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The Effect of Endogenous Growth Model of Renewable Hydrogen Energy on the Environment by Using the Theory of "Learning by Doing"

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Abstract: This paper studies a new endogenous growth model about the conversion of renewable energy into hydrogen energy by using the theory of "learning by doing", introduces consumption and environmental factors into the utility function, and theoretically analyzes the relationship among energy, environment and economic growth. By analyzing the optimal growth path of the model, the internal relations among environmental pollution, capital accumulation, endogenous technological progress and economic sustainable development are discussed. Finally, it is found that after adding environmental pollution to the endogenous growth model, when the capital development is fully effective, the inter-temporal substitution elasticity is less than 1, and the regeneration capacity of the environment is large enough, the sustainable optimal growth of the economy can be achieved.

Keywords: learning by doing; endogenous growth; optimal growth path

1 Introduction

In recent years, how to achieve sustainable economic growth and development, to realize the replacement and combination of renewable energy and non-renewable energy, and to improve environmental quality has been a topic of great concern. With the increasing pressure of global energy supply and the great pressure of greenhouse gas emission reduction, the energy transformation is imminent. Tahvaonen and Salo [1] studied the transition between non-renewable energy and renewable energy at different stages of economic development. Acemoglu et al. [2] studied the importance of substitutability between non-renewable and renewable inputs in guiding endogenous technological change. Saad Mekhilef et al. [3] pointed out that Malaysia, as a member of the developing countries and the association of Southeast Asian Nations (ASEAN), is researching endless and inexhaustible alternative energy sources, such as solar energy, wind energy, micro-hydropower and biomass energy. With the development of society, the proportion of renewable energy in energy use is increasing. However, in the development and utilization of renewable energy, many times the cost has not been reduced, and the benefits are not high. So we consider converting part of renewable energy into hydrogen energy, which is more richer, greener and lower-carbon. Liu Jian [4] analyzed the application potential of hydrogen energy in China's energy industry in the future based on the current situation of hydrogen energy production and consumption in China. Mostafa Rezaei et al. [5] examined the economics of hydrogen production from wind energy in Afghanistan. The conversion of renewable energy to hydrogen energy can not only reduce costs, but also meet the requirements of environmental protection. Multi-energy progresses and energy efficiency improves, which in turn affects the sustainability of energy development.

Learning by doing is a concept in the classical economic growth model, and the main content of the technology endogenous growth model. Technology is considered as an endogenous variable because one of the most important sources of technological change is learned from observation and practice rather than developed through specialized research. The

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concept of learning by doing can be traced back to Arrow [6], which is an important determinant of workers' knowledge. Arrow used the old-fashioned method in his research. Levhari [7] showed that the results can be extended to any first-order homogeneous production function. This idea was optimized and integrated into the neoclassical growth model by Sheshinski [8] and Romer [9]. In the "learning by doing" model, experience and learning are expressed in the form of tangible elements input, so as to realize the endogenous of technological progress. Stephane bouche [10] integrated capital and technology into the growth model through "learning by doing", and then studied the endogenous discount in the growth model.

Most research of domestic and foreign scholars on economic growth focus on economic growth and environmental pollution, as well as energy substitution and economic growth, but there is few research on combining economic pollution and energy substitution in economic growth model. There is even less research in the field of endogenous growth to replace the waste part of renewable energy with renewable hydrogen energy for reuse. On the basis of the existing literature, this paper refers to the method of Stephane Bouche [10] to study the relationship between "learning by doing", endogenous discouning and economic development, and draws on the idea of Sun Gang [11] and Peng Shuijun [12] to analyze sustainable development problems by using growth theory. Based on AK model [13], this paper introduces environmental quality as an endogenous factor into the optimal growth model. By analyzing the optimal growth path of the model, the internal relations of negative externalities of environmental pollution, capital accumulation, endogenous technological progress and sustainable economic development are explored.

The structure of this paper is as follows: the second section introduces the model, including production function, energy, material capital, environmental quality and utility function. In the third section, the model is solved and made a comparative static analysis of equation. The fourth section summarizes the article.

2 Model

2.1 Production function

Suppose a closed social-economic system is composed of R & D sectors and final product sectors. The production function takes the form of the Cobb-Douglas production function, standardizes labor force to 1, and adds energy and environment as input factors into the production function. Set the production function of the final product department as:

$$Y = AK^{\alpha}R^{\beta}Q^{\gamma}z, \tag{1}$$

where A is the production technology, K is the capital stock, R represents the renewable energy input, Q represents the renewable hydrogen energy. α, β ,γ are the output elasticity of hydrogen energy converted from capital, renewable energy and renewable energy respectively, and $\alpha + \beta + \gamma = 1$. z is pollution intensity, and $0 \leq z \leq 1$. Pollution flow P is an increasing function of output level and pollution intensity, $P\left(Y, z\right) = Yz^{\theta}$, $\theta > 0$ represents the degree of pollution. If $z = 1$, then the economic output is not affected by the environment, that is, the impact of the environment is not considered, so there is the largest potential output; If $z < 1$, then the economic output is affected by the environment, that is, the impact of the environment is considered, the actual output will be lower than the potential output.

2.2 Energy

There are abundant renewable energy sources in nature, such as solar energy, wind energy, micro-hydro power, and biomass energy. Renewable energy is widely distributed and renewable, so it is the main direction of energy development in the future. In the R & D sector, the resource stock equation of renewable energy is as follows:

$$\dot{S}_R = \xi R\left(1 - \frac{R}{M}\right) - S_1, \tag{2}$$

where ξ is the exponential growth rate of renewable energy, M is the environmental carrying capacity, and S_1 represents the amount of renewable energy input in the production process.

Renewable energy is the focus of future energy development. In the process of development and utilization, many times the cost has not been reduced, and the benefits are not high. So we consider converting part of renewable energy into richer, greener and lower-carbon hydrogen energy. On the one hand, hydrogen can be produced in various ways, and on the other hand, the waste part of renewable energy can be converted to extract hydrogen energy. So the resource stock equation of renewable hydrogen energy in the R & D sector is:

$$\dot{S}_Q = \eta Q + \zeta R - S_2, \tag{3}$$

where η is the exponential growth rate of renewable hydrogen energy, ζ is the conversion rate of renewable energy to renewable hydrogen energy, that is, the highest level of resource stocks maintained by the natural environment, and S_2 represents the amount of renewable hydrogen energy input in the production process.

2.3 Physical capital

Since the depreciation rate of physical capital has no effect on the economic growth in steady state, the depreciation rate of capital is not considered in this paper. The physical capital accumulation equation is:

$$\dot{K} = Y - C, \tag{4}$$

where C is consumption.

2.4 Environmental quality

Environmental quality E can be measured by the difference between the actual environmental quality and its upper limit value, so E always takes a negative value. The motion equation of environmental quality is:

$$\dot{E} = -P\left(Y, z\right) - \varphi E = -Yz^{\theta} - \varphi E, \tag{5}$$

where $\varphi > 0$ is the environmental self-purification rate, indicating the maximum regeneration speed of the environment.

2.5 Utility function

Assuming that individual consumers are homogeneous, and use the same utility function. The utility obtained by consumers is not only related to the current consumption C, but also affected by the environmental quality E. In essence, the utility function is multivariate function. We add environmental factors to the utility function to obtain a standard additively separable and fixed elasticity utility function:

$$u\left(C, E\right) = \frac{C^{1-\epsilon}}{1-\epsilon} - \frac{E^{\omega+1}}{1+\omega} \tag{6}$$

where $\epsilon > 0$ is the marginal utility elasticity coefficient and $\omega > 0$ is the environmental awareness parameter.

3 Solution

According to the above assumptions, the dynamic optimization problem with utility maximization as the objective function is constructed:

$$\begin{array}{l} \arg_{C,z} \max U\left(C, E\right) = \int_0^{\infty} u\left(C, E\right) e^{-\rho t} dt \\ s.t. Y = AK^{\alpha} R^{\beta} Q^{\gamma} z \\ \dot{K} = Y - C \\ \dot{E} = -P(Y, z) - \varphi E = -Yz^{\theta} - \varphi E \\ \dot{S}_R = \xi R \left(1 - \frac{R}{M}\right) - S_1 \\ \dot{S}_Q = \eta Q + \zeta R - S_2 \end{array} \tag{7}$$

where ρ is the discount rate, indicating the degree of consumer preference for consumption, and $\alpha + \beta + \gamma = 1$.

According to the model, we can establish Hamilton function:

$$J = U\left(C, E\right) + \lambda_1 \left(Y - C\right) + \lambda_2 \left(-Yz^{\theta} - \varphi E\right) + \lambda_3 \left[\xi R \left(1 - \frac{R}{M}\right) - S_1\right] + \lambda_4 \left(\eta Q + \zeta R - S_2\right) \tag{8}$$

among them, λ_1, λ_2, λ_3, λ_4 are Lagrange multipliers, which can be interpreted as the shadow prices of physical capital K, environmental quality E, renewable energy S_R, and renewable hydrogen energy S_Q, and they are state variables; C, z, R, Q are control variables.

The maximization first-order conditions are:

$$\frac{\partial J}{\partial C} = C^{-\epsilon} - \lambda_1 = 0, \tag{9}$$

$$\frac{\partial J}{\partial z} = AK^{\alpha}R^{\beta}Q^{\gamma}\left[\lambda_1 - \lambda_2\left(\theta+1\right)z^{\theta}\right] = 0, \tag{10}$$

$$\frac{\partial J}{\partial R} = \frac{\beta Y}{R}\left(\lambda_1 - \lambda_2 z^{\theta}\right) + \lambda_3\xi - \frac{2\lambda_3\xi}{M}R + \lambda_4\zeta, \tag{11}$$

$$\frac{\partial J}{\partial Q} = \frac{\gamma Y}{Q}\left(\lambda_1 - \lambda_2 z^{\theta}\right) + \lambda_4\eta. \tag{12}$$

After transforming the equations, we can get:

$$\lambda_1 = C^{-\epsilon}, \tag{13}$$

$$\lambda_2 = \frac{\lambda_1 z^{\theta}}{\theta+1}, \tag{14}$$

$$\lambda_3 = \left[\frac{\beta\theta Y}{R(\theta+1)}\lambda_1 + \lambda_4\eta\right]\frac{M}{\left(2R-M\right)\xi}, \tag{15}$$

$$\lambda_4 = \frac{\gamma Y}{Q\eta}\left(\lambda_2 z^{\theta} - \lambda_1\right). \tag{16}$$

Euler equations are:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial J}{\partial C} = \left[\rho - \frac{\alpha\theta Y}{\left(\theta+1\right)K}\right]\lambda_1, \tag{17}$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial J}{\partial E} = \left(\rho+\varphi\right)\lambda_2 - E^{\omega}, \tag{18}$$

$$\dot{\lambda}_3 = \rho\lambda_3 - \frac{\partial J}{\partial S_R} = \rho\lambda_3, \tag{19}$$

$$\dot{\lambda}_4 = \rho\lambda_4 - \frac{\partial J}{\partial S_Q} = \rho\lambda_4. \tag{20}$$

Thus,

$$g_{\lambda_1} = \rho - \frac{\alpha\theta Y}{\left(\theta+1\right)K}, \tag{21}$$

$$g_{\lambda_2} = \rho + \varphi - \frac{E^{\omega}}{\lambda_2}, \tag{22}$$

$$g_{\lambda_3} = \rho, \tag{23}$$

$$g_{\lambda_4} = \rho. \tag{24}$$

The cross-sectional conditions are:

$$\lim_{t\to\infty}\lambda_1 K e^{-\rho t} = 0, \lim_{t\to\infty}\lambda_2 E e^{-\rho t} = 0, \lim_{t\to\infty}\lambda_3 S_R e^{-\rho t} = 0, \lim_{t\to\infty}\lambda_4 S_Q e^{-\rho t} = 0. \tag{25}$$

Combining the first-order conditions Eq.(9) - (12) and Euler equations Eq.(17) - (20), the growth rate of each economic variable can be derived:

$$g_{\lambda_1} = -\epsilon g_C, \tag{26}$$

$$g_{\lambda_2} = g_{\lambda_1} - \theta g_z, \tag{27}$$

$$g_{\lambda_3} = g_Y + g_{\lambda_1} + g_{\lambda_4} - g_R, \tag{28}$$

$$g_{\lambda_4} = g_Y + g_{\lambda_1} - g_Q, \tag{29}$$

$$\omega g_E = g_{\lambda_2}, \tag{30}$$

$$g_E = g_Y + \theta g_z, \tag{31}$$

$$g_Y = g_A + \alpha g_K + \beta g_R + \gamma g_Q + g_z. \tag{32}$$

In the path of the balanced growth, Y, K, C have equal growth rates, that is, $g_Y = g_K = g_C$. Assuming that the growth rate of production technology is constant, let $g_A = a$. By combining the growth rate of economic variables Eq.(26) - (32), we get the equilibrium growth rates of each variable:

$$\frac{\dot{C}}{C} = g_C = \frac{1}{\epsilon}\left[\frac{\alpha\theta Y}{(\theta+1)K} - \rho\right], \tag{33}$$

$$g_z = -\frac{\epsilon+\omega}{(1+\omega)\theta} g_C, \tag{34}$$

$$g_R = (1-\epsilon) g_C, \tag{35}$$

$$g_Q = (1-\epsilon) g_C - \rho, \tag{36}$$

$$g_E = \frac{1-\epsilon}{1+\omega} g_C, \tag{37}$$

$$g_Y = g_K = g_C = (\alpha - \gamma\rho)\left[(1-\alpha) - (1-\epsilon)(\beta+\gamma) + \frac{\epsilon+\omega}{(1+\omega)\theta}\right]^{-1} \tag{38}$$

For Eq.(33), in the steady state, since the equation allows $\frac{Y}{K}$ to grow at the same rate, the consumption growth rate may be positive. From Eq.(34) - (38), we can know that the critical conditions for the existence of the sustainable optimal growth path are:

$$\alpha - \gamma\rho > 0, \tag{39}$$

$$1 - \epsilon < 0, \tag{40}$$

$$(\epsilon-1)(\alpha-\gamma\rho) < \varphi\left[(1-\alpha)(1+\omega) - (1-\epsilon)(\beta+\gamma)(1+\omega) + \frac{\epsilon+\omega}{1+\omega}\right] \tag{41}$$

Among them, Eq.(39) guarantees that along the optimal growth path, the growth of output, consumption and capital accumulation is unlimited, that is, $g_Y = g_K = g_C > 0$. According to Eq.(37) ,the preference restriction of condition $\epsilon > 1$ can ensure that consumers with rational expectation will not destroy the environment below the threshold, that is $g_E < 0$, otherwise, when the environment is destroyed below the threshold, it will not be repaired.

Our hypothesis ensures that the ratio of pollution emission to environmental quality in the production process is greater than zero, so that the growth rate of environmental quality is less than the self-purification rate of the environment. Thus, from the motion equation of environmental quality Eq.(5), we can get:

$$-\frac{Yz^{\theta}}{E} = \frac{\dot{E}}{E} + \varphi > 0 \tag{42}$$

If the regeneration speed or self-purification rate of the environment is sufficiently large, so that Eq. (41) meets Eq.(42),thereby avoiding the excessive growth of the relative environmental regeneration capacity, then the sustainable optimal growth path can be obtained.

In order to study the specific influence of parameters on the optimal growth rate, we make a comparative static analysis of Eq.(38):

$$\frac{\partial g_C}{\partial \theta} = \frac{(\alpha-\gamma\rho)(1+\omega)(\epsilon+\omega)}{\theta\epsilon(1+\omega)(\beta+\gamma)+\epsilon+\omega} > 0, \tag{43}$$

$$\frac{\partial g_C}{\partial \rho} = -\frac{\gamma\theta(1+\omega)}{\theta\epsilon(1+\omega)(\beta+\gamma)+\epsilon+\omega} < 0, \tag{44}$$

$$\frac{\partial g_C}{\partial \epsilon} = -\frac{[\theta(1+\omega)(\epsilon+\omega)+1](\alpha-\gamma\rho)(1+\omega]\theta}{[\theta\epsilon(1+\omega)(\beta+\gamma)+\epsilon+\omega]^2} < 0, \tag{45}$$

$$\frac{\partial g_C}{\partial \omega} = -\frac{\theta\left(\alpha - \gamma\rho\right)\left(1-\epsilon\right)}{\left[\theta\epsilon\left(1+\omega\right)\left(\beta+\gamma\right)+\epsilon+\omega\right]^2} > 0, \tag{46}$$

The formula $\frac{\partial g_C}{\partial \theta} > 0$ indicates that the stricter the environmental standards, the more conducive to increasing the rate of economic growth. The formula $\frac{\partial g_C}{\partial \rho} < 0$ indicates that the smaller the time discount rate, the stronger the sustainable development consciousness of the representative consumers, and the higher the long-term economic growth rate. The formula $\frac{\partial g_C}{\partial \epsilon} < 0$ indicates that increasing consumers' preference for inter-temporal consumption will make them give up their current consumption. The formula $\frac{\partial g_C}{\partial \omega} > 0$ indicates that the larger the environmental awareness parameter, the higher the consumer's preference for environmental quality, and the higher the steady-state growth rate.

4 Conclusions

After adding the problem of environmental pollution to the endogenous growth model, the sustainable optimal growth of the economy can be achieved when the capital development is fully effective, the elasticity of cross period substitution is less than 1, and the regeneration capacity of the environment is large enough. The following conclusions can be drawn along the growth path: (1) From Eq.(38), we can get that the unlimited growth of per capita output, consumption and capital accumulation is sustainable, that is, $g_Y = g_K = g_C > 0$. (2) From Eq.(34), we find that the growth rate of environmental pollution intensity $g_z < 0$, that is to say, with the continuous innovation of clean technology, the emission of pollutants is bound to decrease, then the environmental quality is constantly improved. (3) Eq.(40) ensures the growth rate of environmental quality $g_E < 0$. The larger the environmental awareness parameter, the higher the consumer's preference for environmental quality, and the higher the steady-state growth rate. (4) The smaller the time discount rate of consumers in the economy, the easier to meet the conditions of sustainable development.

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