ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.18(2014) No.3,pp.195-199

An Interbank Market Network Model Based on Complex Network Theory

Lu Chen *, Hongxin Yao, Chuiqing Kong
Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu 212013, China
(Received 5 March 2014 , accepted 9 July 2014)

Abstract: As a complex financial system, we can study the topology of the interbank market. The community structure feature of the interbank network is identified through empirical analysis, so the local world evolving model is proposed. By using continuum theory and mean field theory, the degree distribution is obtained theoretically and the degree distribution is further studied through simulation analysis. The result shows that comparing with the previous scale free model, the model this paper proposed is generic and reality.

Keywords: interbank market ; multi-local world model ; degree distribution ; complex network

1 Introduction

The interbank market is also called the interbank lending market in which banks in daily cash shortage or surplus can exchange their capital for a short term. It is an important part of modern financial system, and is indispensable to the sound and stable operation of the entire financial system. In the interbank market, creditor banks, the credit lending relationship between creditor banks form a credit lending network. As a kind of complex economic system, we can study the topological features of the interbank market with the theory of complex network.

Foreign scholars have found some structure characteristics as a complex network through the empirical research of the actual interbank market network. See for an overview, the degree distribution of most of these real networks follow a power law, some networks exhibit a small-world phenomenon [1-4]. In recent years, foreign scholars have also studied the system risk under different network topology of the interbank market, the risk of transmission in interbank market, and bankruptcy problem in the interbank market. Interbank market began to develop in China in 1984, and using the theory of complex network to study the structure of the interbank network started relatively late in our country, related literature in this aspect is not much.

Wan creatively put forward the double power law structure and Li proposed a tiered network model [5]. Based on the complex network theory Li Hao etc. have established the interbank market network extension random - mixed scale-free network, which integrates random network with scale-free network. The model takes both the growth and preferential attachment mechanism into account, in the network evolution process the changes of connection not only consider external nodes, but also considers the internal nodes [6].

The scale free model calculates the probability of the selection for each node in the whole network, in which the connections are according to the probability. But in the real banking network, due to the difference in the type, the size and the credit lending capacity between different banks, as Boss in his literature [3] mentioned: Austrian banking system is composed of seven departments of savings banks (S), Raiffeisen (agricultural) banks (R), Volksbanken (VB), joint stock banks (JS), state mortgage banks (SM), housing construction savings and loan associations (HCL), and special purpose banks (SP). In the real interbank network it has such a phenomenon: some banks in the network clustering densely with a lot of connections between them, so the network may form several communities. Comparing to the internal connections of the same group, the connections between different groups may be too sparse. Based on the above analysis, the preferential attachment mechanism is not valid for the entire network, but effective in their local world, so this world to establish a multi-local model to describe the evolution of the interbank market network and study its structural characteristics.

*Corresponding author. *E-mail address*: alanchenlu@163.com

IJNS.2014.12.15/841

2 The model for the interbank market

Assuming that each bank is taken as a node, and see the credit lending relationship between banks as the edges connecting between these nodes in the network, thus the nodes and edges compose the interbank market network. The initial setting of the network: there are m local worlds; every local has m_0 nodes and e_0 edges. In the scale free network model, the preferential attachment mechanism is based on the priority in the whole interbank network, which is in the growth process each new node chooses one node of the network to link according to the connection probability; this is called choosing the best in global. Contrast to this, the presence of many local world in the real world, nodes in each local world generally prefer to link to the nodes from its local world. Both the preferential attachment mechanisms in the complex networks portray the reality is one-sided. Actually according to the real situation, the nodes can be divided into two types for the best choice: global preferential attachment and local preferential attachment. In the real, based on difference of the property and the external environment the nodes can choose the connection with different preferential attachment. So following the above analysis, we establish a new interbank market network called the multi-local world evolution model, which considering five factors in the dynamic evolution process: the increase of new nodes, the increase of the local worlds, the adding and deleting of edges in the local world, and the increase of edges between different local worlds. At each time step the following situations may occur in the lending network: a new bank adds in, new lending contrast between banks generates and cancels, lending relationship occurs between different types of banks. According to the above analysis, the growth of the local world evolving network model is as follows:

1) Adding a local world of m_0 nodes and e_0 edges with the probability of p.

2) Adding a new node with the probability of q : first randomly select a local world, then select a node in the selected local world according to the principle of preferential attachment, then connect the new node with the selected node, repeats for m_1 times. The probability of being selected is as the follows:

$$\prod(k_i) = \frac{k_i}{\sum_{j\in\Omega} k_j} \tag{1}$$

3) Adding m_2 edges in a local world with the probability of r: first randomly select a local world and select a node in the selected local world, then choose the best node in the remaining nodes and connect the two selected nodes, repeat for m_2 times. The probability is the same as equation (1) .

4) Deleting m_3 edges in the local world with the probability of$s$: first randomly select a local world and in which randomly selects a node, then selected a neighbor node of the selected node according to the equation below, delete the edge linking the two nodes, repeated for $m_3$times.

$$\prod'(k_i) = \frac{1-\prod(k_i)}{N_\Omega(t)-1}$$

$N_\Omega(t)$ is the number of nodes in the local world Ω.

5)Establishing $m_4$edges between two local worlds with the probability of u : at first randomly select two local worlds, in every local world select a node according to equation(1), connect the two selected nodes, repeat for $m_4$times. These parameters are limited by $p+q+r+s+u=1$, and repeat until the size of the network is large enough.

3 Degree distribution

Supposing node i joined in the network at the s^th time step, $k_i(t)$ is the node is degrees at the s^th time step. And based on the continuity theory and mean-field theory, we can get variation caused at each step with each operation as the follows:

1)Adding a local world of m_0 nodes and e_0 edges with the probability of p. In this operation there is no new edge, so the degree does not change:

$$\frac{\partial k_i}{\partial t} = 0 \tag{2}$$

2)Adding a new node with m_1 edges with the probability of$q$.

$$\frac{\partial k_i}{\partial t} = \frac{m_1 q}{m+pt}\frac{k_i(t)}{\sum_{j\in\Omega} k_j} \tag{3}$$

3)Adding $m_2$edges in a local world with the probability of r.

$$\frac{\partial k_i}{\partial t} = \frac{m_2 r}{m+pt}[\frac{1}{N_\Omega(t)} + (1-\frac{1}{N_\Omega(t)})\frac{k_i}{\sum_{j\in\Omega} k_j}] \tag{4}$$

4)Deleting $m_3$edges in the local world with the probability of s.

$$\frac{\partial k_i}{\partial t} = -\frac{m_3 s}{m+pt}[\frac{1}{N_\Omega(t)} + (1-\frac{1}{N_\Omega(t)})(\frac{1}{N_\Omega(t)-1})(1-\frac{k_i}{\sum_j \in \Omega k_j})] \quad (5)$$

5)Establishing $m_4$edges between two local worlds with the probability of$u$.

$$\frac{\partial k_i}{\partial t} = m_4 u[\frac{2}{m+pt}\frac{k_i}{\sum_{j\in\Omega} k_j} - \frac{1}{(m+pt)^2}\frac{k_i}{\sum_{j\in\Omega} k_j}] \quad (6)$$

Combining equation (1)-(5), and we can get the change of node is degree:

$$\begin{aligned}\frac{\partial k_i}{\partial t} =& \frac{m_1 q}{m+pt}\frac{k_i(t)}{\sum_{j\in\Omega} k_j} + \frac{m_2 r}{m+pt}[\frac{1}{N_\Omega(t)} + (1-\frac{1}{N_\Omega(t)})\frac{k_i}{\sum_{j\in\Omega} k_j}] - \\ & \frac{m_3 s}{m+pt}[\frac{2}{N_\Omega(t)} + (1-\frac{1}{N_\Omega(t)})\frac{k_i}{\sum_j \in \Omega k_j}] + [\frac{m_4 u[2(m+pt)-1]}{(m+pt)^2}\frac{k_i}{\sum_{j\in\Omega} k_j}\end{aligned} \quad (7)$$

At the tth time step, the average total degree of any local network and the average total number of the nodes in the local world Ω respectively is:

$$\sum_{j\in\Omega} k_j = 2t(pe_0 + qm_1 + rm_2 - sm_3 + um_4)/(m+tp), \;\; N_\Omega(t) = (m_0 m + m_0 pt + qt)/(m+pt)$$

Let $\alpha = 2(pe_0 + qm_1 + rm_2 - sm_3 + um_4)$ and when t is large enough through equation (8)-(10) we can get:

$$\begin{aligned}\frac{\partial k_i}{\partial t} =& \frac{k_i}{t}[\frac{m_2 r(q+m_0 p - p)}{(m_0 p + q)\alpha} + \frac{m_3 sp}{(m_0 p+q)\alpha} + \frac{2m_4 u}{\alpha}] + \\ & \frac{1}{t}[\frac{m_1 q}{\alpha} + \frac{m_2 r}{m_0 p + q} + \frac{m_2 r(q+m_0 p - p)}{(m_0 p+q)\alpha} + \frac{m_3 sp}{(m_0 p+q)\alpha} - \frac{2m_3 s}{m_0 p + q} + \frac{2m_4 u}{\alpha}]\end{aligned} \quad (8)$$

Define

$$\gamma_1 = \frac{m_1 q}{\alpha} + \frac{m_2 r(q+m_0 p - p)}{(m_0 p+q)\alpha} + \frac{m_3 sp}{(m_0 p+q)\alpha} + \frac{2m_4 u}{\alpha}$$

$$\gamma_2 = \frac{m_1 q}{\alpha} + \frac{m_2 r}{m_0 p+q} + \frac{m_2 r(q+m_0 p - p)}{(m_0 p+q)\alpha} + \frac{m_3 sp}{(m_0 p+q)\alpha} - \frac{2m_3 s}{m_0 p+q} + \frac{2m_4 u}{\alpha}$$

We get the equation $\frac{\partial k_i}{\partial t} = \gamma_1 \frac{k_i}{t} + gamma_2 \frac{1}{t}$,the equation can be solved according to the initial conditions $k_i(s) = m_1$ and the solution is:

$$k_i(t) = (m_1 + \frac{\gamma_2}{\gamma_1})(\frac{t}{s})^{\gamma_1} - \frac{\gamma_1}{\gamma_2} \quad (9)$$

With the definition of degree distribution:

$$P\{k_i(t) < k\} = P\{t_i > t(\frac{m_1 + \frac{\gamma_2}{\gamma_1}}{k + \frac{\gamma_2}{\gamma_1}})^{\frac{1}{\gamma_1}}\} = 1 - P\{t_i \le t(\frac{m_1 + \frac{\gamma_2}{\gamma_1}}{k + \frac{\gamma_2}{\gamma_1}})^{\frac{1}{\gamma_1}}\} = 1 - \frac{1}{[3m + t(1+2p)]}(\frac{m_1 + \frac{\gamma_2}{\gamma_1}}{k + \frac{\gamma_2}{\gamma_1}})^{\frac{1}{\gamma_1}} t \quad (10)$$

Let $P(k) = \frac{\partial P[k_i(t)<k]}{\partial k}$ and $\gamma = 1 + 1/\gamma_1$, the $P(k)$ is:

$$P(k) = \frac{t}{\gamma_1[3m + t(1+2p)]}\frac{(m_1 + \frac{\gamma_2}{\gamma_1})^{\frac{1}{\gamma_1}}}{(k + \frac{\gamma_2}{\gamma_1})^{\gamma}} \quad (11)$$

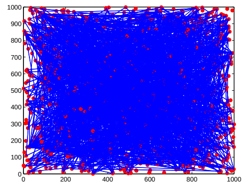
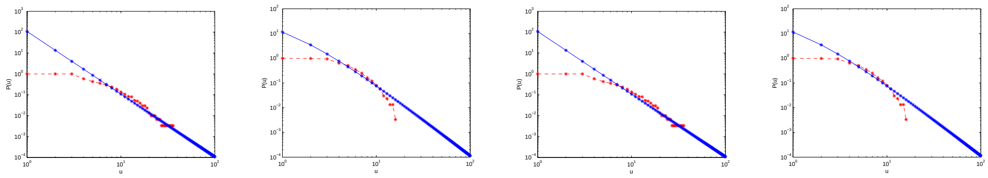


Fig. 1: structure graph of the model



(a) q=r=0.3 (b) q=r=0.3 (c) q=r=0.3 (d) q=r=0.3

Fig. 2: Degree distribution

4 Simulation and analysis

Suppose three local world exist initially in the network,$m = 3$, the parameters are$m_0 = 5, m_1 = 2, m_2 = 2, m_3 = 1, m_4 = 1$.The fig.1 below shows the structure graph of the interbank market network.

Here we will firstly talk about 4 special cases of the model we propose.

1) if$m = 1, q = 1$, there is only one local world in the network, the growth only brings new nodes into the network, and the index$\gamma = 3 + \alpha/m$. Here the MLV model degrades into a typical BA model.

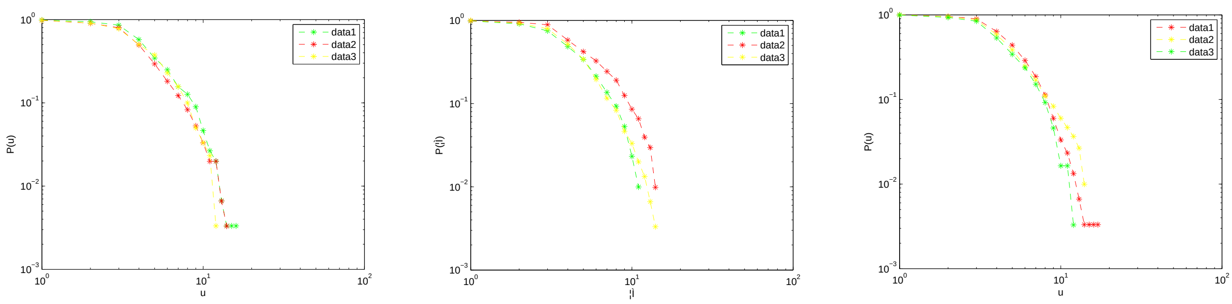
2) if $m = 1$,$p = s = u = 0$,then there is only one local world and the growth only considers the addition of nodes and edges. The index $\gamma = 3 + \alpha q/[(m_1 - m_2)q + m_2]/m$, which means the relationships between the existing nodes in the network are very important.

3) if $p = r = s = 0$, there are some local worlds at the beginning and addition of edges in the same local world, and the index $\gamma = 2 + (m_1 + \alpha)q/[(m_1 - 2m_4) + 2m_4]$.

4) if $rm_2 = 2sm_3$,$b = \alpha$,we see $P(k) \propto (k + \alpha)^{-\gamma}$, this shows that the attraction of the nodes is important to the growth of the network.

Fig.2 shows the degree distribution of the model under these 4 special cases discussed above. It is obvious that they all follow a power law distribution.

By comparing graph a and b in Fig.3, we can see the degree distribution in Fig.3(a) increases more than in Fig.3(b) with the increase of q and the decrease of$r$, thus we know the attraction of nodes plays a more important role than the edges between the existing nodes. By comparing graph Fig.3(a) and Fig.3(c) we can see the degree distribution also increase so the links between different local world also bring the increase of degree. In the daily operation, the banks usually become



(a) p=0.1,q=0.3,r=0.3,s=0.2 (b) p=0.1,q=0.3,r=0.3,s=0.2 (c) p=0.1,q=0.3,r=0.2,s=0.2,u=0.1

Fig. 3: Degree distribution

liquidity shortage or surplus for a short term, so the banks will choose a bank to borrow or lending according to their own needs. Initially, there is only a small number of banks and a small amount of lending, but with the bank's development and economic progress and the development of globalization, more and more banks to start joining the interbank market. Based on their size and type banks can be divided into different communities, competition and cooperation relationship are existed between communities, each bank can change its links for their own community development need. From the analysis above we know in the interbank market both the addition of banks and the increase of links between existing banks affect the structure of the interbank networks. Furthermore the addition of banks makes a more profound effect on the structure than the increase of links between existing banks.

5 Conclusion

Based on community character of the interbank market, we propose an interbank market network model, the world's multi-local network evolution model. And through theoretical analysis of the model we prove that the degree distribution follows a power law distribution, and simulation results further verify the result. Furthermore we find the addition of new banks makes a more profound effect on the structure than the increase of links between existing banks which can guide the practice application. In this paper, there are also some disadvantages, the selection of the nodes only consider the preferential attachment mechanism not the banks credit which has important significance in the actual network. This is interpreted as a competitive relationship in the process of establishing relationship between banks which can be reflected by the fitness of the network model. Future research will take fitness into consideration, how to determine the value of fitness and study the structural features with a given fitness distribution.

Acknowledgments

This research was supported by the National Nature Science Foundation of China (No. 71271103, No.71371087, No.71271107) and the Priority Academic Program Development of Jiangsu Higher Education Institutions.

References

[1] X F Wang, X Li, G R Chen. Complex theory and application. *Tsinghua University Press, Beijing*, 2006.

[2] Simone L, Gabriele T. Systemic risk on different interbank network topologies. *physica A*, 391(2012):4331-4341.

[3] S W Li, J M He, Y M Zhuang. Fitness model for tiered structure in the interbank market. *Complexity* , 23(2005):379-389.

[4] S W Li, J M He.The impact of bank activities on contagion risk in interbank networks.*Advances in Complex Systems*, 15(1250086).

[5] Tedeschi G, Mazloumian A,Gallegati M, Helbing D. Bankruptcy Cascades in Interbank Markets. *PLoS ONE*,7(12)(2012): e52749.doi:10.1371/journal.pone.0052749

[6] S W Li, J M He, Y M Zhuang. A network model of the interbank market. *Phys A* , 389(2010):5587C5593.

[7] S Y Wang. Interbank contagion and risk immunization:an quantitative framework based on network theory.*Shanghai Jiaotong University, Shanghai*, 2007.

[8] H Li, T Q Chen, J M H. The Interbank Market Network Structure Evolution Model based on the Complex Network Theory. *Journal of Beijing institute of technology (social science edition)*, Beijing, (14) 2012:71-76.

[9] Lelyveld I, Liedorp F. Interbank contagion in the Dutch banking sector [EB/OL]. *http://ssrn.com/abstract=632805*, 2011.

[10] Harrison S L A, Tudela M. Tiering in UK payment systems. *Bank of England Financial Stability Review*, 62(2005):63-72.

[11] Upper C, Worms A. Estimating bilateral exposures in the German interbank marketIs there a danger of contagion? *European Economic Review*,48(4)(2004):827-849.

[12] Boss M, Elsinger H, Summer M, Thurner S. Network topology of the interbank market. *Quant Finance*, 4(2004):677-684.

[13] Xuedi Wang, Yan Cheng. Adaptive Control of the Complex Network . *International Journal of Nonlinear Science*,8(3)(2009):313-317.

[14] Hongxing Yao, Jiejie Cui. Stability of a Time-delay and Impulsive Financial Model. *International Journal of Nonlinear Science*, 9(2)(2010):229-232.

[15] Jing Wu. An Empirical Study of In?uential Factors of Debt Financing .*International Journal of Nonlinear Science*, 3(3)(2007):208-212.