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Robustness of Directed Networks under Localized Attack

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Abstract: Researchers are paying more and more attention to directed networks due to its strong practical significance. However, most of the researches on localized attack focus on undirected networks. Here, we present two ways of local attack based on the direction of edges attaching a node. The first one is to select a node as the root node and then start the attack from the root node, next attack neighbor nodes in the direction of its out-edge direction, and then attack neighbor nodes in the direction of their out-edge direction in the next step and so on until the $1 - p$ part of the node is removed, and finally the network is calculated the size of giant strongly connected component (GSCC) while the second is to select a node as the root node and then it is affected, followed by neighbors in the direction of its in-edge direction, and then neighbors in the direction of their in-edge direction, and so on until the p part of the node is left, and finally the network is calculated the size of GSCC. We apply these two local attacks to a single directed ER network and two coupling networks with two directed ER networks coupling together. Finally we obtain simulation results and analyze the relationship between average degree (average in-degree and average out-degree) and network robustness. Simulation results suggest that average in-degree and out degree have an important effect on network robustness.

Keywords: Robustness; Localized attack; Directed networks

1 Introduction

With the development of society and science, complex networks have caused great interest among scientists in the past few decades [1–16]. Complex networks are visible everywhere in life. The World Wide Web (WWW), a virtual network, can be seen as a network of web pages and hyperlinks. Human society can also be seen as a network of relationships between people and people. Other examples are biological networks [17], social network [18, 19], transportation network [20] and so on. The random network model initially proposed by Erdös and Rényi (ER) in 1960 [4]. Its degree distribution satisfies Poisson distribution while another common degree distribution is the power law distribution. The degree distribution of ER network satisfies the Poisson distribution while the degree distribution of SF network satisfies the power law distribution. Recently, coupling complex networks gradually become main research fields on complex network, such as interdependent networks [3, 21–23], interconnected networks [24, 25], multiplex networks [26, 27], temporal networks [28], multilayer networks [29], a network of networks [1]. According to the attack method, it can be divided into three types: random attack, hub-targeted attack, and localized attack. For the case of a random attack, each node in the network is removed with the same probability. For the case of target attack, a node with a large degree is more likely to be attacked than one with a small degree. There are many papers about exploring network robustness through target attacks and random attacks. Buldey et al. studied the catastrophic cascade of failures in interdependent networks and find that a broader degree distribution decrease the robustness of interdependent networks when it suffer random failure [16]. Feng et al. studied the percolation on multi-layer interdependent networks with degree-degree correlation and solved the critical value of the network through a simplified self-consistent probability framework [30]. Domenico et al. studied navigability of interconnected networks under random failures by using the coverage time of random walks and shown an individual network. And results indicate that system is more vulnerable to random failures than the interconnected network by introducing addition dimensions decreasing the vulnerability to random failures [24]. In addition, some papers are about

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local attacks. Shao et al. used generating function methods to researching the percolation on complex networks, when the network is suffering from a localized attack [31]. Dong et al. introduced treelike network and starlike network by introducing a probability function for the case of a target attack [32]. Min et al. present the vulnerability of interdependent infrastructure systems under spatially localized attacks and find space-local attacks cause fewer vulnerabilities than random equivalent failures [33].

But most of the previous researches on local attacks are about undirected networks. However, more and more evidences show that the real world cannot ignore its directionality. The directed network has also made a lot of progress in random attacks. Liu et al. studied the robustness of interdependent network through the generation function and found that the directionality inside the network increases the vulnerability of the interdependent network and exhibits hybrid phase transitions phenomena [34]. Dorogovtsev et al. described how to find the giant strongly connected component of the directed network and found that compared with the giant weakly connected component, under the random attack, the giant strongly connected component has less resilient [35]. Jin investigated the cascading vulnerability of directed and weighted networks and found out which power exponent of load-capacity function value can make the power law network and Poisson network more robust [36].

In this paper, we discuss the robustness of directed networks under localized attacks. This paper introduce two different failure modes according to the direction of the directed network, and simulation results about robustness of a single directed network and multiple connected directed networks are analyzed.

2 Model and results

Here we study the robustness of a single directed network and multiple interconnected directed networks under local attacks respectively. Due to the directionality of the edges, local attacks have two different forms. (i) Randomly selecting a node as the root node, and their neighbor nodes following out direction in the next step, and until the $1-p$ fraction of the node is removed. For this case, the neighbor nodes of a root node are found by the direction of the out-edge, as shown in Fig. 1(a). (ii) Randomly selecting a node as the root node, including its neighbors followed by in direction, are removed, and until the $1-p$ fraction of the node is removed. For this case, the neighbor node of a node is determined by the direction of the in-edge, as shown in Fig. 1(b).

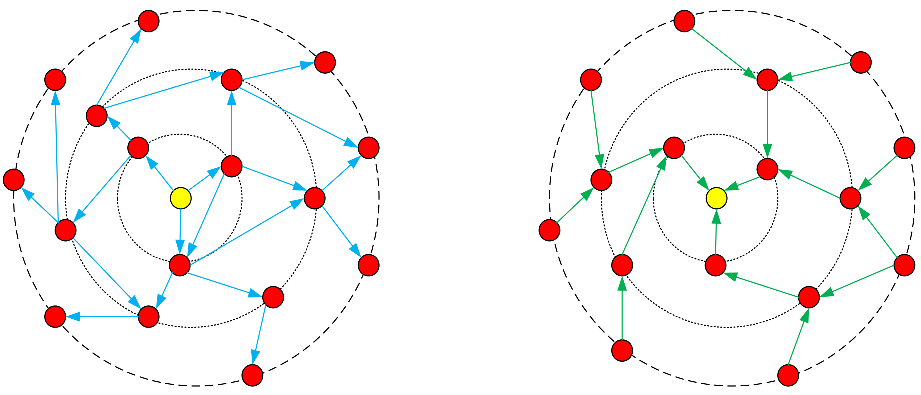

Figure 1: Let's set the l-th dotted circle in the figure as the l layer. Note that the $l+1$ layer node cannot be in the l layer. (a) Localized attack based on the direction of the out-edge of the node: randomly selecting a node as the root node (the yellow dot is the root node), and their neighbor nodes following out direction in the next step, and until the $1-p$ fraction of the node is removed. For this case, the neighbor nodes of a root node are found by the direction of the out-edge. (b) Localized attack based on the direction of the in-edge of the node: randomly selecting a node as the root node (the yellow dot is the root node), including its neighbors followed by in direction, are removed, and until the $1-p$ fraction of the node is removed. For this case, the neighbor node of a node is determined by the direction of the in-edge.

2.1 Single directed ER network

Here, we study the robustness of a single directed network in two failure modes and take the directed ER network as an example, the number of nodes in the network is set to N, and the average in-degree and out-degree are described by $<k_{in}>$, $<k_{out}>$ respectively. The average in-degree and average out-degree of the directed network are set to $<k>$ because they are the same, and then we performed two forms of local attacks on the network as above. Simulation results show that the proportion of giant strongly connected component (GSCC) in the entire network, as shown in Fig. 2, one can observe that the relationship between GSCC and p according to the out-edge attacking mode (Fig. 2(a)) and the in-edge attacking mode (Fig. 2(b)). The result of in the attack in the way of the out-edge is same as the result of the attack in the way of the in-edge. As the p value increases, the GSCC obtained in both cases is getting larger and larger. In Fig. 3, we notice that p_c value decreases as the average degree increases, which means that as the average degree increases, the robustness of the network becomes stronger.

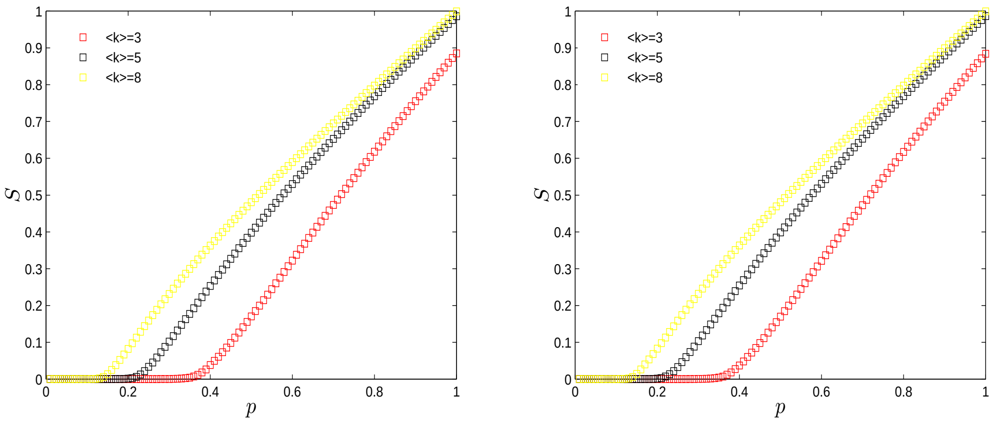

Figure 2: S as a function of p with a single directed network in two local attack modes. (a) Simulation results for in the way of the out-edge. N =10000, the number of iterations is 100. (b) Simulation results according to the way of in-edge. The parameters N =10000, results are average over 100 iterations.

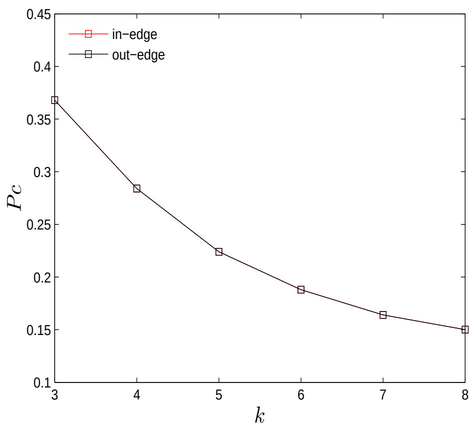

Figure 3: p_c value of a single directed network under different average degrees. The parameters N =10000, results are average over 100 iterations.

2.2 Two interconnected directed networks

In this section, we study the robustness of two coupling directed networks. Without loss of generality, we have studied the robustness of the two network connections (Fig. 4), the average degree (average out-degree and average in-degree) of the two networks is equal and the degree distribution between the two networks satisfies the Poisson distribution. The two networks are attacked in localized attack in the in-edge mode method or the out-edge mode method. We set k_{in}^{ii} as the networks inner average in-degree of a given node in network i, k_{out}^{ii} as the inner average out-degree of a given node in network i, k_{in}^{ij}(i$\neq$ j) is the outer average in-degree of a given node in network j from network i to network j. k_{out}^{ij}(i$\neq$ j) is the outer average out-degree of a given node in network i from network i to network j. For the convenience of description, we set the average in- and out-degree of a single network to $<k_1>$, and the average in- and out-degree between networks to $<k_2>$. As shown in Fig. 5, the result of the attack in the way of the out-edge is same as the result of in the way of the in-edge. As the p value increases, the GSCC obtained in both cases is getting larger and larger.

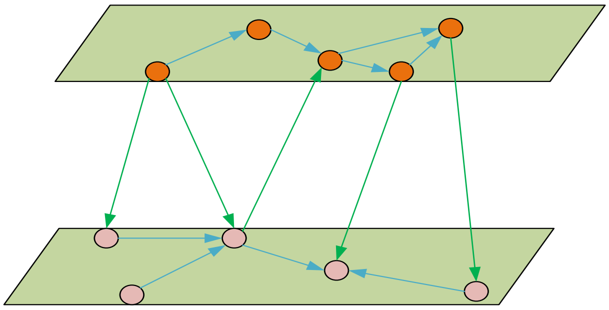

Figure 4: A system of two interconnected directed networks. The number of nodes of the two networks is N_1 and N_2, respectively. The out-degree and in-degree between the two networks obey distribution. The two directed networks are connected by directed edges, where the blue edge is the inner edge of the network and the green edge is the edge between the networks.

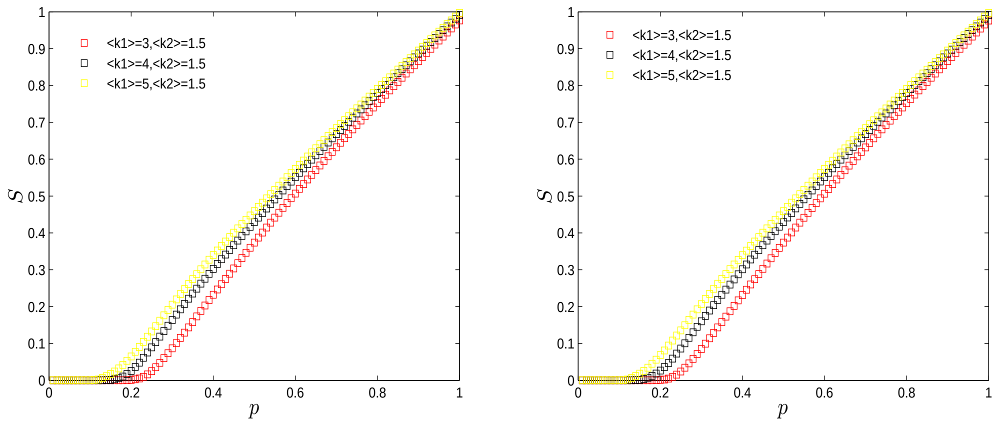

Figure 5: S as a function of p with a single directed network in two local attack modes. (a) Simulation results for in the way of the out-edge. (b) Simulation results according to the way of in-edge. The parameters N =10000, results are average over 100 iterations.

3 Conclusions

In this paper, we study the robustness of single directed ER network and two interconnected directed ER networks under two kinds of localized attack, which following in-degree and out-degree direction respectively. The simulation results imply that the two methods of local attack have a similar effect on a single directed ER network and two interconnected directed ER networks,respectively. In a single directed ER network, simulation results suggest the system becomes more robustness as the average degree increasing. And, the similar results can be get for two coupling directed ER networks.

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