**ISSN** 1749-3889 (print), 1749-3897 (online)
**International Journal of Nonlinear Science**
Vol.19(2015) No.1,pp.14-19

# Tracking the State of Hyperchaotic 4D System with Time Delay by Using the Genesio-Tesi System via a Single Input

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*(Received 22 May 2014, accepted 28 December 2014)*

**Abstract:**Based on Lyapunov stability theory, this paper presents a partial synchronization procedure to track the signal of the delay hyperchaotic 4D system by using the Genesio-Tesi system via a single nonlinear controller. Special Lyapunov function which constructed is novel. Four cases are considered to verify the proposed partial synchronization scheme. The method is simple and easy to achieve, theoretical analysis and numerical simulations are presented to demonstrate feasibility and effectiveness of the partial synchronization approach.

**Keywords:** Lyapunov stability theory; hyperchaotic 4D system; Genesio-Tesi system; partial synchronization

## 1 Introduction

In recent years, the study of the nonlinear chaotic dynamics is a popular problem in the field of the nonlinear science and great progress has been made in the research of nonlinear chaotic dynamics. Since 1963, Lorenz found the first chaotic system [1] when he studied atmospheric convection, chaos and hyperchaos have been extensively studied due to its theoretical and practical applications in fields of communications, circuit, laser, neural work, mathematics and so on [2-5].

Synchronization is a fundamental phenomenon that enables coherent behavior in coupled systems. In 1990, Pecora and Carroll [6] introduced a method to synchronize two identical chaotic systems with different initial conditions. Since then, chaos synchronization was studied such as in secure communication, neural work and information processing [7-10]. Recently, synchronization is applied into complex dynamical networks [11-13]. The synchronization of chaotic dynamical systems has been one of the most interesting topics in nonlinear science and many theoretical and experimental results have been obtained. Up to now, a variety of synchronization have been proposed in dynamical systems such as drive-response synchronization, coupled synchronization, lag synchronization, impulsive synchronization, Adaptive synchronization, variable structure (sliding mode), among many others [14-22]. Tracking problem can be used achieve the synchronization between non-linear oscillators with different structures and orders, where the variable states of the slave system are forced to follow the trajectories of the master system. M.Hasler et al. studied partial synchronization of chaotic systems between two or more similar chaotic systems [23]. Vieira et al.[24] showed that partial synchronization does not necessarily precede complete synchronization. Taborov et al. discussed partial synchronization in a system of three coupled logistic maps [25]. These results show that partial synchronization depends on the type of basic map constituting the coupled system. However, the mechanism of the occurrence of partial synchronization remains unclear. On the other hand, most of theoretical results about synchronization phenomena focus on structurally equivalent low-dimensional dynamical systems, but this paper introduces partial synchronization into the high-dimensional chaotic system. We will study the problem of tracking the delay hyperchaotic 4D system by controlling the Genesio-Tesi system via single nonlinear controller [26, 27]. The contribution of this paper is that we construct Lyapunov function is novel and successfully synchronize high-dimensional dynamical systems by using the Genesio-Tesi system via single controller. Four cases are considered and some numerical simulations are presented to demonstrate the effectiveness of the proposed scheme.

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IJNS.2015.02.15/847

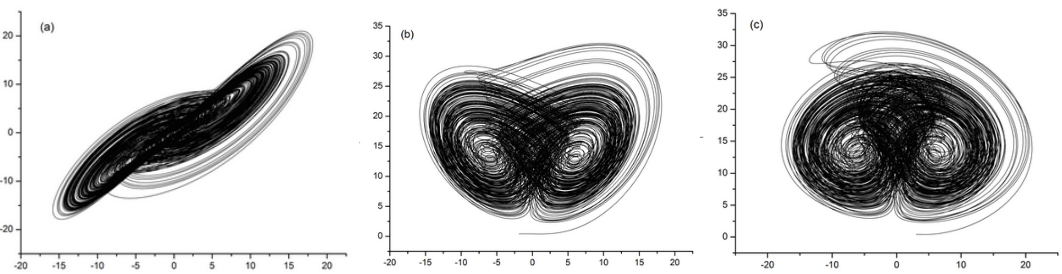


Figure 1: 2D overview hyperchaotic attractor of Eq. (1) with $a$=20, $b = 10.6$, $c = 2.8$, $r = 3.7$, $\tau = 1$,
(a) $(x_1, x_2)$, (b) $(x_1, x_3)$, (c) $(x_2, x_3)$.

The rest of the paper is organized as follows. The hyperchaotic 4D system and Genesio-Tesi system are discussed in Section 2. In Section 3, Based Lyapunov stability theory, a systematic design procedure is proposed to simulate the activity of the delay hyperchaotic 4D system by using the Genesio-Tesi system, four cases are considered to illustrate the feasibility of the partial synchronization scheme. Finally, brief conclusion of this paper is drawn in Section 4.

# 2 System description

## 2.1 The hyperchaotic 4D system

The hyperchaotic 4D system is described as

$$\dot{x}_1 = a(x_2 - x_1), \quad \dot{x}_2 = x_1 + bx_2 - x_1x_3 + x_4(t - \tau), \quad \dot{x}_3 = x_1^2 - cx_3, \qquad \dot{x}_4 = -rx_1, \tag{1}$$

where $\tau > 0$ is the time delay. When $\tau = 0$ , system (1) is the hyperchaotic system constructed by Cai et al[26]. It is well known, time delay is ubiquitous in most physical, chemical, biological, neural and other natural systems. As the dynamical systems given by DDEs have an infinite dimensional state space, usually the attractors of the solutions are high dimensional. By the Galerkin approximation technique, an algorithms considered by Ghosh for calculating Lyapunov exponents for system with time delay, when $a = 20$,$b = 10.6$, $c = 2.8$,$r = 3.7$ and the time delay $\tau$ is chosen as 1, system (1) has two positive Lyapunov exponents, i.e.,$\lambda_1 = 0.6513$, $\lambda_2 = 0.1394$, and the hyperchaotic attractors of system (1) are shown in Fig.1.

## 2.2 The Genesio-Tesi system

The Genesio-Tesi system, proposed by Genesio and Tesi, is a chaotic system, which includes a simple square part and three simple ordinary differential equations that depend on three positive real parameters. The dynamic equations of the system can be written as follows

$$\dot{y}_1 = y_2, \qquad \dot{y}_2 = y_3, \qquad \dot{y}_3 = -a_1y_1 - b_1y_2 - c_1y_3 + y_1^2, \tag{2}$$

where $y_1, y_2, y_3$are state variables, and $a_1, b_1, c_1$ are real constants. If the parameters are taken as $a_1 = 6.0$, $b_1 = 2.88$, $c_1 = 1.2$ , system (2) has chaotic behavior, the chaotic attractors of system (2) are shown in Fig.2.

# 3 Scheme and Model

In this section, we will propose a systematic design procedure to track the activity of the delay hyperchaotic 4D system by using the Genesio-Tesi system, the control scheme can be approached via an improved adaptive track. The mechanism can be understood as partial synchronization between dynamical models. This method only needs one single controller. A single control input $u$ is added to the second equation of system (2) and the first state $y_1$ of the controlled Genesio-Tesi chaotic model is used to simulate the dynamical properties of the hyperchaotic 4D system. Thus, the controlled Genesio-Tesi chaotic system is given as follows

$$\dot{y}_1 = y_2, \quad \dot{y}_2 = y_3 + u, \quad \dot{y}_3 = -a_1y_1 - b_1y_2 - c_1y_3 + y_1^2, \tag{3}$$

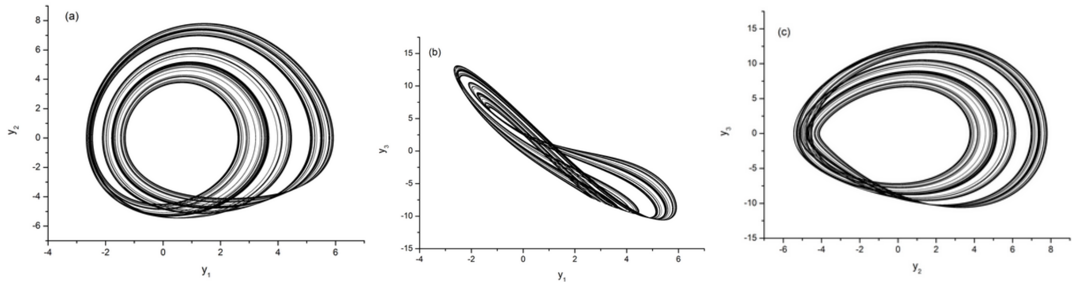

Figure 2: 2D overview hyperchaotic attractor of Eq. (2) with $a_1$=6.0, $b_1 = 2.88$, $c_1 = 1.2$,
(a) $(y_1, y_2)$, (b) $(y_1, y_3)$, (c) $(y_2, y_3)$.

The error system is denoted as

$$E = x - y_1, \tag{4}$$

where $x$ is one of the observed states of system (1). The error function $E = x - y_1$ will be stabilized to certain value when the output variables $x$ and $y_1$ is close to each other, which indicates a kind of partial synchronization. To realize the partial synchronization between the two systems ,we need to choose a suitable control law $u$ and make $E$ generally converge to zero with time increasing.

To realize partial synchronization , the positive and changeable Lyapunov function could be measured by

$$V = E^2 + (\dot{E} + x^2 E)^2 \tag{5}$$

where the variable $x$ is anyone output variable of the system to be controlled, the over dot denotes the differential coefficient vs. time. The differential coefficient of Lyapunov function as shown in Eq.(5) vs. time is approached by

$$\begin{aligned} \tfrac{dV}{dt} &= 2E\dot{E} + 2(\dot{E} + x^2 E)(\ddot{E} + 2x\dot{x}E + x^2\dot{E}) \\ &= -2x^2 V + 2(\dot{E} + x^2 E)(\ddot{E} + 2x^2\dot{E} + x^4 E + 2x\dot{x}E + E), \end{aligned} \tag{6}$$

As a result, negative condition in Eq.(6) can be simplified as shown in Eq.(7),

$$\frac{dV}{dt} = -2x^2 V < 0. \tag{7}$$

only if the condition in Eq.(8) is satisfied

$$2(\dot{E} + x^2 E)(\ddot{E} + x^2\dot{E} + x^4 E + 2x\dot{x}E + E) = 0. \tag{8}$$

According to the Lyapunov stability theory, the errors of corresponding variables will be stabilized to a certain threshold. As a result, the observed state $x$ of system (1) and the salved state $y_1$ of the system (2) with a controller will reach synchronization completely. In the following, four cases will be considered and some numerical simulation results will be presented to demonstrate the effectiveness of the proposed scheme. In all of following numerical simulations, the fourth-order Runge-Kutta algorithm is used for calculating the nonlinear equations with time step $h = 0.001$. The parameters of the systems are chosen as $a = 20$, $b = 10.6$, $c = 2.8$, $r = 3.7$,$\tau = 1$, and $a_1 = 6.0$, $b = 2.88$, $c_1 = 1.2$. The initial values of the hyperchaotic 4D system and the Genesio-Tesi system are set to be $(1.0, -2.0, 0.1, 0.3)$ and (2.0, 1.0, 0.2), respectively.

**Case 1: Simulating the activity of $x_1$ using $y_1$**

In this case,the state $x_1$ of the hyperchaotic system will be considered as the drive signal, and the first state $y_1$ of the controlled Genesio-Tesi chaotic model is used as the salve signal to simulate the hyperchaotic behavior. Thus, the error of is denoted by $e_1 = x_1 - y_1$ . According to the formulas(8), the corresponding controller $u_1$ can be derived as

$$\begin{aligned} u_1 = &\, a[x_1 + bx_2 - x_1x_3 + x_4(t-\tau) - a(x_2 - x_1)] - y_3 + x_1^4(x_1 - y_1) \\ &+ 2x_1^2[a(x_2 - x_1) - y_2] + 2ax_1(x_2 - x_1)(x_1 - y_1) + (x_1 - y_1). \end{aligned} \tag{9}$$

The evolutions of the outputs of $x_1$ and $y_1$ , as well as the corresponding error variable $e_1$ , are calculated to demonstrate the effectiveness of the proposed partial synchronization scheme. The results are shown in Figs.3 and 4.

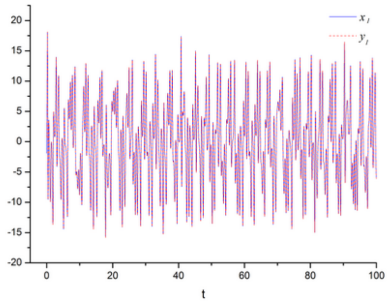


Figure 3: The evolution of $x_1$ and $y_1$

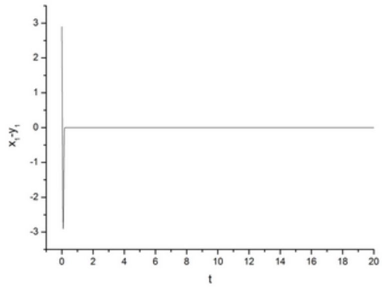


Figure 4: Synchronization error of $e_1$

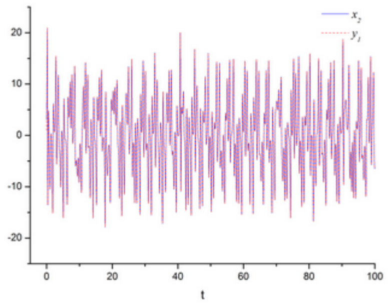


Figure 5: The evolution of $x_2$ and $y_1$

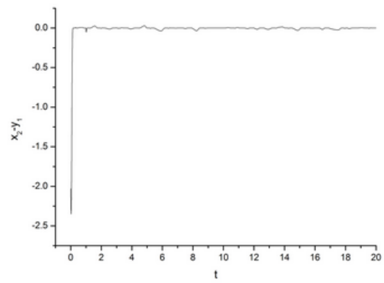


Figure 6: Synchronization error of $e_2$

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**Case 2: Simulating the activity of $x_2$ using $y_1$**

In this case, the first state $y_1$ of the controlled Genesio-Tesi system is used to simulate the dynamical properties of the hyperchaotic behavior state $x_2$. Thus, the error of corresponding variable is denoted by $e_2 = x_2 - y_1$ . According to the formulas (8), the corresponding controller $u_2$ can be deduced as

$$\begin{aligned} u_2 = & a(x_2 - x_1) + b[x_1 + bx_2 - x_1x_3 + x_4(t-\tau)] - a(x_2 - x_1)x_3 \\ & - x_1(x_1^2 - cx_3) - rx_1(t-\tau) - y_3 + 2x_2^2[x_1 + bx_2 - x_1x_3 + x_4(t-\tau) - y_2] \\ & + x_2^4(x_2 - y_1) + 2x_2[x_1 + bx_2 - x_1x_3 + x_4(t-\tau)](x_2 - y_1) + (x_2 - y_1). \end{aligned} \tag{10}$$

The evolutions of $x_2$ and $y_1$ , as well as the corresponding error variable $e_2$ , are calculated to demonstrate the effectiveness of the proposed partial synchronization. The results are shown in Figs.5 and 6.

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**Case 3: Simulating the activity of $x_3$ using $y_1$**

In this case, the state $x_3$ of the hyperchaotic system will be considered as the drive signal, and the first state $y_1$ of the controlled Genesio-Tesi chaotic model is used as the salve signal to simulate the chaotic behavior. Thus, the error of is denoted by $e_3 = x_3 - y_1$ . Accordingly, the corresponding controller $u_3$ can be derived as

$$\begin{aligned} u_3 = & [2ax_1(x_2 - x_1) - c(x_1^2 - cx_3) - y_3] + 2x_3^2(x_1^2 - cx_3 - y_2) \\ & + x_3^4(x_3 - y_1) + 2x_3(x_1^2 - cx_3)(x_3 - y_1) + (x_3 - y_1). \end{aligned} \tag{11}$$

The evolutions of $x_3$ and $y_1$ , as well as the corresponding error variable $e_3$ , are calculated to demonstrate the effectiveness of the proposed partial synchronization. The results are shown in Figs.7 and 8.

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**Case 4: Simulating the activity of $x_4$ using $y_1$**

In this case, $y_1$ is used to simulate the state $x_4$ of the Genesio-Tesi system. The error is denoted by $e_4 = x_4 - y_1$ . According to the conditions in (8), the controller can be written as

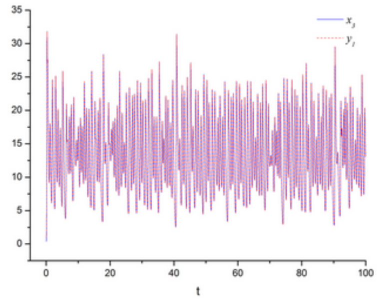


Figure 7: The evolution of $x_3$ and $y_1$

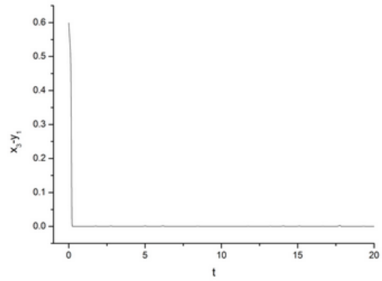


Figure 8: Synchronization error of $e_3$

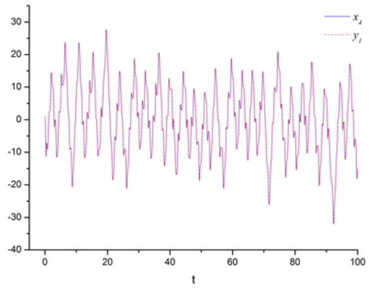


Figure 9: The evolution of $x_4$ and $y_1$

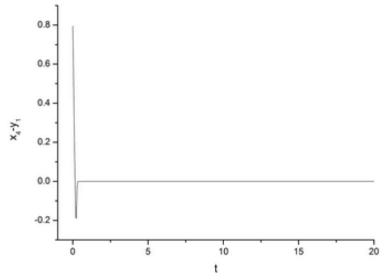


Figure 10: Synchronization error of $e_4$

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\begin{aligned}
u_4 = &-ar(x_2 - x_1) - y_3 + 2x_4^2(-rx_1 - y_2) + x_4^4(x_4 - y_1) \\
&- 2rx_1x_4(x_4 - y_1) + (x_4 - y_1).
\end{aligned} \tag{12}
$$

The evolutions of $x_4$ and $y_1$ , as well as the corresponding error variable $e_4$, are calculated to demonstrate the effectiveness of the proposed partial synchronization. The results are shown in Figs.9 and 10.

# 4 Conclusions

We have studied the partial synchronization of chaotic dynamical systems. Based on Lyapunov stability theory, an improved scheme is proposed to track the hyperchaotic 4D system by using the Genesio-Tesi system via single nonlinear controller. we construct Lyapunov function is simple and easy to realize. Numerical simulations are provided to show the effectiveness and feasibility of the developed method. The proposed control scheme can be applied to synchronize other chaotic systems.


# Acknowledgments

This work is supported by National Natural Science Foundation of China ( Nos.11472238, 61273106 and 61379064) and The Natural Science Foundation of the Jiangsu Province of China ( No. BK2012672).


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