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On Dynamical Cournot Game on a Graph

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Abstract: Cournot dynamical game is studied on a graph. The stability of the system is studied. Prisoner's dilemma game is used to model natural gas transmission.

Keywords: dynamical Cournot game; graph; gas; water; electricity transportation

1 Introduction

Game theory [1] was first introduced by von Neumann and Morgenstern in 1944 as a mathematical model. It is the study of ways in which strategic interactions among rational players produce outcomes with respect to the preferences of the players. Each player in a game faces a choice among two or more possible strategies. A strategy is a predetermined program of play that tells the i player which action to take in response to every possible strategy other players may use . Transportation of natural gas electricity and water allocation for different purposes was studied recently by Ilkihc [2]. He modeled it as static Cournot game on a graph. We study the dynamic game. The dynamic equations are derived. The stability of the interior equilibrium solution is studied. Prisoner's dilemma game is used to model the source countries-transit countries- markets interaction on a graph.

2 A Cournot dynamic game on a graph

Transportation of natural gas electricity and water allocation for different purposes is an important problem. However most of its studies are static while the problem is dynamic. The vertices of the graph representing the problem are pf three types markets $m_i, i = 1, 2, ..., k_1$ firms (producers) $f_j,\ j = 1, 2, ..., k_2$ and transit countries through which the lines pass. The profit function of firm f_j is given by

$$\Pi_j = \sum_i \alpha_i q_{ij} - \gamma_j s_j^2/2 - \sum_i \beta_i q_{ij} c_i, \tag{1}$$

$$s_j = \sum_l q_{lj}, c_i = \sum_l q_{il}, \tag{2}$$

where q_{ij} are the production quantity from firm j to market i and α_i, β_i, γ_j are positive constants. The sum over i is on all markets connected to firm j. Conversly the sum in c_i is over all firms supplying market i.

The dynamic Cournot game with bounded rationality is given by [3, 4]

$$dq_{ij}/dt = b_j q_{ij}(\partial\Pi_j/\partial q_{ij}). \tag{3}$$

The parameters b_j are proportionality parameters.

For general graph Cournot bounded rationality game the dynamic equations are

$$dq_{ij}/dt = b_j q_{ij}[\alpha_i - \gamma_j \sum_l q_{lj} - \beta_i q_{ij} - \beta_i \sum_k q_{ik}]. \tag{4}$$

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Here we will consider a simple graph consisting of two firms and two markets. The first firm supplies both markets while second firm supply only second market. In this case the above system takes the form

$$\begin{aligned} dq_{11}/dt &= q_{11}[\alpha_1 - \gamma_1(q_{11}+q_{21}) - 2\beta_1 q_{11}], \\ dq_{21}/dt &= q_{21}[\alpha_2 - \gamma_1(q_{11}+q_{21}) - \beta_2(2q_{21}+q_{22})], \\ dq_{22}/dt &= q_{22}[\alpha_2 - \gamma_2 q_{22} - \beta_2(2q_{22}+q_{21})], \end{aligned} \tag{5}$$

after rescaling the system becomes:

$$\begin{aligned} dq_{11}/dt &= q_{11}[1 - r_1 q_{11} - q_{21}], \\ dq_{22}/dt &= q_{22}[1 - r_2 q_{22} - q_{21}], \\ dq_{21}/dt &= q_{21}[1 - r_3 q_{21} - r_4 q_{11} - r_5 q_{22}]. \end{aligned} \tag{6}$$

Applying Routh-Hurwitz conditions [5] the unique interior equilibrium of the above system is locally asymptotically stable if the following conditions are satisfied:

$$\begin{aligned} a_1 &> 0,\ a_3 > 0,\ a_1 a_2 > a_3, \qquad (7) \\ a_1 &= r_1 + r_2 + r_3, \\ a_2 &= r_1 r_2 + r_1 r_3 + r_2 r_3 - r_4 - r_5, \\ a_3 &= r_1 r_2 r_3 - r_1 r_5 - r_2 r_4. \end{aligned}$$

For the special case $r_1 = r_2$ the above results simplifies significantly. The unique interior equilibrium solution become

$$\begin{aligned} q_{11} &= q_{22} = (1 - q_{21})/r_1, \\ q_{21} &= (r_1 - r_4 - r_5)/(r_1 r_3 - r_4 - r_5). \end{aligned} \tag{8}$$

The existence and stability conditions are

$$\begin{aligned} r_1 &> r_4 + r_5, \\ r_3 &> 1. \end{aligned} \tag{9}$$

The approximate solutions for system (6) displayed in Figs. 2.1-2.2.

In Fig.2.1 we take $q_{11}(0) = 0.1, q_{22}(0) = 0.2, q_{21}(0) = 0.3, r_1 = 0.01, r_2 = 0.1, r_3 = 1.1, r_4 = -0.3,$and $r_5 = 0.4$.

In Fig.2.2 we take $q_{11}(0) = 0.1, q_{22}(0) = 0.2, q_{21}(0) = 0.3, r_1 = 0.2, r_2 = 0.5, r_3 = 1.5, r_4 = -0.3,$and $r_5 = 0.4$.

In Fig.2.1 we find that the interior equilibrium point is unstable where the conditions (7) is not satisfied.

In Fig.2.2 we find that the interior equilibrium point $(1.13636, 0.454545, 0.772727)$ is locally asymptotically stable where the conditions (7) is satisfied.

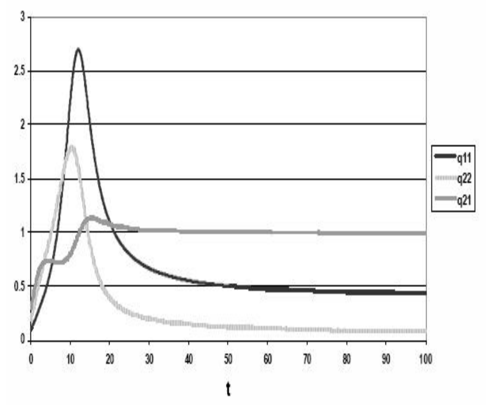

Fig 2.1

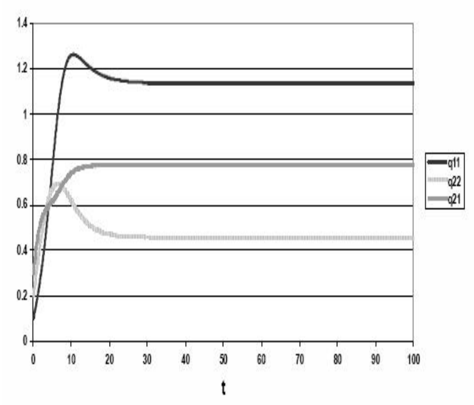

Fig 2.2

3 Cooperation on graphs

In natural gas, electricity and water transport a new situation exists namely transit countries. Those where transmission lines pass e.g. Bellarus in the line Russia-Bellarus-EU gas line. In this case a Prisoner's Dilemma (PD) game [6] exists between both producers and end users from one side and transit countries on the other. In PD game there are two strategies namely to cooperate or to defect. Typically the payoff matrix for such a game is given by

$$\begin{bmatrix} R & S \\ T & U \end{bmatrix}, T > R > U > S. \tag{10}$$

The standard dominant strategy is to defect hence both sides lose a lot. A way to solve this problems is through side payments paid by both producers and end users to transit countries.

Games on graphs has been studied by May and Sigmund [7]. They have shown that in the case of Prisoner's dilemma game cooperation can exist easier than the standard game.

4 Conclusions

Dynamical games representing natural gas, electricity and water are studied using Cournot game on a graph. Existence and stability of the unique interior equilibrium solution is investigated. Prisoner's dilemma game is used to model the producers- transit countries-end users interactions. The existence of graphs will improve the possibility of cooperation between these countries.

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