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Global Synchronization of Complex Dynamical Networks with Internal Delay and Distributed-Delay Coupling via Aperiodically Intermittent Control

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Abstract: In this paper, the global synchronization problem for a class of complex dynamical networks with internal delay and distributed-delay coupling is investigated via aperiodically intermittent control. The network topology is assumed to be directed. Based on the reduction to absurdity and mathematical induction method, some sufficient conditions to guarantee global synchronization are derived analytically. Different from previous works, the control type here is aperiodic intermittent with changeable control period, which expands the scope of practical applications of intermittent control strategy. Finally, a numerical example is provided to demonstrate the effectiveness of the theoretical results.

Keywords: Complex dynamical networks; Internal delay; Distributed-delay coupling; Global synchronization; Aperiodically intermittent control

1 Introduction

Over the past decades, much effort has been devoted to the study of complex dynamical networks, which consist of a large set of interacting dynamical nodes connected by links. The main reason is that many real-world systems can be modeled as complex dynamical networks, such as ecosystems, neural networks, biomolecular networks, and social networks [1–3]. As a typical collective behavior, synchronization, which means that all dynamical nodes in a complex dynamical network achieve a common behavior, has become a hot research topic in various fields of science and engineering [4, 5]. This is partly due to its wide potential applications in many areas, including secure communication [6], parallel image processing [7], and pattern recognition [8]. Hitherto, several different synchronization patterns have been introduced and intensively investigated, such as complete synchronization [5], phase synchronization [9], generalized synchronization [10], and cluster synchronization [11].

In practical applications, it is usually expected that complex dynamical networks could realize synchronization or synchronize with a given trajectory by themselves. In the real world, however, due to the existence of weak coupling or heterogeneity [12], it is very hard for complex dynamical networks to achieve the objective automatically. In view of this fact, many effective control methods including state feedback control [13], adaptive control [14], impulsive control [15], and intermittent control [16], have been proposed to force the states of all dynamical nodes in a complex dynamical network into a desired objective trajectory. Intermittent control as a discontinuous control technique has been widely used in engineering fields for its practical and easy implementation in engineering control [16, 17]. In the implementation of intermittent control, the control is activated only during a sequence of disjoint nonzero time intervals [17]. Hence, in comparison with continuous feedback control such as state feedback control and adaptive control [13, 14], intermittent control can decrease the control cost and reduce the amount of information transmission.

Recently, a periodically intermittent control scheme (i.e., the control time of the control scheme is periodic, and in each period, the time is composed of work time and rest time [17]) has been adopted to investigate the synchronization problem for chaotic systems as well as complex dynamical networks, see [16–25] and references therein. For instance, the lag synchronization problem is explored for a class of neural networks with mixed delays via periodically intermittent control in [20]. In [22], the exponential synchronization problem for a class of complex networks with finite distributed delays

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coupling was considered via periodically intermittent control. In [16], the pinning synchronization problem for complex dynamical networks with or without time delays was analyzed via periodically intermittent control. In [24, 25], Liu et al. and Cai et al. investigated the cluster synchronization problem for complex dynamical networks under periodically intermittent pinning control.

However, the requirement of periodicity of intermittent control strategy in previous works [16–25], i.e., each control period is fixed (or equal to each other), is unreasonable. For instance, the generation of wind power in smart grid relies on the real world, which is obviously aperiodically intermittent [26, 27]. In addition, in practical applications, each control period of intermittent control strategy is needed to be changeable and therefore adjusted in accordance to actual situations. Therefore, it is significant and of prime importance to consider the synchronization problem for complex dynamical networks via aperiodically (or nonperiodically) intermittent control. Unfortunately, few results have been reported on this issue. In [26], aperiodically intermittent pinning control for the exponential synchronization of linearly coupled networks with delayed dynamical nodes was studied. In [27], the quasi-synchronization of nonlinear coupled networks with delayed dynamical nodes and parameter mismatches by using aperiodically intermittent pinning control was discussed.

Time delay is a very familiar phenomenon in various systems [16–25, 28–30]. Owing to the finite speeds of information transmission and processing, a signal traveling through a dynamical network usually is associated with time delay [16, 22–25, 28–30]. Hence, it is imperative to investigate the effect of time delay on synchronization of complex dynamical networks. In general, there exist two types of time delays in dynamical networks. One is internal delay occurring inside each individual dynamical node, i.e., each network node is a delayed dynamical system [28]. The other is coupling delay caused by the information exchange among network nodes [28]. Thus, realistic modeling of complex dynamical networks requires both the internal delay and coupling delay to be taken into account. Note that many real-world networks such as biological neural networks, metabolic pathways and ecosystems, normally have a long spatial distance between two coupled cells or network nodes, thus there will be a distribution of conduction velocities along these connected paths [22, 29, 30]. Accordingly, to better describe the coupling delay in a complex dynamical network, a more satisfactory way is to incorporate distributed delay. In view of these facts, in this paper we will study the synchronization problem for complex dynamical networks with internal delay and distributed-delay coupling via aperiodically intermittent control. To the best of our knowledge, there is still no theoretical result concerning this problem.

Motivated by the above analysis, this paper is concerned with global synchronization of a class of complex dynamical networks with internal delay and distributed-delay coupling via aperiodically intermittent control. Some sufficient conditions to guarantee global synchronization are derived by utilizing the reduction to absurdity and mathematical induction method. It is noted that the control type here is aperiodically intermittent, which takes the periodically intermittent control proposed in previous works as a special case. Additionally, the coupling matrix can be asymmetric and reducible. Finally, a numerical example is given to demonstrate the effectiveness of the proposed control method.

2 Problem formulation and preliminaries

Consider a general complex network consisting of N identical delayed dynamical nodes with distributed-delay coupling, which is described by [22]:

$$\dot{x}_i(t) = f(t, x_i(t), x_i(t-\sigma(t))) + c \sum_{j=1, j\neq i}^{N} b_{ij}\Gamma\Big(\int_{t-\tau}^{t} \rho(t-s)x_j(s)\,\mathrm{d}s - x_i(t)\Big), \tag{1}$$

where $i \in \mathfrak{I} = \{1, 2, \cdots, N\}$, $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^\top \in \mathbb{R}^n$ is the state variable of node i, $f : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector-valued function describing the dynamics of each isolated (uncoupled) node. The time delay $\sigma(t)$ is bounded satisfying $0 \leq \sigma(t) \leq \sigma$, which denotes the internal delay occurring inside the dynamical node [28]. $\rho(t)$ is a real-valued nonnegative bounded function defined on $[0, \tau]$, which reflects the influence of the past states on the current dynamics [20]. The positive constant $c > 0$ is the coupling strength, $\Gamma = (\gamma_{ij})_{n\times n}$ is the inner connecting matrix describing the individual coupling between nodes, $B = (b_{ij})_{N\times N}$ is the coupling matrix representing the network's topological structure, in which b_{ij} is defined as follows: if there is a directed link from node j to node i ($j \neq i$), then $b_{ij} > 0$; otherwise, $b_{ij} = 0$. This implies that the network is directed and the coupling matrix B is asymmetric. Additionally, the diagonal elements of matrix B are defined by $b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij}$, $i \in \mathfrak{I}$, and thus one has $\sum_{j=1}^{N} b_{ij} = 0$, $i \in \mathfrak{I}$. The initial conditions of the network (1) are given by $x_i(s) = \phi_i(s) \in C([-\tau^*, 0], \mathbb{R}^n)$, $i \in \mathfrak{I}$, where $C([-\tau^*, 0], \mathbb{R}^n)$ denotes the set of all $n-$dimensional continuous functions defined on $[-\tau^*, 0]$ with $\tau^* = \max\{\sigma, \tau\}$.

In the case that the network (1) reaches synchronization, i.e., $\lim_{t\to+\infty} \|x_i(t) - \pi(t)\| = 0$, $i \in \mathfrak{I}$, due to the fact that $b_{ii} = -\sum_{j=1,j\neq i}^{N} b_{ij}$, $i \in \mathfrak{I}$, then $\pi(t)$ satisfies the following dynamic equation:

$$\dot{\pi}(t) = f(t,\pi(t),\pi(t-\sigma(t))) - cb_{ii}\Gamma\Big(\int_{t-\tau}^{t} \rho(t-s)\pi(s)\,\mathrm{d}s - \pi(t)\Big), \quad i \in \mathfrak{I}. \tag{2}$$

Here, $\pi(t)$ is called the synchronous state. Evidently, the synchronous state $\pi(t)$ is uniform for all $i \in \mathfrak{I}$. Therefore, similar to [22, 23, 28], the assumption that $b_{11} = b_{22} = \cdots = b_{NN} = -a < 0$ is imposed for realizing the synchronization. In practice, the coupling matrix B can be chosen as follows [23]: $B = a\tilde{G}$ with $\tilde{G} = (\tilde{g}_{ij})_{N\times N} = (g_{ij}/(\sum_{j=1,j\neq i} g_{ij}))_{N\times N}$, where $G = (g_{ij})_{N\times N}$ be any square matrix satisfying $g_{ij} \geq 0$ $(i \neq j)$ and $g_{ii} = -\sum_{j=1,j\neq i}^{N} g_{ij} \neq 0$, $i \in \mathfrak{I}$. In this situation, it is easy to see that the diagonal elements of B are all equal to $-a$ satisfying the above assumption. Accordingly, the dynamic equation for $\pi(t)$ becomes the following form:

$$\dot{\pi}(t) = f(t,\pi(t),\pi(t-\sigma(t))) + ac\Gamma\Big(\int_{t-\tau}^{t} \rho(t-s)\pi(s)\,\mathrm{d}s - \pi(t)\Big). \tag{3}$$

In this paper, our main aim is to design some proper controllers such that the states of all nodes $x_i(t)$, $i \in \mathfrak{I}$ in the network (1) will be globally asymptotically synchronized with the synchronous state $\pi(t)$, i.e.,

$$\lim_{t\to+\infty} \|x_i(t) - \pi(t)\| = 0, \quad i \in \mathfrak{I}. \tag{4}$$

In order to achieve the synchronization objective (4), we introduce intermittent control strategy to nodes in the network (1). Accordingly, the controlled delayed dynamical network is given by:

$$\dot{x}_i(t) = f(t,x_i(t),x_i(t-\sigma(t))) + c\sum_{j=1,j\neq i}^{N} b_{ij}\Gamma\Big(\int_{t-\tau}^{t} \rho(t-s)x_j(s)\,\mathrm{d}s - x_i(t)\Big) + u_i(t), \tag{5}$$

where $i \in \mathfrak{I}$ and the control input $u_i(t)$ is an intermittent controller designed as

$$u_i(t) = -d_i(t)(x_i(t) - \pi(t)), \quad i \in \mathfrak{I}. \tag{6}$$

in which $d_i(t)$ is the intermittent feedback control gain described dy

$$d_i(t) = \begin{cases} d, & t_m \leq t < t_m + \delta_m, \\ 0, & t_m + \delta_m \leq t < t_{m+1}, \end{cases} \tag{7}$$

where $m \in Z^+=\{1,2,\cdots\}$, the control time sequence $\{t_m\}_{m=1}^{+\infty}$ satisfies $0 = t_1 < t_2 < \cdots < t_m < \cdots$ and $\lim_{m\to+\infty} t_m = +\infty$, $d > 0$ is a positive constant called control gain. The time span $[t_m, t_{m+1})$ is the time of the mth period, and $t_{m+1}-t_m$ is called the mth control period; $[t_m, t_m + \delta_m)$ is the mth work time, and $\delta_m > 0$ is called the mth control width (control duration); $[t_m + \delta_m, t_{m+1})$ is the mth rest time, and $(t_{m+1} - t_m) - \delta_m > 0$ is called the mth rest width (rest duration).

Remark 1 *It can be observed that the control time of the controller (6) is aperiodic, and each control period* $[t_m, t_{m+1})$ *is composed of "work time* $[t_m, t_m + \delta_m)$*" and "rest time* $[t_m + \delta_m, t_{m+1})$*". The controller is imposed to the network during the work time, but it is removed during the rest time. This kind of control strategy is called aperiodically (or nonperiodically) intermittent control [26]. Obviously, the above requirement of* t_m *and* δ_m *has a large scope. Especially, when* $t_{m+1} - t_m \equiv \mathrm{T}$ *and* $\delta_m \equiv \delta$, $m \in Z^+$, *the intermittent control type becomes the periodic one, which has been widely adopted in recent years (see [16–25] and the references therein).*

To derive the main results, the following assumptions are needed.

($\mathbf{A}_1$) (see [12]) For the vector-valued function $f(t,x(t),x(t-\sigma(t)))$, there exist a constant L_1 and a positive constant L_2 such that

$$\begin{aligned}[x(t)-y(t)]^\top[f(t,x(t),x(t-\sigma(t))) - f(t,y(t),y(t-\sigma(t)))] \leq{}& L_1\,[x(t)-y(t)]^\top[x(t)-y(t)] \\ &+L_2[x(t-\sigma(t))-y(t-\sigma(t))]^\top[x(t-\sigma(t))-y(t-\sigma(t))]\end{aligned}$$

for any $x(t), y(t) \in \mathbb{R}^n$.

($\mathbf{A}_2$) (see [22]) The kernel function $\rho : [0,\tau] \to [0,+\infty)$ is a real-valued nonnegative continuous function and satisfies

$$\int_0^\tau \rho(s)\mathrm{d}s = 1.$$

Remark 2 *In fact, there are many functions satisfying* ($\mathbf{A}_2$)*, such as* $\rho(t) = (1/\tau)$*,* $\rho(t) = \exp\{-t\}/(1-\exp\{-\tau\})$ *and* $\rho(t) = (2/\tau^2)(\tau - t)$ *for* $t \in [0, \tau]$*.*

3 Main results

In this section, we discuss global synchronization of the delayed dynamical network (1) under the aperiodic intermittent controller (6). Based on the reduction to absurdity and mathematical induction method, some sufficient conditions to guarantee the global synchronization will be derived.

For convenience, let $\mathrm{T}_0 = \hat{\mathrm{T}}_0 = t_1$, $\mathrm{T}_m = t_{m+1} - t_m$, $\hat{\mathrm{T}}_m = \sum_{j=0}^{m} \mathrm{T}_j$, and $\theta_m = \delta_m/\mathrm{T}_m$, $m \in Z^+$, where θ_m is called the mth control rate. Then, we get $t_m = \hat{\mathrm{T}}_{m-1}$ and $\delta_m = \theta_m \mathrm{T}_m$, $m \in Z^+$. Define error variables as $e_i(t) = x_i(t) - \pi(t)$, $i \in \mathfrak{I}$, then from (3) and (5)-(7), we can derive the following error dynamical system:

$$\begin{cases} \dot{e}_i(t) = \tilde{f}(t, x_i, \pi, x_i^{\sigma}, \pi^{\sigma}) + c \displaystyle\sum_{j=1, j\neq i}^{N} b_{ij} \Gamma\Big(\int_{t-\tau}^{t} \rho(t-s) e_j(s)\, \mathrm{d}s - e_i(t) \Big) - d e_i(t), & \hat{\mathrm{T}}_k \le t < \hat{\mathrm{T}}_k + \theta_{k+1}\mathrm{T}_{k+1}, \\ \dot{e}_i(t) = \tilde{f}(t, x_i, \pi, x_i^{\sigma}, \pi^{\sigma}) + c \displaystyle\sum_{j=1, j\neq i}^{N} b_{ij} \Gamma\Big(\int_{t-\tau}^{t} \rho(t-s) e_j(s)\, \mathrm{d}s - e_i(t) \Big), & \hat{\mathrm{T}}_k + \theta_{k+1}\mathrm{T}_{k+1} \le t < \hat{\mathrm{T}}_{k+1}, \end{cases} \tag{8}$$

where $i \in \mathfrak{I}$, $\tilde{f}(t, x_i, \pi, x_i^{\sigma}, \pi^{\sigma}) = f(t, x_i(t), x_i(t - \sigma(t))) - f(t, \pi(t), \pi(t - \sigma(t)))$, and $k = 0, 1, 2, \cdots$. Clearly, if the error variables satisfy $\lim\limits_{t\to+\infty} \|e_i(t)\| = 0$, $i \in \mathfrak{I}$, then the controlled delayed dynamical network (5) can realize global synchronization.

Theorem 1 *Suppose that all the rest widths* $(T_m - \delta_m)$*,* $m \in Z^+$*, are bounded, then the controlled delayed dynamical network (5) can realize global synchronization if there exists a positive constant* $\eta_1 > q$ *such that the following conditions hold:*

$$\text{(i) } 0 < (p + \eta_1) \le 2d, \quad \text{(ii) } \lim_{k\to+\infty} \Big[-\varepsilon\Big(\sum_{j=0}^{k} \mathrm{T}_j\Big) + (\eta_1 + p)\Big(\sum_{j=0}^{k} (1-\theta_j)\mathrm{T}_j\Big)\Big] = -\infty,$$

where $p = 2L_1 + ac\lambda_{\max}(\Gamma\Gamma^\top - (\Gamma + \Gamma^\top))$*,* $q = 2L_2 + ac$*, and* $\varepsilon > 0$ *is the unique positive solution of the equation* $\varepsilon - \eta_1 + 2L_2\exp\{\varepsilon\sigma\} + ac\exp\{\varepsilon\tau\} = 0$*.*

Proof. Let $V_i(t) = \dfrac{1}{2} e_i^\top(t) e_i(t)$, $i \in \mathfrak{I}$, then when $\hat{\mathrm{T}}_k \le t < \hat{\mathrm{T}}_k + \theta_{k+1}\mathrm{T}_{k+1}$, $k = 0, 1, 2, \cdots$, using ($\mathbf{A}_1$)-($\mathbf{A}_2$) and Lemma 1, the time derivative of $V_i(t)$ along the trajectories of (8) can be calculated as follows:

$$\begin{aligned} \dot{V}_i(t) &= e_i^\top(t) \Big[\tilde{f}(t, x_i, \pi, x_i^{\sigma}, \pi^{\sigma}) + c \sum_{j=1, j\neq i}^{N} b_{ij} \Gamma\Big(\int_{t-\tau}^{t} \rho(t-s) e_j(s)\, \mathrm{d}s - e_i(t) \Big) - d e_i(t) \Big] \\ &= e_i^\top(t) \tilde{f}(t, x_i, \pi, x_i^{\sigma}, \pi^{\sigma}) + c \sum_{j=1, j\neq i}^{N} b_{ij} \int_{t-\tau}^{t} \rho(t-s) e_i^\top(t) \Gamma e_j(s)\, \mathrm{d}s - c \sum_{j=1, j\neq i}^{N} b_{ij} e_i^\top(t) \Gamma e_i(t) - d e_i^\top(t) e_i(t) \\ &\le L_1 e_i^\top(t) e_i(t) + L_2 e_i^\top(t - \sigma(t)) e_i(t - \sigma(t)) - ac e_i^\top(t) \Gamma e_i(t) - 2d V_i(t) + \frac{c}{2} \sum_{j=1, j\neq i}^{N} b_{ij} \int_{t-\tau}^{t} \rho(t-s) \Big(e_i^\top(t) \Gamma\Gamma^\top e_i(t) + e_j^\top(s) e_j(s) \Big)\, \mathrm{d}s \\ &= (2L_1 - 2d) V_i(t) + 2L_2 V_i(t - \sigma(t)) + \frac{ac}{2} e_i^\top(t) (\Gamma\Gamma^\top - (\Gamma + \Gamma^\top)) e_i(t) + \frac{c}{2} \sum_{j=1, j\neq i}^{N} b_{ij} \int_{0}^{\tau} \rho(u) e_j(t-u)^\top e_j(t-u)\, \mathrm{d}u \\ &\le (2L_1 - 2d) V_i(t) + 2L_2 V_i(t - \sigma(t)) + \frac{ac}{2} \lambda_{\max}(\Gamma\Gamma^\top - (\Gamma + \Gamma^\top)) e_i^\top(t) e_i(t) + c \sum_{j=1, j\neq i}^{N} b_{ij} \int_{0}^{\tau} \rho(u) V_j(t-u)\, \mathrm{d}u \\ &= (p - 2d) V_i(t) + 2L_2 V_i(t - \sigma(t)) + c \sum_{j=1, j\neq i}^{N} b_{ij} \int_{0}^{\tau} \rho(u) V_j(t-u)\, \mathrm{d}u \\ &= (p + \eta_1 - 2d) V_i(t) - \eta_1 V_i(t) + 2L_2 V_i(t - \sigma(t)) + c \sum_{j=1, j\neq i}^{N} b_{ij} \int_{0}^{\tau} \rho(u) V_j(t-u)\, \mathrm{d}u. \end{aligned} \tag{9}$$

It follows from condition (i) that when $\hat{T}_k \le t < \hat{T}_k + \theta_{k+1}T_{k+1}$, $k = 0, 1, 2, \cdots$,

$$\dot{V}_i(t) \le -\eta_1 V_i(t) + 2L_2 V_i(t-\sigma(t)) + c\sum_{j=1, j\neq i}^{N} b_{ij}\int_0^{\tau}\rho(u)V_j(t-u)\,\mathrm{d}u, \quad i \in \mathfrak{T}. \tag{10}$$

Similarly when $\hat{T}_k + \theta_{k+1}T_{k+1} \le t < \hat{T}_{k+1}$, $k = 0, 1, 2, \cdots$, we can obtain

$$\begin{aligned}\dot{V}_i(t) &= e_i^{\top}(t)\Big[\tilde{f}(t, x_i, \pi, x_i^{\sigma}, \pi^{\sigma}) + c\sum_{j=1, j\neq i}^{N} b_{ij}\Gamma\Big(\int_{t-\tau}^{t}\rho(t-s)e_j(s)\,\mathrm{d}s - e_i(t)\Big)\Big]\\ &\le \Big(2L_1 + ac\lambda_{\max}(\Gamma\Gamma^{\top} - (\Gamma + \Gamma^{\top}))\Big)V_i(t) + 2L_2V_i(t-\sigma(t)) + c\sum_{j=1, j\neq i}^{N} b_{ij}\int_0^{\tau}\rho(u)V_j(t-u)\,\mathrm{d}u.\\ &= pV_i(t) + 2L_2V_i(t-\sigma(t)) + c\sum_{j=1, j\neq i}^{N} b_{ij}\int_0^{\tau}\rho(u)V_j(t-u)\,\mathrm{d}u, \quad i \in \mathfrak{T}.\end{aligned} \tag{11}$$

In the following, using (10), (11) and condition (ii), we prove that $\lim_{t\to+\infty} V_i(t) = 0$, for all $i \in \mathfrak{T}$.

Denote $\psi(z) = z - \eta_1 + 2L_2\exp\{z\sigma\} + ac\exp\{z\tau\}$. Since $\eta_1 > q > 0$, we get $\psi(0) < 0$, $\psi(+\infty) > 0$, and $\psi'(z) > 0$. According to the continuity and the monotonicity of $\psi(z)$, the equation $\psi(z) = z - \eta_1 + 2L_2\exp\{z\sigma\} + ac\exp\{z\tau\} = 0$ has an unique positive solution $\varepsilon > 0$. Take $M_0 = \sup_{-\tau^*\le s\le 0}\{\max_{i\in\mathfrak{T}} V_i(s)\}$, $Q_i(t) = \exp\{\varepsilon t\}V_i(t)$, where $t \ge -\tau^*$ and $i \in \mathfrak{T}$. Let $W_i(t) = Q_i(t) - hM_0$, where $h > 1$ is a constant and $i \in \mathfrak{T}$. Note that $\hat{T}_0 = 0$, then it is easy to see that

$$W_i(t) < 0, \quad \text{for all } t \in [-\tau^*, \hat{T}_0] \text{ and } i \in \mathfrak{T}. \tag{12}$$

Next, we prove that

$$W_i(t) < 0, \quad \text{for all } t \in [\hat{T}_0, \theta_1 T_1) \text{ and } i \in \mathfrak{T}. \tag{13}$$

We utilize the reduction to absurdity. Otherwise, by (12), there exist a $\ell \in \mathfrak{T}$ and $t^* \in [\hat{T}_0, \theta_1 T_1)$ such that

$$W_\ell(t^*) = 0, \quad \dot{W}_\ell(t^*) \ge 0, \quad W_j(t^*) \le 0, \quad \forall j \in \mathfrak{T}\setminus\{\ell\}, \tag{14}$$

and for all $i \in \mathfrak{T}$

$$W_i(t) < 0, \quad -\tau^* \le t < t^*. \tag{15}$$

Using (10), we obtain

$$\begin{aligned}\dot{W}_\ell(t^*) &= \varepsilon Q_\ell(t^*) + \exp\{\varepsilon t^*\}\dot{V}_\ell(t^*)\\ &\le \varepsilon Q_\ell(t^*) - \eta_1 Q_\ell(t^*) + 2L_2\exp\{\varepsilon t^*\}V_\ell(t^*-\sigma(t^*)) + c\sum_{j=1, j\neq\ell}^{N} b_{\ell j}\int_0^{\tau}\rho(u)\exp\{\varepsilon t^*\}V_j(t^*-u)\,\mathrm{d}u.\end{aligned} \tag{16}$$

From (14) and (15), we can get that $Q_\ell(t^*) = hM_0$, $Q_\ell(t) < hM_0$, $-\tau^* \le t < t^*$, and for all $j \in \mathfrak{T}\setminus\{\ell\}$ $Q_j(t) \le hM_0$, $-\tau^* \le t \le t^*$.

This means that $V_\ell(t^*) = hM_0\exp\{-\varepsilon t^*\}$, $V_\ell(t) < hM_0\exp\{-\varepsilon t\}$, $-\tau^* \le t < t^*$, and for all $j \in \mathfrak{T}\setminus\{\ell\}$ $V_j(t) \le hM_0\exp\{-\varepsilon t\}$, $-\tau^* \le t \le t^*$. Hence, $\exp\{\varepsilon t^*\}V_\ell(t^*-\sigma(t^*)) < \exp\{\varepsilon\sigma\}hM_0 = \exp\{\varepsilon\sigma\}Q_\ell(t^*)$ and $\exp\{\varepsilon t^*\}(\sup_{t*-\tau\le s\le t^*} V_j(s)) \le \exp\{\varepsilon\tau\}hM_0 = \exp\{\varepsilon\tau\}Q_\ell(t^*)$, $\forall\, j \in \mathfrak{T}\setminus\{\ell\}$.

According to $(\mathbf{A}_2)$, it follows that

$$\begin{aligned}\dot{W}_\ell(t^*) &< (\varepsilon - \eta_1 + 2L_2\exp\{\varepsilon\sigma\})Q_\ell(t^*) + c\exp\{\varepsilon\tau\}Q_\ell(t^*)\sum_{j=1, j\neq\ell}^{N} b_{\ell j}\int_0^{\tau}\rho(u)\,\mathrm{d}u\\ &= (\varepsilon - \eta_1 + 2L_2\exp\{\varepsilon\sigma\} + ac\exp\{\varepsilon\tau\})Q_\ell(t^*) = 0.\end{aligned} \tag{17}$$

This contradicts the second inequality in (14), which shows that (13) holds. Together with (12), we can obtain that

$$V_i(t) < hM_0\exp\{-\varepsilon t\}, \quad \text{for all } t \in [-\tau^*, \theta_1 \mathrm{T}_1) \text{ and } i \in \mathfrak{T}. \tag{18}$$

Denote $\varrho = p + \eta_1$. Now, we prove that

$$H_i(t) = Q_i(t) - hM_0\exp\{\varrho\,(t - \theta_1\mathrm{T}_1)\} < 0, \quad \text{for all } t \in [\theta_1\mathrm{T}_1, \hat{\mathrm{T}}_1) \text{ and } i \in \mathfrak{T}. \tag{19}$$

Otherwise, there exist a $\ell \in \mathfrak{T}$ and $t^{**} \in [\theta_1\mathrm{T}_1, \hat{\mathrm{T}}_1)$ such that

$$H_\ell(t^{**}) = 0, \quad \dot{H}_\ell(t^{**}) \geq 0, \quad H_j(t^{**}) \leq 0, \quad \forall j \in \mathfrak{T}\setminus\{\ell\}, \tag{20}$$

and for all $i \in \mathfrak{T}$

$$H_i(t) < 0, \quad \theta_1\mathrm{T}_1 \leq t < t^{**}. \tag{21}$$

For $\sigma(t^{**}) > 0$, if $\theta_1\mathrm{T}_1 \leq t^{**} - \sigma(t^{**}) < t^{**}$, it follows from (20) and (21) that

$$\exp\{\varepsilon t^{**}\}V_\ell(t^{**} - \sigma(t^{**})) < \exp\{\varepsilon\sigma\}hM_0\exp\{\varrho\,(t^{**} - \theta_1\mathrm{T}_1)\} = \exp\{\varepsilon\sigma\}Q_\ell(t^{**})$$

and if $-\tau^* \leq t^{**} - \sigma(t^{**}) < \theta_1\mathrm{T}_1$, it follows from (18) and (20) that

$$\exp\{\varepsilon t^{**}\}V_\ell(t^{**} - \sigma(t^{**})) < \exp\{\varepsilon\sigma\}hM_0 \leq \exp\{\varepsilon\sigma\}Q_\ell(t^{**}).$$

Therefore, we always have

$$\exp\{\varepsilon t^{**}\}V_\ell(t^{**} - \sigma(t^{**})) < \exp\{\varepsilon\sigma\}Q_\ell(t^{**}), \quad \text{for } \sigma(t^{**}) > 0.$$

Similarly, with the same analysis, we can derive that

$$\exp\{\varepsilon t^{**}\}(\sup_{t^{**}-\tau\leq s\leq t^{**}} V_j(s)) \leq \exp\{\varepsilon\tau\}Q_\ell(t^{**}), \quad \forall j \in \mathfrak{T}\setminus\{\ell\}.$$

Hence, using (11), we get

$$\begin{aligned}
\dot{H}_\ell(t^{**}) &= \varepsilon Q_\ell(t^{**}) + \exp\{\varepsilon\, t^{**}\}\dot{V}_\ell(t^{**}) - \varrho hM_0\exp\{\varrho(t^{**} - \theta_1\mathrm{T}_1)\} \\
&\leq \varepsilon Q_\ell(t^{**}) + pQ_\ell(t^{**}) - \varrho Q_\ell(t^{**}) + 2L_2\exp\{\varepsilon t^{**}\}V_\ell(t^{**} - \sigma(t^{**})) + c\sum_{j=1, j\neq\ell}^{N} b_{\ell j}\int_0^\tau \rho(u)\exp\{\varepsilon t^{**}\}V_j(t^{**} - u)\,\mathrm{d}u \\
&< (\varepsilon + p - \varrho + 2L_2\exp\{\varepsilon\sigma\} + ac\exp\{\varepsilon\tau\})Q_\ell(t^*) = 0,
\end{aligned}$$

which contradicts the second inequality in (20). Therefore (19) holds, i.e., for all $t \in [\theta_1\mathrm{T}_1, \hat{\mathrm{T}}_1)$ and $i \in \mathfrak{T}$,

$$Q_i(t) < hM_0\exp\{\varrho\,(t - \theta_1\mathrm{T}_1)\} \leq hM_0\exp\{\varrho\,(1 - \theta_1)\mathrm{T}_1\}.$$

Combining with inequalities (12) and (13), we get

$$Q_i(t) < hM_0\exp\{\varrho\,(1 - \theta_1)\mathrm{T}_1\}, \quad \text{for all } t \in [-\tau^*, \hat{\mathrm{T}}_1) \text{ and } i \in \mathfrak{T}.$$

Similar to the proofs of (13) and (19), we can prove that

$$Q_i(t) < hM_0\exp\{\varrho\,(1 - \theta_1)\mathrm{T}_1\}, \quad \text{for all } t \in [\hat{\mathrm{T}}_1, \hat{\mathrm{T}}_1 + \theta_2\mathrm{T}_2) \text{ and } i \in \mathfrak{T}$$

and

$$\begin{aligned}
Q_i(t) \quad &< \quad hM_0\exp\{\varrho(1 - \theta_1)\mathrm{T}_1\}\exp\{\varrho(t - \hat{\mathrm{T}}_1 - \theta_2\mathrm{T}_2)\} \\
&= \quad hM_0\exp\Big\{\varrho(t - (\theta_1\mathrm{T}_1 + \theta_2\mathrm{T}_2))\Big\}, \quad \text{for all } t \in [\hat{\mathrm{T}}_1 + \theta_2\mathrm{T}_2, \hat{\mathrm{T}}_2) \text{ and } i \in \mathfrak{T}.
\end{aligned}$$

By mathematical induction, we can derive the following estimation of $Q_i(t)$ for any nonnegative integer k and $i \in \mathfrak{T}$.

For all $\hat{\mathrm{T}}_k \leq t < \hat{\mathrm{T}}_k + \theta_{k+1}\mathrm{T}_{k+1}$, $k = 0, 1, 2, \cdots$ and $i \in \mathfrak{T}$,

$$Q_i(t) < hM_0 \exp\Big\{\varrho\Big(\sum_{j=0}^{k}(1-\theta_j)\mathrm{T}_j\Big)\Big\}. \tag{22}$$

And for all $\hat{\mathrm{T}}_k + \theta_{k+1}\mathrm{T}_{k+1} \leq t < \hat{\mathrm{T}}_{k+1}$, $k = 0, 1, 2, \cdots$ and $i \in \mathfrak{T}$,

$$Q_i(t) < hM_0 \exp\Big\{\varrho\Big(t - \Big(\sum_{j=0}^{k+1}\theta_j\mathrm{T}_j\Big)\Big)\Big\} \leq hM_0 \exp\Big\{\varrho\Big(\sum_{j=0}^{k+1}(1-\theta_j)\mathrm{T}_j\Big)\Big\}. \tag{23}$$

Recalling that $Q_i(t) = \exp\{\varepsilon t\}V_i(t)$, it follows from (22) and (23) that for all $i \in \mathfrak{T}$ and $t \in [\hat{\mathrm{T}}_k, \hat{\mathrm{T}}_k + \theta_{k+1}\mathrm{T}_{k+1})$, $k = 0, 1, 2\cdots$,

$$V_i(t) < hM_0\exp\{-\varepsilon t\}\exp\Big\{\varrho\Big(\sum_{j=0}^{k}(1-\theta_j)\mathrm{T}_j\Big)\Big\} \leq hM_0 \exp\Big\{-\varepsilon\Big(\sum_{j=0}^{k}\mathrm{T}_j\Big) + \varrho\Big(\sum_{j=0}^{k}(1-\theta_j)\mathrm{T}_j\Big)\Big\}, \tag{24}$$

and for all $i \in \mathfrak{T}$ and $t \in [\hat{\mathrm{T}}_k + \theta_{k+1}\mathrm{T}_{k+1}, \hat{\mathrm{T}}_{k+1})$, $k = 0, 1, 2\cdots$,

$$V_i(t) < hM_0\exp\{-\varepsilon t\}\exp\Big\{\varrho\Big(\sum_{j=0}^{k+1}(1-\theta_j)\mathrm{T}_j\Big)\Big\} < hM_0 \exp\{\varrho(1-\theta_{k+1})\mathrm{T}_{k+1}\}\exp\Big\{-\varepsilon\Big(\sum_{j=0}^{k}\mathrm{T}_j\Big) + \varrho\Big(\sum_{j=0}^{k}(1-\theta_j)\mathrm{T}_j\Big)\Big\}. \tag{25}$$

Since all the rest widths $(\mathrm{T}_m - \delta_m)$, $m \in Z^+$, are bounded, we can assume that $\sup_{m\in Z^+}\{\mathrm{T}_m - \delta_m\} = \varpi_0$, where $\varpi_0 > 0$ is a positive constant. Denote $\Upsilon_0 = hM_0\exp\{\varrho\varpi_0\}$, we have from (24) and (25) that

$$V_i(t) < \Upsilon_0 \exp\Big\{-\varepsilon\Big(\sum_{j=0}^{k}\mathrm{T}_j\Big) + \varrho\Big(\sum_{j=0}^{k}(1-\theta_j)\mathrm{T}_j\Big)\Big\}, \quad \text{for all } t \in [\hat{\mathrm{T}}_k, \hat{\mathrm{T}}_{k+1}),\ \ k = 0, 1, 2\cdots \text{ and } i \in \mathfrak{T}. \tag{26}$$

According to condition (ii), one has $\lim_{t\to+\infty} V_i(t) = 0$, for all $i \in \mathfrak{T}$. The proof is completed. ■

Remark 3 *From the proof of Theorem 1, we can see that in the work time* $[\hat{\mathrm{T}}_k, \hat{\mathrm{T}}_k + \theta_{k+1}\mathrm{T}_{k+1})$*, the control, which benefits network synchronization, is imposed to the network; while in the rest time* $[\hat{\mathrm{T}}_k + \theta_{k+1}\mathrm{T}_{k+1}, \hat{\mathrm{T}}_{k+1})$*, the control is removed, which means that this time span does not promote (may be harmful for) the synchronization. Therefore, for achieving synchronization, one can simply make the work time as long as possible and the rest time as short as possible.*

Define the following indices

$$AE(j) = -\varepsilon\mathrm{T}_j + (p+\eta_1)(1-\theta_j)\mathrm{T}_j, \quad j = 0, 1, 2, \cdots,$$

then it can be observed from (26) that the aggregated effects of intermittent control in the time of the $(j+1)$th period $[\hat{\mathrm{T}}_j, \hat{\mathrm{T}}_{j+1})$ can be characterized by the index $AE(j)$. Evidently, when $AE(j) < 0$ (> 0), the aggregated effects of the intermittent control are beneficial (harmful) for the synchronization. It should be stressed that, it is not necessary to ensure that $AE(j) < 0$ for all $j = 0, 1, 2, \cdots$, that is to say, it is allowable that $AE(j) \geq 0$ for some time span $[\hat{\mathrm{T}}_j, \hat{\mathrm{T}}_{j+1})$, provided that condition (ii) in Theorem 1 holds.

Suppose that η_1 is given as $\eta_1^* > q$. Substituting $\eta_1 = \eta_1^*$ into the equation $\varepsilon - \eta_1 + 2L_2\exp\{\varepsilon\sigma\} + ac\exp\{\varepsilon\tau\} = 0$ yields $\varepsilon = \varphi(\eta_1^*)$, then the following result can be obtained readily from Theorem 1.

Corollary 1 *Suppose that all the rest widths* $(T_m - \delta_m)$*,* $m \in Z^+$*, are bounded and* η_1 *is given as* $\eta_1^* > q$*. Then the controlled delayed dynamical network (5) can realize global synchronization if the following conditions hold:*

$$\text{(i) } d \geq \frac{(p+\eta_1^*)}{2} > 0, \quad \text{(ii) } \lim_{k\to+\infty}\Big[-\varphi(\eta_1^*)\Big(\sum_{j=0}^{k}\mathrm{T}_j\Big) + (\eta_1^*+p)\Big(\sum_{j=0}^{k}(1-\theta_j)\mathrm{T}_j\Big)\Big] = -\infty,$$

where $p = 2L_1 + ac\lambda_{\max}(\Gamma\Gamma^\top - (\Gamma+\Gamma^\top))$ *and* $q = 2L_2 + ac$*.*

In real-world applications, condition (ii) of Corollary 1 is not easy to be verified. In the following, two special cases will be discussed, which will simplify the validation of the condition.

Case 1 *Periodically intermittent control scheme.*

When each control period and each control rate are fixed, i.e., $\mathrm{T}_m \equiv \mathrm{T}$ and $\theta_m \equiv \theta$ for all $m \in Z^+$, where T and θ are both positive constants, then the control becomes periodically intermittent control [17]. For this case, we can obtain the following result from Corollary 1.

Corollary 2 *Suppose that η_1 is given as $\eta_1^* > q$. If the following conditions hold:*

$$\text{(i) } d \geq \frac{(p+\eta_1^*)}{2} > 0, \quad \text{(ii) } 1 - \frac{\varphi(\eta_1^*)}{p+\eta_1^*} < \theta < 1,$$

where p and q are defined in Corollary 1, then the controlled delayed dynamical network (5) can realize global synchronization.

In this case, it is easy to observe that the index $AE(j) = -\varphi(\eta_1^*)\mathrm{T} + (p+\eta_1^*)(1-\theta)\mathrm{T} = (-\varphi(\eta_1^*) + (p+\eta_1^*)(1-\theta))\mathrm{T}$, $j = 0, 1, 2, \cdots$. Therefore, condition (ii) in Corollary 2 implies that $AE(j) < 0$ holds for any $j = 0, 1, 2, \cdots$, and so condition (ii) in Corollary 1 is satisfied. In [22], complex dynamical networks with finite distributed delays coupling under periodically intermittent control are investigated, and similar results are also derived.

Case 2 *Aperiodically intermittent control scheme.*

Suppose that $\inf_{m \in Z^+}\{\theta_m\} = \theta_{\inf} > 0$ for the aperiodically intermittent control strategy, where $\theta_{\inf}$ is a positive constant. Then the following result is easily obtained from Corollary 1.

Corollary 3 *Suppose that η_1 is given as $\eta_1^* > q$. If the following conditions hold:*

$$\text{(i) } d \geq \frac{(p+\eta_1^*)}{2} > 0, \quad \text{(ii) } 1 - \frac{\varphi(\eta_1^*)}{p+\eta_1^*} < \theta_{\inf} < 1,$$

where p and q are defined in Corollary 1, then the controlled delayed dynamical network (5) can realize global synchronization.

For this case, the index $AE(j) = -\varphi(\eta_1^*)\mathrm{T}_j + (p+\eta_1^*)(1-\theta_j)\mathrm{T}_j \leq (-\varphi(\eta_1^*) + (p+\eta_1^*)(1-\theta_{\inf}))\mathrm{T}_j$, $j = 0, 1, 2, \cdots$. Since condition (ii) in Corollary 4 means that $AE(j) < 0$ holds for any $j = 0, 1, 2, \cdots$, condition (ii) in Corollary 1 holds. Evidently, this intermittent control type takes the aforementioned type of intermittent control (Case 1) as a special case.

Remark 4 *In [16–25], the synchronization problem for chaotic systems as well as complex dynamical networks via periodically intermittent control was investigated. However, the designed controllers in [16–25] are periodically intermittent with fixed control period and control width (i.e., $\mathrm{T}_m \equiv \mathrm{T}$ and $\delta_m \equiv \delta$ for all $m \in Z^+$). Obviously, this requirement is unreasonable and limits the application scopes of the theoretical results. In this paper, based on aperiodically intermittent control technique, some global synchronization criteria are developed for complex dynamical networks with internal delay and distributed-delay coupling. To the best of our knowledge, result on synchronization of complex dynamical networks with internal delay and distributed-delay coupling via aperiodically intermittent control has not yet been reported. In our network model, the internal delay can be constant or time-varying, and even non-differentiable; the coupling matrix are not demanded to be symmetric or irreducible. Therefore, our theoretical results are more general and expand the scope of practical applications of intermittent control strategy.*

Remark 5 *From Corollary 4, it can be seen that only the control rate $\theta_{\inf}$, rather than either the control width δ_m or the control period T_m, affects the control performance. This means that, for achieving the synchronization, each control period T_m, $m \in Z^+$ can be arbitrarily selected according to the actual requirement, only if condition* (ii) *in Corollary 4 holds. This facilitates the potential practical applications of the theoretical results in engineering fields.*

Remark 6 *For illuminating how to design suitable aperiodic intermittent controllers in real-world applications for the achievement of the synchronization for a given delayed dynamical network (1) and a given synchronous state $\pi(t)$, we take example for the application of Corollary 4, the following steps are provided:*

Step 1. For a given η_1^*, calculate the value of $\varphi(\eta_1^*)$, and then choose control gain d and control rates θ_m, $m \in Z^+$ such that conditions (i) and (ii) of Corollary 4 are satisfied.

Step 2. Select control periods T_m, $m \in Z^+$ according to the actual requirement.

Step 3. Based on the above chosen d, θ_m, T_m, design aperiodic intermittent controller $u_i(t)$ described in (6).

4 Numerical simulations

In this section, a numerical example is given to illustrate the effectiveness of the theoretical results derived above.

Consider the controlled delayed dynamical network (5) consisting of 100 identical delayed Chua oscillators, which is described by

$$\dot{x}_i(t) = f(t, x_i(t), x_i(t-\sigma(t))) + c\sum_{j=1, j\neq i}^{100} b_{ij}\Gamma\Big(\int_{t-\tau}^{t} \rho(t-s)x_j(s)\,\mathrm{d}s - x_i(t)\Big) + u_i(t), \tag{27}$$

where $i = 1, 2, \cdots, 100$, $\Gamma = I_3$, $c = 1$, $\tau = 0.25$, $\rho(t) = 8 - 32t$ for $t \in [0, 0.25]$. The coupling matrix B is of the form

$$B = \begin{pmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 1 \\ 1 & 0 & \cdots & 0 & -1 \end{pmatrix}_{100\times 100}$$

and then $a = 1$. The dynamics of the delayed Chua oscillator is given by [16]

$$\dot{x}_i(t) = f(t, x_i(t), x_i(t-\sigma(t))) = Ax(t) + f_1(x_i(t)) + f_2(x_i(t-\sigma(t))), \tag{28}$$

where $x_i(t)=(x_{i1}(t), x_{i2}(t), x_{i3}(t))^\top \in \mathbb{R}^3$, $f_1(x_i(t))=(-\frac{1}{2}\alpha_0(a_1 - a_2)(|x_{i1}(t) + 1| - |x_{i1}(t) - 1|), 0, 0)^\top \in \mathbb{R}^3$, $f_2(x(t-\sigma(t))) = (0,\ 0,\ -\beta_0\eta_0 \sin(v_0 x_1(t-\sigma(t))))^\top \in \mathbb{R}^3$, $A = \begin{pmatrix} -\alpha_0(1+a_2) & \alpha_0 & 0 \\ 1 & -1 & 1 \\ 0 & -\beta_0 & -\omega_0 \end{pmatrix}$, and α_0=10, β_0= 17.53, ω_0= 0.1636, a_1= −1.4325, a_2=−0.7831, $v_0 = 0.5$, $\eta_0 = 0.2$, and $\sigma(t) = 0.02$. It is easy to check that

$$\begin{aligned}
&(x(t) - y(t))^\top (f(t, x(t), x(t-\sigma(t))) - f(t, y(t), y(t-\sigma(t))))\\
&\le \frac{1}{2}(x(t) - y(t))^\top (A + A^\top)(x(t) - y(t)) + |\alpha_0(a_1 - a_2)|(x_1(t) - y_1(t))^2 + \beta_0\eta_0 v_0|x_3(t) - y_3(t)|\,|x_1(t-\sigma(t)) - y_1(t-\sigma(t))|\\
&\le \lambda_{\max}(\tilde{A})(x(t) - y(t))^\top (x(t) - y(t)) + (\beta_0\eta_0 v_0)/(2\epsilon_1)(x(t-\sigma(t)) - y(t-\sigma(t)))^\top (x(t-\sigma(t)) - y(t-\sigma(t)))\\
&= L_1(x(t) - y(t))^\top (x(t) - y(t)) + L_2(x(t-\sigma(t)) - y(t-\sigma(t)))^\top (x(t-\sigma(t)) - y(t-\sigma(t))),
\end{aligned}$$

where $\tilde{A} = (A + A^\top)/2 + \mathrm{diag}(|\alpha_0(a_1 - a_2)|, 0, \epsilon_1(\beta_0\eta_0 v_0)/2)$, and $L_1 = \lambda_{\max}(\tilde{A})$, $L_2 = (\beta_0\eta_0 v_0)/(2\epsilon_1)$ can be determined by selecting an appropriate parameter $\epsilon_1 > 0$. Therefore, $(\mathbf{A}_1)$ is satisfied.

Let $\epsilon_1 = 2$, then one has $L_1 = 10.9151$, $L_2 = 0.4383$, and so $p = 20.8302$, $q = 1.8766$. Based on Corollary 4 (ii), the relationship curve between the parameter η_1^* and the control rate $\theta_{\inf}$ is depicted in Fig. 1. If $\eta_1^* = 30$ is selected as a special case, by conditions (i) and (ii) of Corollary 3, we get

$$d \ge 25.4151, \quad 0.7796 < \theta_{\inf} < 1.$$

For simplicity, select $d = 25.5$, $\theta_m = 0.80$, and $\mathrm{T}_m = t_{m+1} - t_m = 0.2m$, $m \in Z^+$, then condition (42) holds. Fig. 2 shows the time evolutions of the state variables $x_{i1}(t)$, $x_{i2}(t)$, $x_{i3}(t)$ $(1 \le i \le 100)$ for system (40) with different initial values under the aperiodic intermittent controllers (6)-(7), which indicates the controlled delayed dynamical network (40) is asymptotically synchronized.

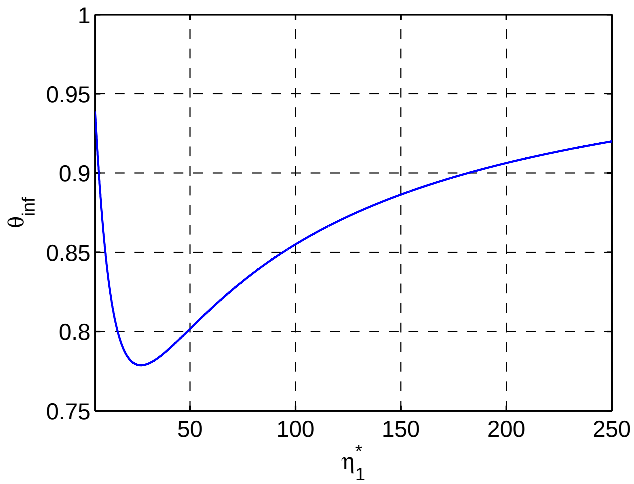

Figure 1: The relationship curve between the parameter η_1^* and the control rate $\theta_{\inf}$.

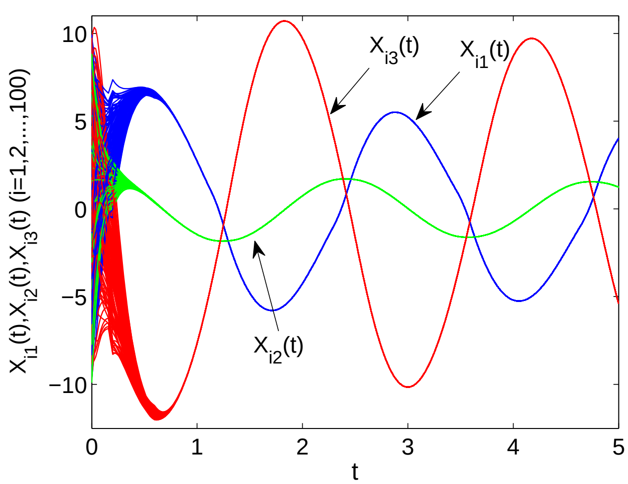

Figure 2: Time evolutions of the state variables $x_{i1}(t)$, $x_{i2}(t)$, $x_{i3}(t)$, $1 \le i \le 100$ for system (40) under the aperiodically intermittent control.

5 Conclusion

In this paper, the aperiodically intermittent control was generalized to investigate the synchronization problem for a class of complex dynamical networks with internal delay and distributed-delay coupling. Some sufficient conditions to guarantee global synchronization are derived by utilizing the reduction to absurdity and mathematical induction method. Finally, numerical simulations are given to show the feasibility of the theoretical results.

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