Stabilized Control of Fiber-optic Signals in the Nonlinear Schrödinger Equation

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Abstract: In this paper, the four-order Runge-Kutta method is to study the nonlinear Schrödinger equation with external perturbation. By adding a proper controller, the chaotic fiber-optic signal will arrive at a stable state. More new complex dynamical behaviors can be found.

Keywords: Fiber-optic signal; Nonlinear Schrödinger equation; four-order Runge-Kutta method

1 Introduction

The nonlinear Schrödinger equation

\[ iu_t + u_{xx} + u |u|^2 = 0 \]  \hspace{1cm} (1)

is widely used in many areas of physics, such as the evolution of nearly monochromatic, high intensity laser beam propagation and one-dimensional waves in deep wave. It also describes the evolution of the slowly varying envelope of an optical plus [1–3]. The NLS equation plays an important role in understanding optic fibers which is of importance to the fiber-based telecommunications [4].

On the external perturbation conditions, the process of fiber-optic signal transmission can be depicted as the following equation

\[ iu_t + u_{xx} + u |u|^2 = d \cos(\omega x) \exp(ict), \]  \hspace{1cm} (2)

where \( d \) and \( \omega \) are the amplitude and frequency of a certain perturbation respectively. The \( c \) is the coefficient of linear term.

In this paper, we will add the controller which has the same function with the damping. The organization of the paper is as follows. In Section 2, we the chaotic behavior of system (2) and the chaos control in controlled system (3) are studied. Last section is the conclusion.

2 Chaos and control system

Let \( u = \phi(x)e^{ict} \) and substituting it into equation (2), we get

\[ \phi'' + \phi^3 + c\phi = d \cos(\omega x) \]  \hspace{1cm} (4)

We make the transformation \( \phi \rightarrow x_1, \phi' \rightarrow x_2 \), then equation (4) can be transformed into first-order non-autonomous equations

\[ \begin{align*}
x_1' & = x_2, \\
x_2' & = -x_1^3 + cx_1 + d \cos(\omega x). 
\end{align*} \hspace{1cm} (5) \]

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We will discuss the behaviors of the fiber-optic signal transmission under perturbation by the four-order Runge-Kutte method. The study is carried out by taking $c = 1, \omega = 0.05$ and setting $d$ as the variable with the initial condition $[1.0, 0.0]$. The results are given by maximum Lyapunov exponents and Phase plane diagram.

According to Lyapunov exponents (see Fig. 1). We can observe that the value of Lyapunov exponents is positive, so the system easily converts to chaos even if there is small perturbation. The chaos is undesirable. So adding an appropriate controller is needed to satisfy the practical applications of fiber-optic communication.

![Figure 1: The Maximum Lyapunov exponents of $x(d \in [0, 30])$](image)

We have performed computer simulations on the perturbed system (5).

![Figure 2: Phase trajectories](image)

Next we study control of fiber-optic signals in the nonlinear Schrödinger equation.
Suppose $u = \phi(x)e^{ict}$ and substitute it into (3), then equation (3) takes the form as follows:

$$\phi'' + \phi^3 + c\phi = d\cos(\omega x) - \epsilon\phi'.$$

(6)

So (6) can be written as the following form:

$$x_1' = x_2,$$

$$x_2' = -x_1' + cx_1 + d\cos(\omega x) - \epsilon x_2.$$

(7)

Now we will study the chaotic control of system (7) with $c = 1$, $d = 2$, $\omega = 0.05$. According to Lyapunov exponents (see Fig.3), we can obtain that the behavior of (7) is still chaotic within $\epsilon \in (0, 0.052)$ for the controller being too weak to inhibit the chaos. Chaos of system (6) can be suppressed to the stable station with the larger $\epsilon$. It is easy to see that the signal can not propagate normally and might leak from the media, which is called escape.

![Figure 3](image-url)

Figure 3: (a) The Maximum Lyapunov exponents of $x(\epsilon \in [0, 30])$;(b) Enlarging view of (a) ($\epsilon \in (0, 0.1)$)

We have performed computer simulations on the perturbed system (7).

3 Conclusions

Recently, there has been much more interest in completely integrable the nonlinear Schrödinger equation with external perturbation. In this paper, we conclude that chaos may occur easily in this system due to the absence of damping in system (2). This phenomenon will cause the distortion in the process of information transmission. The efficiency of this controller was demonstrated. The complex fiber-optic transmission system of the perturbed NLSE was controlled.

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Figure 4: Phase trajectories: (a) $\varepsilon = 0.055$, (b) $\varepsilon = 1.55$, (c) $\varepsilon = 8.55$, (d) $\varepsilon = 12.5$

References


IJNS homepage: http://www.nonlinearscience.org.uk/