



# Modeling and Empirical Analysis of Energy Substitution System Based on Nonlinear Dynamics on America

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**Abstract:** In this paper, a new energy substitution dynamic model is proposed to describe the coupling relationship between carbon emissions, GDP and installed capacity of renewable energy. The model expressed in nonlinear differential equations form reveals complex dynamical behavior when the appropriate parameters are selected. The existence of a chaotic attractor is verified by the Lyapunov exponent spectrums and bifurcation diagram. Notable in the study is that, the system is dissipative. The genetic algorithm is used to identify the actual parameters of the system based on real data. The corresponding control measures are given for American energy markets, by analyzing the American energy substitution system.

**Keywords:** Energy substitution; Installed capacity; Dynamical behavior; Chaos; Genetic algorithm

## 1 Introduction

Direct emissions of carbon from Canadian forest fires were estimated for all Canada and for each ecozone for the period 1959-1999. The estimates were based on a data base of large fires for the country and calculations of fuel consumption for each fire using the Canadian Forest Fire Behaviour Prediction System [1]. By using autoregressive distributed lag bounds testing approach of cointegration examines the long run and causal relationship issues between economic growth, carbon emissions, energy consumption and employment ratio in Turkey [2]. Every industry in China is preparing to realize this national reduction target. Some attempts have been made to achieve low-carbon development in a few industries, but relatively little work has linked low-carbon development to tourism [3]. Based on the status quo of carbon emissions in China, this paper introduces dynamic evolutionary factors, advance rate, critical time and evolutionary coefficients of carbon emissions, deducing relative theories, such as Change Trends Theorem and Evolutionary Theorem. Least-square method is used to establish the dynamic evolutionary system of carbon emissions [4]. In this paper we test the Environment Kuznet's Curve (EKC) hypothesis for 43 developing countries. We suggest examining the EKC hypothesis based on the short- and long-run income elasticities [5].

The atmospheric  $CO_2$  concentration is increasing, due primarily to fossil-fuel combustion and deforestation. Sequestering atmospheric in agricultural soils is being advocated as a possibility to partially offset fossil-fuel emissions [6]. In this paper we investigates the effect of energy consumption and output on carbon emissions in the United States [7]. We investigates the existence and direction of Granger causality between economic growth, energy consumption, and carbon emissions in China, applying a multivariate model of economic growth, energy use, carbon emissions, capital and urban population [8]. This paper uses multivariate co-integration causality tests to investigate the correlations between carbon dioxide emissions, energy consumption and economic growth in China [9]. The capacity adequacy control of power supply, reserve and distributed generation in micro-grid under electricity market environment are systematically annotated and it is suggested to fully utilize the complementary features in technology and economy among them to improve the economic efficiency of NRDG (Non-Renewable Distributed Generation) configuration [10]. This paper applies the panel unit root, heterogeneous panel cointegration and panel-based dynamic OLS to re-investigate the co-movement and relationship between energy consumption and economic growth for 30 provinces in mainland China from 1985 to 2007. The

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empirical results show that there is a positive long-run cointegrated relationship between real GDP per capita and energy consumption variables [11].

Facing the energy crisis and the requirement of green power, China is making great efforts to fulfill the international cooperation on renewable energy electricity aimed at protecting the security of electricity supply and economy development [12]. It reflects on the relevance of "systems-theoretic" approaches to the interdependent policy issues relating to the dynamics of science, technology and innovation and their relationship to economic growth [13]. The survey conduct to evaluate the causality between energy consumption, GDP growth and carbon emissions for eight Asia-Pacific countries from 1971 to 2005 using the panel data. The results indicate that there are long-run equilibrium relationships between these variables [14]. Unlike previous renewable energy consumption-growth studies, this study examines the relationship between renewable and non-renewable energy consumption and economic growth for 80 countries within a multivariate panel framework over the period 1990-2007 [15]. This paper investigated the short-run causal relationships and the long-run equilibrium relationships among carbon dioxide emissions, economic growth, technical efficiency, and industrial structure for three African countries [16]. We build a general equilibrium model with renewable non-polluting and non-renewable polluting resources to analyze the interaction and compatibility between economic growth and a cleaner environment [17]. Shifting to renewable sources of electricity is imperative in achieving global reductions in carbon emissions and ensuring future energy security [18]. Over the past decade, concern over potential global warming has focused attention on the emission of greenhouse gases into the atmosphere, and there is an active debate concerning the desirability of reducing emissions [19].

In this paper, the ANN artificial neural network method will be used to identify the parameters of the evolution system. It is a computational intelligence technique with a good nonlinear fitting capability, and it can be used to identify unknown parameters. For applications of the ANN method in energy systems, one can turn to [20]-[22]. The rest of this paper is structured as follows. Section 2 establishes a universal alternative energy evolution model. Section 3 gives the empirical analysis. Section 4 discusses the America market and proposes a process of parameter identification. Section 5 outlines the conclusion drawn from the study.

## 2 Main results

The model contains three variables:  $x(t)$  is the time-dependent variable of carbon emissions.  $y(t)$  is the time-dependent variable of economic growth(GDP).  $z(t)$  is the time-dependent variable of installed capacity of renewable energy.

### 2.1 Energy substitution dynamic system

We built energy substitution dynamic model:

$$\begin{cases} \dot{x} = a_1x(1 - x/M) + a_2y - a_3z \\ \dot{y} = -b_1x - b_2y + b_3z(z/N - 1) \\ \dot{z} = c_1e^{-x}\dot{x} + c_2y(y/P - 1) \end{cases} \quad (1)$$

where  $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, M, N, P$  are positive constants,  $t \in I$ .  $I$  is a given economic period.  $a_1$  is the development coefficient of  $x(t)$ ,  $a_2$  is the influence coefficient of  $y(t)$  to  $x(t)$ ,  $a_3$  is the influence coefficient of  $z(t)$  to  $x(t)$ .  $M$  is the inflexion (local maximum point) of  $x(t)$ .  $b_1$  is the influence coefficient of  $x(t)$  to  $y(t)$ ,  $b_2$  is the development coefficient of  $y(t)$ ,  $b_3$  is the influence coefficient of  $z(t)$  to  $y(t)$ ,  $N$  is the peak value of during a given period;  $c_1$  is the influence coefficient of  $x(t)$  to  $z(t)$ ,  $c_2$  is the influence coefficient of  $y(t)$  to  $z(t)$ ,  $P$  is the inflexion of  $y(t)$  to  $z(t)$ .

The first line of Eq. (1) indicates that the rate of carbon emissions is proportional to  $a_2y$  and there is a negative correlation with  $a_3z$ . The first term indicates that when the carbon emissions is higher than the inflexion  $M$ , Government, business and consumers are aware of the importance of protecting the environment and begin to develop policies to reduce the use of fossil fuels and to use renewable energy more. It will restrain the growth of carbon emissions.

The second line of Eq. (1) explains that the rate of economic growth is negative correlation with  $x(t)$  and  $y(t)$ . The early investment to  $z(t)$  will counteract the development of  $y(t)$ . With technology progress and integrated development of  $z(t)$ ,  $z(t)$  will promote in turn.

The last line of Eq. (1) illustrates the rate of the installed capacity of renewable energy is positively related to  $c_2y(y/P - 1)$ . When the use of renewable energy is low, so it will inhibit the rate of the installed capacity of renewable energy and will promote otherwise. Through the simulation of the actual data, we can get the relationship between  $z(t)$  and  $x(t)$  is generally started to increase quickly, and then gradually smooth. By calculation we can get that there is a

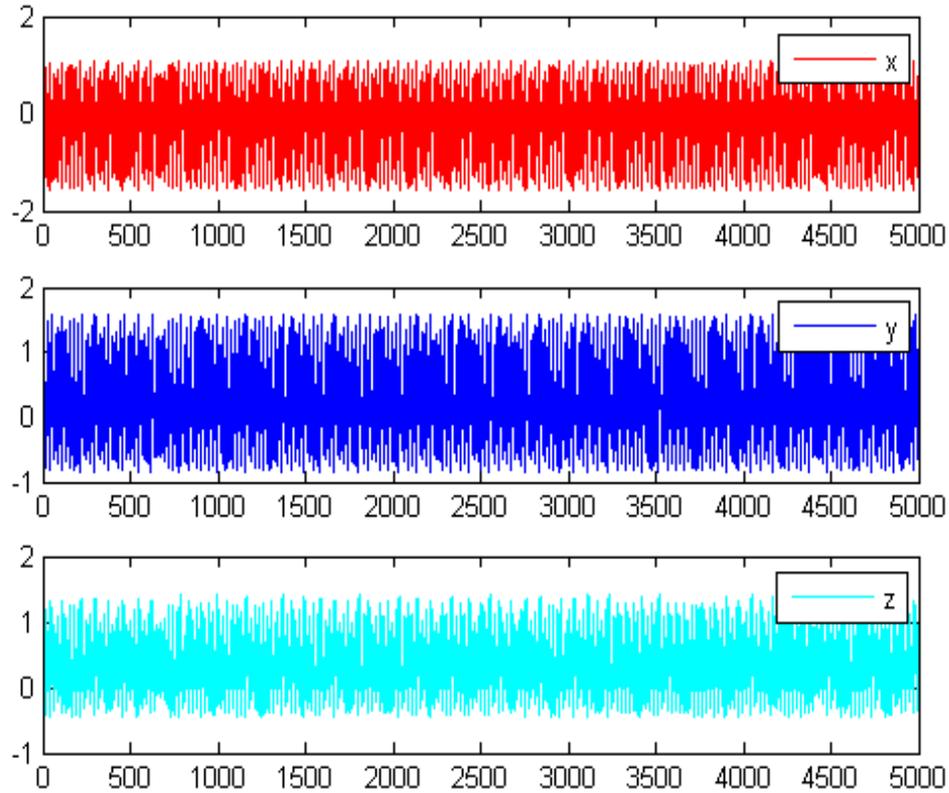


Figure 1: Time evolution series of  $x(t), y(t), z(t)$

positive correlation between  $\dot{x}$  and  $\dot{z}$ . We can explain like this, with the rapid increase of carbon emissions, people realize the importance of protecting the environment, so the development of nonrenewable energy is rapid.

### 2.2 Equilibrium point analysis

In this section, we will analyze the dynamics of the evolution system (1).

Let

$$\begin{cases} \dot{x} = a_1x(1 - x/M) + a_2y - a_3z = 0 \\ \dot{y} = -b_1x - b_2y + b_3z(z/N - 1) = 0 \\ \dot{z} = c_1e^{-x}\dot{x} + c_2y(y/P - 1) = 0 \end{cases} \quad (2)$$

For simplicity, we fix parameters:  $a_1 = 0.15, a_2 = 0.6, a_3 = 0.5, b_1 = 0.6, b_2 = 0.29, b_3 = 0.2, c_1 = 0.42, c_2 = 0.9, M = 1.8, N = 0.8, P = 1.5$ . We can get four equilibrium points  $O(0, 0, 0), S_1(0.0381, 1.5, 1.8112), S_2(5.3847, 0, -3.2171), S_3(6.8011, 1.5, -3.8688)$ ,  $O(0, 0, 0)$  and  $S_3(6.8011, 1.5, -3.8688)$  are stable, Others are unstable saddle focus. we take initial condition  $x(0), y(0), z(0)$  as  $(0.2, 0.5, 0.8)$  after debugging many times. By using numerical simulation, time evolution series  $x(t), y(t), z(t)$  are shown in Fig.1 and chaotic attractor phenomena of the dynamic evolution system is observed as shown in Fig.2,3,4,5. The existence of the chaotic attractor indicates the complexity of the dynamic evolution system.

### 2.3 Bifurcation and Lyapunov exponent

**Theorem 1** Let  $c_1$  be varied and fix parameters  $a_1 = 0.15, a_2 = 0.6, a_3 = 0.5, b_1 = 0.6, b_2 = 0.29, b_3 = 0.2, c_2 = 0.9, M = 1.8, N = 0.8, P = 1.5$ . Then the following results hold:

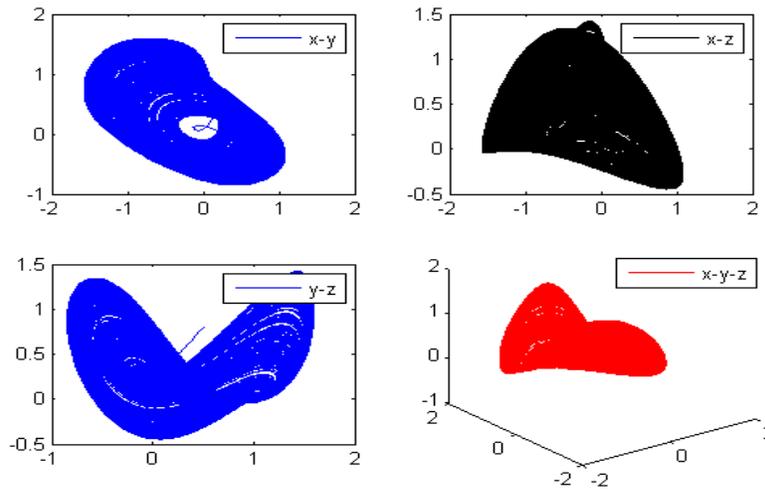


Figure 2: Chaotic attractor of the dynamic evolution system

(i) If  $1.19493 < c_1 < 3.73585$ , then the equilibrium point  $O(0, 0, 0)$  of system 1 is asymptotically stable.

(ii) If  $c_1 = 1.19493$ , the equilibrium point  $O(0, 0, 0)$  is a Hopf bifurcation point.

**Proof.** The characteristic equation of Jacobian matrix of the system (1) at the equilibrium point  $O(0, 0, 0)$  is

$$f(\lambda) = \lambda^3 + (0.5c_1 + 0.14)\lambda^2 + (0.265c_1 + 0.1365)\lambda + 0.297 \quad (3)$$

Let

$$p_1 = 0.5c_1 + 0.14, p_2 = 0.265c_1 + 0.1365, p_3 = 0.297 \quad (4)$$

By the Routh-Hurwitz criterion, all real eigenvalues and all real parts of complex conjugate eigenvalues of equation 3 are negative if and only if the following conditions hold:

$$p_1 > 0, p_1 p_2 - p_3 > 0, p_3 > 0 \quad (5)$$

Hence we can get  $1.19493 < c_1 < 3.73585$ , that means system (1) is unstable if  $0 < c_1 < 1.19493$ . Suppose that the equation 3 have a pair of pure imaginary eigenvalues  $\lambda = \omega i$  ( $\omega > 0$ ) substitute them into equation 3, we can get that  $c_1 = 1.19493, \omega = 0.52337$ . The two sides of the equation 3 derive the  $c_1$ : we can get  $Re(\lambda') = -0.06090932 < 0$ . So If  $c_1 = 1.19493$ , the equilibrium point  $O(0, 0, 0)$  is a Hopf bifurcation point. ■

We fixed  $a_1 = 0.15, a_2 = 0.6, a_3 = 0.5, b_1 = 0.6, b_2 = 0.29, b_3 = 0.2, c_2 = 0.9, M = 1.8, N = 0.8, P = 1.5$ , and varying  $c_1$  from 0.42 to 1.2, the bifurcation diagram and the corresponding Lyapunov exponent spectrums for system (1) are shown in Figure 3 and Figure 4 respectively. It can be seen that the dynamical behaviors of system switch from stable equilibrium point, periodic orbit to chaotic oscillation with decreasing the values of parameter  $c_1$ . It explains the indispensability of the Marshall effect and the system is indeed a chaotic system.

### 3 Empirical analysis

#### 3.1 Data sources

The raw data was collected from EIA, and we select statistical data on carbon dioxide emissions, GDP, and installed capacity of new energy sources ( solar power and wind power) over the period from 2001 to 2014 (See Table 1).

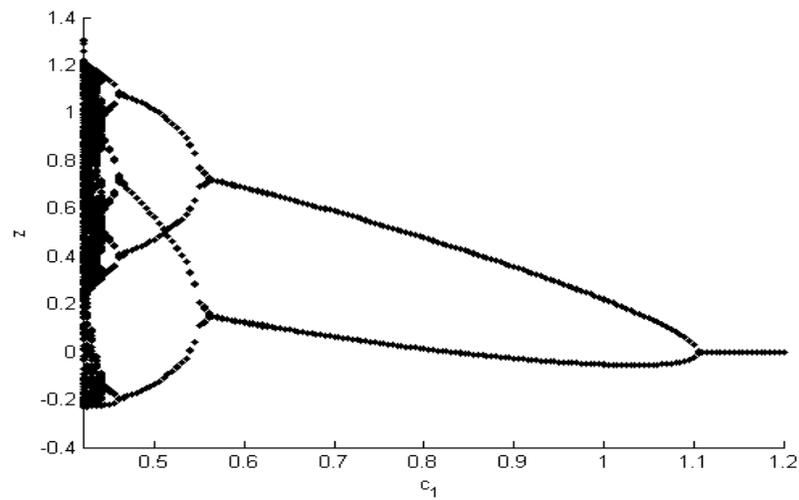


Figure 3: Bifurcation diagram of  $z$  when  $c_1$  adjusted from 0.42 to 1.2. Note that system has a bifurcation at  $c_1 = 1.19493$ .

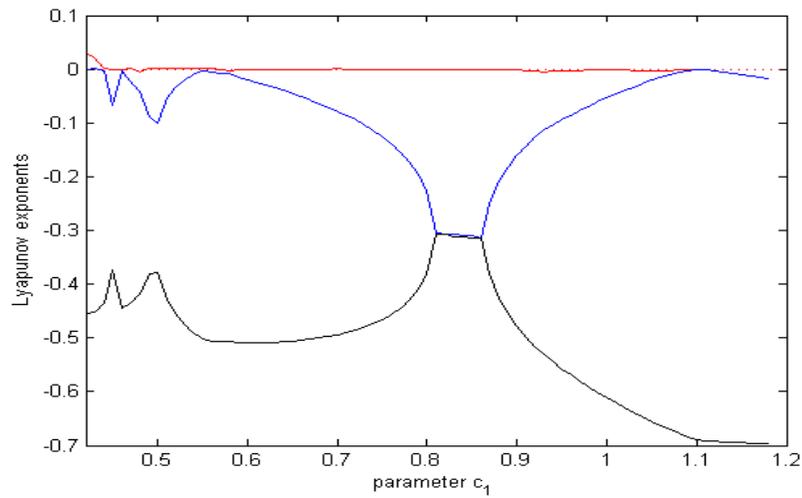


Figure 4: Lyapunov exponent of system (1).

Table 1:  $x(t), y(t), z(t)$  of America date.

Time(year)	$x(t)/10^8$ Tonnes	$y(t)$ Million dollars	$z(t)$ MW
2014	5994.6	159617	84426
2013	5941.4	157103	73371
2012	5786.0	153692	67536
2011	6001.3	150206	51043
2010	6142.7	147838	42314
2009	5908.2	144187	36347
2008	6332.1	148304	25990
2007	6521.5	148737	17334
2006	6412.8	146138	11930
2005	6495.0	142342	9371
2004	6473.3	137735	6861
2003	6343.5	132711	6434
2002	6293.4	129088	4702
2001	6248.4	125597	4267

Table 2: these parameters of identification.

$a_1$	$a_2$	$a_3$	$b_1$	$b_2$	$b_3$	$c_1$	$c_2$	$M$	$N$	$P$
0.0056	0.3047	0.1471	0.1226	0.1486	0.3148	0.9576	0.5223	0.5156	0.6525	0.6099

### 3.2 Genetic algorithm parameter identification

The genetic algorithm of artificial neural network is an effective method to identify the coefficients of a novel system. We substitute the identified parameters into the system (1), we get the actual substitution energy dynamic system (6):

$$\begin{cases} \dot{x} = 0.0056x(1 - x/0.5156) + 0.3047y - 0.1471z \\ \dot{y} = -0.1226x - 0.1486y + 0.3148z(z/0.6525 - 1) \\ \dot{z} = 0.9576e^{-x}\dot{x} + 0.5223y(y/0.6099 - 1) \end{cases} \quad (6)$$

By the standardized data, we set initial condition (0.2, 0.5, 0.3). Substituting the initial condition into the system (6), the three dimensional phase diagram of the system is shown in the Fig.5. Fig.5 illustrates how the system (6) converges to the equilibrium point  $O(0, 0, 0)$ . It shows that substitution energy system is a complex system and reflects complex dynamical behavior in America.

### 3.3 Policy suggestions

It is obvious in Fig.6 that, the parameter  $c_1 = 0.824$  is the best when the range is [0.824, 0.826]. The reason is that, not only the state of installed capacity is stable, but also the time of the shock is shortest, which explains that appropriate installed capacity incentives contribute to the shift and optimize the evolution of the system.

Fig.7 shows that whether parameter  $c_2$  is too large or too small, it is unfavorable to the long-term development of installed capacity. It can be seen that when the parameter  $c_2 = 0.4$ , the amplitude of the installed capacity fluctuates gradually and eventually diverges to infinity, which is not consistent with the actual situation. This indicates that, the impact of carbon emissions can not be ignored.

It is obvious in Fig.8 that, the parameter  $c_1 = 0.824$  is the best when the range is [0.824, 0.826]. The reason is that, not only the state of GDP is stable, but also the time of the shock is shortest, which suggests that renewable energy was not universally accepted in the early days.

Fig.9 shows that whether parameter is too large or too small, it is unfavorable to the long-term development of GDP. It can be seen that when the parameter  $c_2 = 0.4$ , the amplitude of the installed capacity fluctuates gradually and eventually diverges to infinity, which is not consistent with the actual situation.

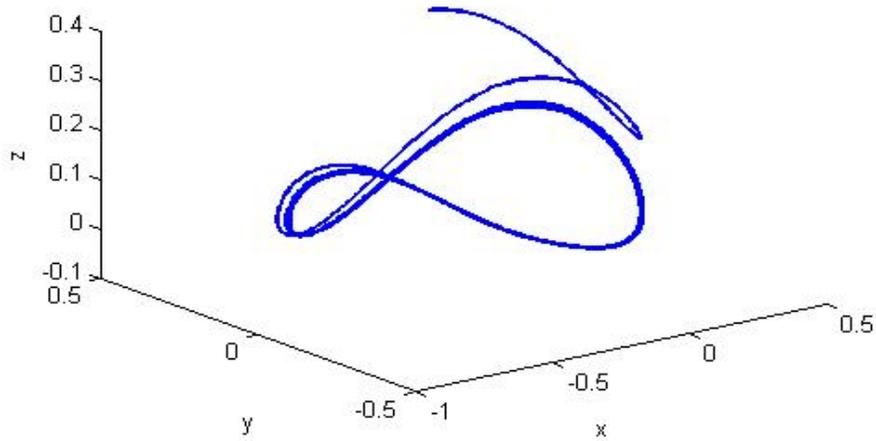


Figure 5: Evolutionary trajectory of parameter-identified system (6)

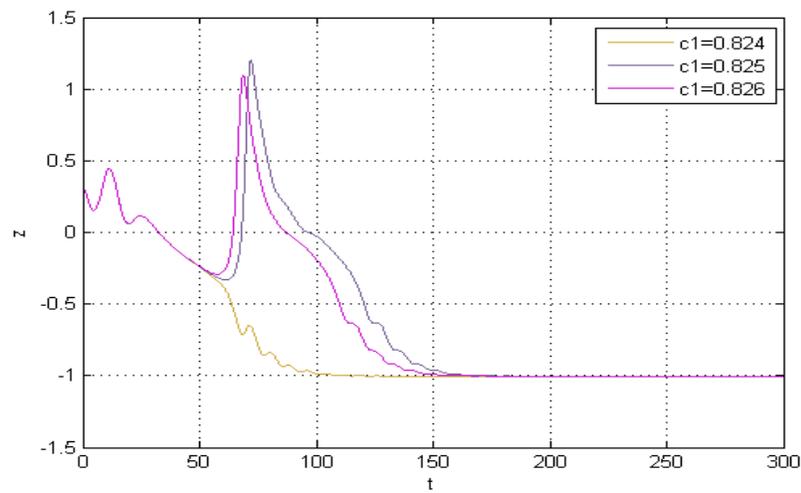


Figure 6: The evolution of the installed capacity with parameter  $c_1$

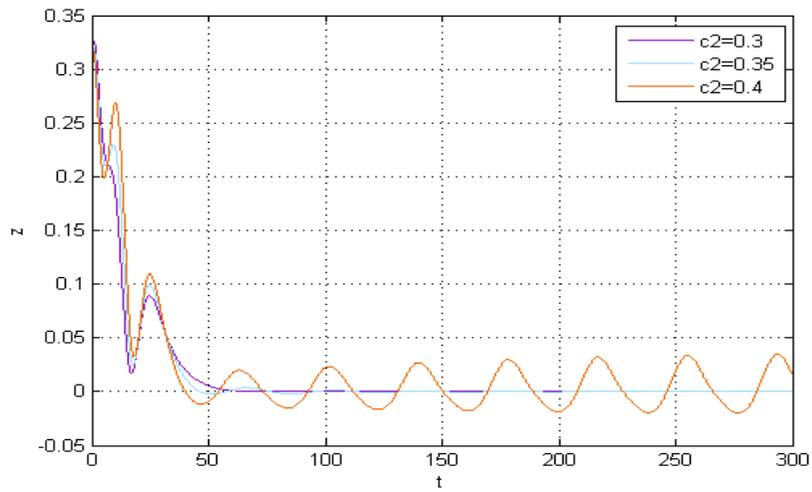


Figure 7: The evolution of the installed capacity with parameter  $c_2$

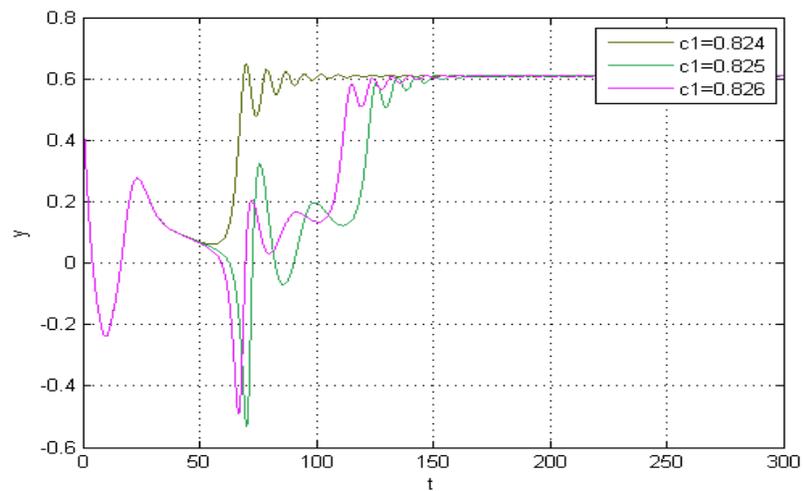


Figure 8: The evolution of GDP with parameter  $c_1$

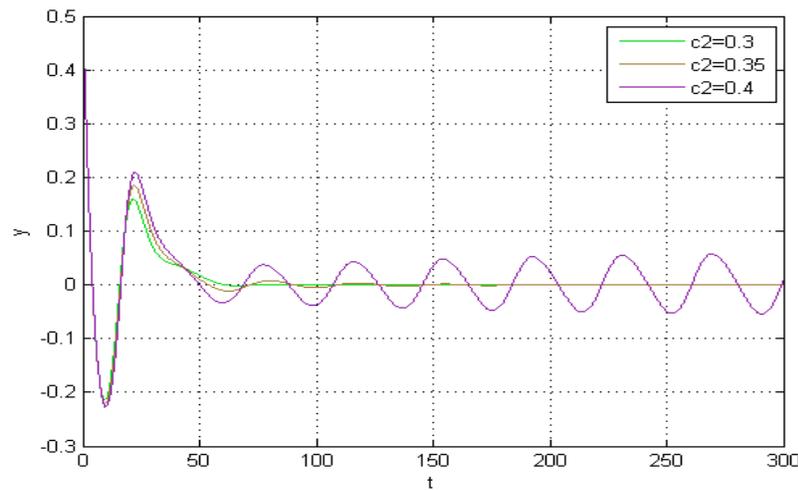


Figure 9: The evolution of GDP with parameter  $c_2$

## 4 Conclusion

In this paper, we applied the model to the America market to study their long-term evolution and found that, the adjustment of the parameters in the model (1), such as  $c_1$ , were completely different in two different markets. In the America market, the state of GDP and installed capacity of renewable energy are stable and the shock is short. So America should enhance investment in renewable energy research and development and the policy restrictions on non-renewable, policy makers could take some specific measures. For example, governments should enlarge subsidies for renewable energy and the government should levy a carbon tax.

In this paper, a new dynamic evolution model reflecting the coupling relationship between carbon emissions, GDP and installed capacity of renewable energy is proposed, and the dynamics of the system are analyzed. We can get some instructions. Government should take effective measures to curb and harness this urgent situation. For example, collecting carbon tax in America is a very good job. policy makers should enact energy policies to ensure the security of energy resource. However, we only discuss the relationship between carbon emissions, GDP and installed capacity of renewable energy. Some other factors, such as carbon tax, subsidy, technical innovation, research and development investment in renewable energy, which play important roles on the evolution behavior of alternative energy system, are not covered in the paper. These factors are going to be considered in future study.

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