution equations. *Communications in Nonlinear Science and Numerical Simulation*, 13(9)(2008):1748-1757 .

[37] A. Bekir. The Exp-function method for Ostrovsky equation. International *Journal of Nonlinear Sciences and Numerical Simulation*, 10(6)(2009):735-739 .

[38] F. Tascan, A. Bekir, M. Koparan. Travelling wave solutions of nonlinear evolution equations by using the first integral method. *Communications in Nonlinear Science and Numerical Simulation*, 14(5)(2009):1810-1815 .

[39] F. Tascan, A. Bekir. Applications of the first integral method to thenonlinear evolution equations. *Chinese Physics B*, 19(8)(2010):080201

[40] A. Bekir, O. Unsal. Periodic and solitary wave solutions of coupled nonlinear wave equations by using first integral method. *Physica Scripta*, 85 065003 (2012).

[41] M. A. Abdou. Exact periodic wave solutions for some nonlinear evolution equations. *International Journal of Nonlinear Science* , 5(2008):1-9.

[42] A. Bekir. New Exact Travelling Wave Solutions for Regularized Long-wave, Phi-Four and Drinfeld-Sokolov Equations. *International Journal of Nonlinear Science* , 6(1)(2008):46-52.

[43] F. Tascan, A. Bekir. Analytic solutions of the (2+1)-dimensional nonlinear evolution equations using the sine-cosine method. *Applied Mathematics and Computation*, 215(8)(2009):3134-3139.

[44] Z. Ma, Y. Hu, J. Lan. Application of Fibonacci tane function to nonlinear differential-difference equations. *Chaos Solitons Fractals*, 36(2)(2008):303-308.