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A New Calculation Formula of the Social Cost of Carbon

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Abstract: In economics, an important concept of climate change is “carbon price”. The social cost of carbon (SCC) is one of the arguments for carbon price. It is the marginal present value cost caused by greenhouse gas emissions. In previous studies, the diffusion of carbon in the atmosphere is mostly linear in the carbon cycle model and without considering the growth potential of carbon emissions. In fact, the diffusion of carbon between different reservoirs, such as atmosphere, ocean and land, is non-linear. In this paper, we have non-linearized the previous carbon cycle structure by introducing a term of power function. Considering that the change of atmospheric CO_2 concentration at time t is proportional to the carbon emissions and the share of CO_2 emission growth potential, we increase the structure of CO_2 emission growth potential. Therefore, the relationship between carbon emissions and atmospheric CO_2 concentration is non-linear. Finally, we construct a new SCC formula through an emission-temperature-damage response function. In addition, this paper uses the new calculation formula to estimate the SCC, and explores the influence of the evolution coefficient of carbon emission on the SCC. Only considering the influence of the evolution coefficient on SCC, we find that the smaller the carbon emission evolution coefficient is, the smaller the SCC estimate is. The result shows that when the evolution coefficient of carbon emission is 0.3, the mean SCC is 33.40 €/tCO_2. Here the SCC has a significant uncertainty, and the density distribution of SCC is a right-skewed distribution with a skewness of 2.93.

Keywords: Social cost of carbon; Carbon cycle; Uncertainty

1 Introduction

The social cost of carbon (SCC) has become a core tool for the formulation of climate change policies, especially with respect to regulatory policies related to greenhouse gas emissions [1,2]. The SCC is the monetized damage from emitting one unit of CO_2 to the atmosphere, and considering the effects of greenhouse gases such as CO_2 in the atmosphere accumulating over time, it also translates the future damage caused by the increased CO_2 of this period into present values. In the current studies, the methods for estimating SCC are mainly cost-benefit method and the marginal cost method. The cost-benefit method is mainly realized by the integrated assessment model (IAM). The marginal cost method to calculate the SCC also uses the modeling idea of the IAMs. The difference is that the IAMs calculate the SCC at optimal level of CO_2 emissions. The marginal cost method is to calculate the SCC caused by the current unit of marginal CO_2 emissions. The climate change integrated assessment model can simulate the “change path” of the optimal carbon emissions over a period of time in order to reflect the change of SCC over time [3]. Among many IAMs, the dynamic integrated model of climate and the economy (DICE) model is one of the earlier and widely used models. Nordhaus and Yang first developed a DICE model to assess social welfare and SCC under climate change [4]. Bijgaart studied a closed formula under the general economic situation and obtained the SCC using the marginal cost method [5].

One of the focuses of SCC research is its uncertainty. Many researchers have been aware of the significant impact of the fat-tailed distribution of climate damage, disaster risk, and uncertainty of discount rate on social welfare and SCC. Weitzman used the theory of “fat tail” probability distribution to study the uncertainty of climate damage caused by global

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warming [6]. Pycroft explicitly considered the impact of extreme sea-level rise on climate damage when estimating the SCC [7]. The evaluation of SCC needs to be discounted for future damage, which involves the discount rate. Johnson and Hope believed that using the low discount rate under the same conditions would result in relatively high marginal social cost caused by short-term greenhouse gas emissions [8].

The SCC is obtained through a dynamic model of climate change related to the global carbon cycle. So the rational establishment of the carbon cycle model is one of the important factors affecting the estimation of SCC. The DICE model has two versions of carbon cycle models. In the early DICE model, the carbon cycle is a solo-reservoir carbon-cycle model. CO_2 exists only in the atmosphere and decays from the atmosphere at a constant rate [9]. Then Nordhaus developed it as a three-reservoir carbon-cycle model, in which CO_2 circulates between the atmosphere, the upper biosphere and the deep ocean[10]. Considering the diffusion of carbon dioxide between the various reservoirs, Gerlagh developed an easy-to-handle carbon cycle system. The total atmospheric CO_2 reserves are obtained by calibrating the reserves of CO_2 stored in the upper ocean [3]. Bijgaart studied the carbon cycle system as a simple " boxes" model in continuous time [5].

This paper extends the research ideas of Bijgaart [5] and adds potential structure of carbon dioxide emissions on the basis of carbon cycle model. We use Monte Carlo method to simulate the data of key parameters, and utilize the new evaluation formula to estimate the SCC. In addition, we analyze the SCC under different carbon emission evolution coefficients and find that there is a great probability of extreme values in the simulated value of SCC, and the density distribution of the SCC is right-skewed.

2 Model

This article refers to the carbon cycle model of the carbon diffusion between different carbon reservoirs by Gerlagh and Bijgaart [3,5]. According to the analysis of the evolution of carbon emissions by Tian and Jin [11], we constructed a nonlinear carbon cycle structure based on the carbon cycle model studied by Bijgaart [5]. And the carbon cycle structure is as follows:

$$\dot{M}_i(t) = a_i E(t)(1 - \frac{E(t)^\beta}{r\, l_e}) - \eta_i M_i(t). \tag{1}$$

The atmospheric CO_2 depreciation is approximated through a dynamic system of "boxes", labeled by $i \in I = \{1, ..., n\}$ and a_i is the vector shares of carbon emissions entering each climate box. Here, we define $M_i(t)$ as the CO_2 stock over and above the pre-industrial level of CO_2 in box i at time t. We use $M(t) = \sum_{i=1} M_i(t)$ to represent the total atmospheric CO_2 at time t. Emissions add to the atmospheric CO_2 stock $M_i(t)$, which depreciates at rate η_i. It assumes that the carbon reserves in the atmosphere during the industrial period are slowly fading and gradually absorbed by the oceans and soil [9]. $E(t)$ is carbon emission at time t. β is the conversion rate of carbon emissions at t, here we set $\beta = 1$. l_e is the largest estimate of carbon emissions for the year 2100. According to the IPCC fifth assessment report, we choose the scenario RCP4.5 [12]. In this scenario, $l_e = 780 G_t C$. r is the evolution coefficient of carbon dioxide emission, $r > 0$. Compared with previous literatures, considering that the process of converting the emissions of CO_2 into the CO_2 concentration in the atmosphere is nonlinear, we add the growth potential structure of CO_2 emission $(1 - \frac{E(t)^\beta}{r\, l_e})$ in Eq.(1). $\dot{M_i}(t)$ is proportional to the carbon emission $E(t)$ and the share of CO_2 emission growth potential $(1 - \frac{E(t)^\beta}{r\, l_e})$. The structure of CO_2 growth potential for emission here is influenced by technological progress, emission reduction technologies and changes in carbon sources. Meanwhile, it is also related to policy regulation and external factors that affect carbon emissions.

We follow most of the IAM literature and assume that the relationship between atmospheric CO_2 concentrations and equilibrium temperatures can be described through a logarithmic curve [13]:

$$T = \varphi(M, c, m) = c\frac{\ln(1 + \frac{M}{m})}{\ln 2}, \tag{2}$$

where T is the temperature rise since industrialization which measured above a pre-industrial baseline; c is a climate-sensitive parameter, that is, the sensitivity parameters of the global average temperature rise when the CO_2 in the atmosphere doubles; m is the pre-industrial stock of atmosphere CO_2. Here the value of m is 275.

Stern Review discussed the uncertainty about the shape of the economic damage function related to the temperature [14]. We use $D = \phi(T) = \omega T^{b_1}$ to represent the relationship between global atmospheric temperature rise and climate damage, where climate damage D is a part of the global output; ω is the economic damage sensitivity parameter and b_1 is the shape parameter, which is equal to 2. It is considered that global economic damages are often assumed to depend on the square of temperature T. From Eq.(2) and temperature rise damage function we can get:

$$D = \phi(T) = \phi(\varphi(M)) = \omega\, c^2 [\frac{\ln(1+\frac{M}{m})}{\ln 2}]^2. \tag{3}$$

And because the relationship between damage and the atmospheric CO_2 concentrations is close to linear, we can get:

$$D(t) = D(t-1) + \varepsilon[\omega \upsilon M(t) - D(t-1)], \tag{4}$$

where ε is the temperature adjustment coefficient, which is equal to 2. It is the delay coefficient of CO_2 concentration converted into temperature rise. The climate damage is assumed to be changed with output, and $\omega\upsilon = \phi'\,\varphi'$. The value of υ is related to the climate sensitivity parameter c, and it can be obtained from Eq.(3).

Therefore, this gives the net present value of future damage caused by one unit of present carbon emissions at time t. Combined with Eqs.(1) and (4), the SCC is expressed as

$$SCC(t) = Y(t)\int_0^{+\infty} e^{-\sigma\tau}\frac{dD(t+\tau)}{dE(t)}d\tau = \omega\upsilon Y(t)\sum_{i\in I}\frac{\alpha_i\varepsilon}{(\eta_i+\sigma)(\varepsilon+\sigma)}(1-\frac{(1+\beta)E(t)^{\beta}}{r\cdot l_e}), \tag{5}$$

where Y is the total annual economic output of the world. Climate damage D is relative to output Y, so that DY equals gross damage. σ is the discount rate, the future damage caused by the increased CO_2 in the t period is discounted into the present value through the discount rate σ. Based on the carbon cycle structure, climate damage function and the discounted economic damage, we finally get a new formula of the SCC as Eq.(5).

3 Data processing

Many climatic and economic parameters are involved in the analysis and calculation of the SCC. This paper simulates and optimizes the parameters using the Monte Carlo simulation method. Each parameter is simulated to get 100,000 simulation parameter values. The simulation parameters are: climate sensitivity parameter c [15]; the sensitivity parameter of the damage ω [16]; the discount rate σ [17]. From the existing literatures, we know that the distribution state of the three parameters is lognormal distribution. This is consistent with the initial distribution of the relevant parameters in the DICE model. Due to the fact that the values simulated from the mean and variance of the lognormal distribution are too angular, we have optimized the parameters. The optimized parameter samples are shown in Table 1.

Table 1: The distribution of model parameters

Parameter	Obs	Mean	Std. Dev	Min	Max
c	100000	3.229947	1.314707	0.6407264	14.95105
$c(new)$	100000	3.2093	1.221949	1.371921	6.560143
ω	100000	0.0041256	0.0039305	0.0001253	0.0816616
$\omega(new)$	100000	0.0039919	0.0032316	0.0006	0.015
σ	100000	0.0227841	0.0178875	0.0011677	0.3099025
$\sigma(new)$	100000	0.0222655	0.0153732	0.0045	0.0719999

The upper and lower bounds of the new simulated values of climate sensitive parameters c (new) are smaller than those of the initial simulation. For the new damage sensitivity parameter ω (new), the upper and lower bounds of the stochastic simulation are reduced from the initial [0.0001253, 0.0816616] to [0.0006, 0.015]. The upper and lower bounds of the new discount rate σ(new) are reduced from the initial [0.0011677, 0.3099025] to [0.0045, 0.0719999]. After optimization, the ranges of parameters simulation are narrowed, which make the final sampling more concentrated and more consistent with the estimation of the sensitivity parameters.

Table 2: Carbon cycle parameters

Model	Share of emissions entering climate box a_i	Climate box i carbon depreciation rate η_i
DICE	(0.029, 0.356, 0.615)	(0, 0.0035, 0.0364)
GL	(0.163, 0.190, 0.589)	(0, 0.0076, 0.0618)
MR-H	(0.142, 0.241, 0.323, 0.206, 0.088)	(0, 0.0032, 0.0125, 0.0532, 0.5882)

Table 2 shows the CO_2 conversion parameter a_i and the decay rate parameter η_i of the three alternative carbon cycle models. For the parameters in the carbon cycle model, this article refers to the parameter values based on the values that estimated by DICE model [18], as estimated by GL [3] and MR-H [19].

4 Empirical analysis

Considering that the CO_2 stock at time t is not only related to the CO_2 emissions in the atmosphere but also to the share of CO_2 emission growth potential, in the carbon cycle model, as the evolution coefficient of carbon emission increases or decreases, the concentration of CO_2 in the atmosphere increases or decreases correspondingly. We have obtained the influence of carbon emission evolution coefficient on SCC. Fig. 1 shows the effect of evolution coefficient r to SCC based on the model of DICE [18], GL [3] and MR-H [19].

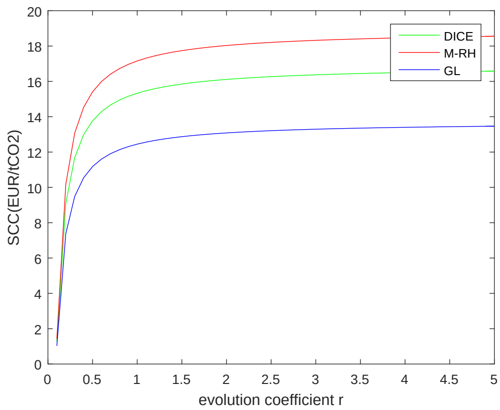

Figure 1: Influence of carbon emission evolution coefficient on SCC

As we can see in Fig. 1, the impact trends of evolution coefficient r to SCC under different models are approximately the same. Assuming that climate sensitive parameters, damage parameters, discount rates are fixed values, when the value of r increases from 0 to 1, the vaules of SCC increase significantly. When the CO_2 evolution coefficient $r > 1$, the vaules of SCC tend to stabilize, so the evolution coefficient of carbon dioxide has little effect on the social cost of carbon. By adjusting the evolution coefficient of carbon dioxide and simulating the uncertain parameters, the simulation values of SCC are calculated through the new calculation formula of SCC, where the output values are reported for the year 2015, in 2010 Euros. We get the following conclusions: assuming the carbon dioxide evolution coefficient $r = 1$, the social cost of a unit of carbon dioxide emission is 43.82 €/tCO_2. On this basis, strengthen the adjustment of CO_2 emission structure through the innovation of technology and the increase of policy regulation and set the CO_2 evolution coefficient $r = 0.3$, then the mean SCC is 33.40 €/tCO_2. When the evolution coefficient of CO_2 dropps to 0.1, the mean SCC is 3.62 €/tCO_2. That is, when the carbon energy structure is greatly optimized and low-carbon technologies are reformed, a smaller value of r will lead to a significant reduction in the estimated value of SCC.

Fig. 2 shows the distribution of the SCC that is simulated by the optimized parameter when $r = 0.3$. The distribution of SCC in Fig. 2 reflects the existence of extreme values of SCC with a certain probability. There are more than 4.6 percent probability that the SCC values above 180 €/tCO_2. The skewness of the density distribution of the SCC is 2.93. This uncertainty is related to the distribution of climate parameters, economic sensitivity parameters and discount rates. Under the influence of various uncertain parameters, it can be seen that the resulting distribution of SCC is strongly right-skewed with a mean SCC of 33.40 €/tCO_2. It also shows that the social cost of extreme disaster damage is enormous. In addition, the reduction of greenhouse gas emissions can be seen as an investment in natural capital, thus generating significant benefits in reducing the social costs of climate change and reducing the risk of extreme climate change. It needs to better regulate the carbon dioxide evolution coefficient to reduce the SCC.

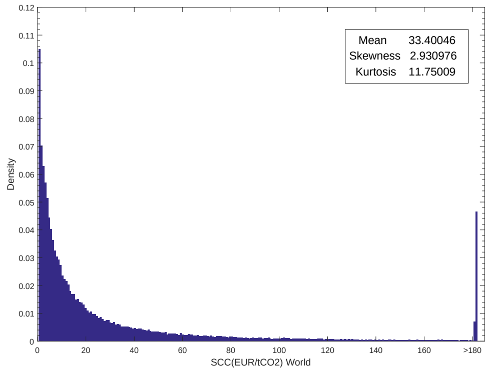

Figure 2: Density distribution of the SCC

5 Conclusions

This paper considers the growth potential of CO_2 emissions in the carbon cycle system and constructs a closed formula for the SCC. Through Monte Carlo simulation, the parameters are optimized to better simulate the uncertainty of the SCC, when $r = 0.3$, the distribution of SCC is strongly right-skewed with a mean SCC of 33.40 €/tCO_2, and more than 4.6 percent probability for a SCC higher than 180 €/tCO_2. It is found that the evolution coefficient of carbon dioxide decreases when the energy structure is optimized. In Fig. 1, when the evolution coefficient is equal to 0.1, the estimated value of SCC is significantly reduced. In this scenario, policy regulation and technology updates will lead to a large amount of abatement costs. Therefore, it is very important to regulate the evolution coefficient of carbon dioxide reasonably, and balance the deficit of the SCC and the abatement costs. The calculation formula of SCC in this paper also provides theoretical and model support for the formulation of carbon taxes and the implementation of low-carbon emission abatement policies.

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