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Parker-Sochacki Method for the Solution of Convective Straight Fins Problem with Temperature-Dependent Thermal Conductivity

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Abstract: In this article, analytical expressions for fin efficiency and effectiveness of convective straight fins with temperature-dependent thermal conductivity are derived using the Parker-Sochacki method. The effect of various parameters of the fin problem on fin efficiency and effectiveness were investigated. The obtained results, presented both graphically and in tables, showed excellent agreement with those obtained from existing notable techniques in the literature.

Keywords: Parker-sochacki; Fin efficiency; Fin effectiveness; Convective straight fins; Thermal conductivity; Temperature-dependent

1 Introduction

Fins are frequently used in enhancing heat transfer on surfaces. The fins achieve the transfer of heat from the source to the fin surface through the process of thermal conduction. Two major parameters therefore affect the efficiency and effectiveness of heat conduction through fins, namely the thermal conductivity and heat transfer coefficient. In this article, analytical expressions for the fin efficiency and fin effectiveness of a convective straight fin with temperature dependent thermal conductivity are derived using the Parker-Sochacki method (PSM) [1] which then allows us to investigate the effect of the aforementioned parameters on efficiency and effectiveness of the fins. The PSM is an extension of power series method for linear initial value problem to nonlinear initial value problems. Recently, this method was extended to solving two-point boundary value problems in [2] which therefore makes it applicable to solving the nonlinear fin problem.

The fin problem is well-known in the literature. We refer to [3, 4] for extensive mathematical description of the fin problem. In recent years, several authors have presented approximate solutions to the nonlinear boundary value problem arising from mathematical modeling of the fin problem using various analytic technique. Using the method of repeated Taylor series expansion, the first known series representation of fin efficiency was presented in [5]. In [6], employing differential transformation method (DTM), a slightly different series solution for the fin efficiency of the convective fins was reported. It apparently becomes imperative to revisit this problem in order to validate the series solution of the fin problem.

Recently, approximate solutions to the fin problem has been sought in the literature through homotopy perturbation method (HPM) [7], homotopy analysis method (HAM) [8] and numerical approach [9]. However, it is well known that these aforementioned methods produce results which are only approximations to the Maclaurin series solution of the fin problem. Therefore, none of these methods, in their own right, is able to confirm the accuracy or otherwise of the known Maclaurin series solutions in [5] and [6]. The main goals of this paper is thus two folds. Firstly we employ a simple and easy to implement Parker-Sochacki method to compute series solution of the fin problem. In the process, the series solution to the fin efficiency given in [5] are validated.

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2 Mathematical formulation of the problem and solution methods

The energy balance equation for a straight fin with heat transfer coefficient h, temperature thermal conductivity k, cross-sectional area A_c, perimeter p, length b is given by [6]

$$A_c \frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] - ph(T_b - T_a) = 0, \tag{1}$$

where in the above, T_a, T_b represent respectively the base surface temperature of the fin and fluid temperature respectively, see e.g. [3, 4]. The variable x denotes the distance measured from the tip of the fin. Under linearity assumption between the thermal conductivity of the fin and fin temperature, it holds $k(T) = k_a[1 + \lambda(T - T_a)]$ where k_a denotes the thermal conductivity at the ambient fluid temperature of the fin and λ a thermal conductivity variation parameter.

Through the variable substitutions

$$\theta = \frac{T - T_a}{T_b - T_a}, \quad \xi = x/b, \quad \beta = \lambda(T_b - T_a), \quad \psi = \sqrt{\frac{hpb^2}{k_a A_c}},$$

the dimensionless equation describing the energy balance of the extended surface of straight fins under the influence of temperature-dependent thermal conductivity is obtained [6] as

$$\frac{d^2\theta}{d\xi^2} + \beta\theta\frac{d^2\theta}{d\xi^2} + \beta\left(\frac{d\theta}{d\xi}\right)^2 - \psi^2\theta = 0 \tag{2}$$

with boundary conditions

$$\frac{d\theta}{d\xi} = 0, \quad \text{when } \xi = 0$$

and

$$\theta = 1, \quad \text{when } \xi = 1$$

where β denotes the variation of thermal conductivity and ψ, the thermo-geometric fin parameter.

2.1 Parker-Sochacki method (PSM)

The Parker-Sochacki method is an iterative procedure for computing Maclaurin series solution to nonlinear first-order ordinary differential equations of the form

$$y^0 = f(t, y), \quad y(0) = y^0, \tag{3}$$

where f is a real-valued polynomial function. The method even extends to higher order nonlinear ODEs. In such cases, the idea is to rewrite, using appropriate variable substitution, the nonlinear problem as a system of first-order, constant coefficient ordinary differential equation with polynomial right hand function $F(y) := f(\cdot, y)$. To apply the method to (3), we assume Maclaurin series for the dependent variable:

$$y(t) = \sum_{i=0}^{\infty} y_i t^i$$

with $y^0 = y(0),\ y_1 = y^0,\ y_2 = \frac{1}{2!}y^0, ...$ Similarly, we write $y(t) = \sum_{i=0}^{\infty} y^i t^i$. Differentiating the original series and shifting index appropriately it holds

$$y(t) = \sum_{i=0}^{\infty} y^i t^i = \sum_{i=0}^{\infty} (i+1) y_{i+1}$$

so that

$$y_{i+1} = \frac{y^i}{i+1} = \frac{F(y_i)}{i+1}.$$

Hence, solution of arbitrary order can be computed using the simple iteration above.

3 Solution of (2) by Parker-Sochacki method(PSM)

In this section, we shall compute the approximate analytical solution of (2) via the Parker-Sochacki method [1]. For that purpose, we rewrite the nonlinear problem (2) as a system of first-order, constant coefficient ordinary differential equation with polynomial right hand. We introduce the variables

$$\theta^0 = v, w = \frac{1}{1+\beta\theta},$$

so that equation (2) reduces to

$$\theta^0 = v, \tag{4}$$
$$v^0 = \psi^2\theta w - \beta w v^2, \tag{5}$$
$$w^0 = -\beta w^2 v \tag{6}$$

subject to initial conditions

$$\theta(0) = a, \quad v(0) = \theta(0) = 0, \quad w(0) = \frac{1}{1+\beta a}.$$

In the above, the constant a is an approximation of $\theta(0)$ the fin temperature at the tip and it is to be determined from the boundary condition $\theta(1) = 1$.

Suppose the solutions $\theta(\xi),\ v(\xi),\ w(\xi)$ to the system (4)-(6) can be written in the form

$$\theta(\xi) = \theta_0 + \theta_1\xi + \theta_2\xi^2 + \theta_3\xi^3 + \dots \tag{7}$$
$$v(\xi) = v_0 + v_1\xi + v_2\xi^2 + v_3\xi^3 + \dots \tag{8}$$
$$w(\xi) = w_0 + w_1\xi + w_2\xi^2 + w_3\xi^3 + \dots \tag{9}$$

Then the coefficients $\theta_i,\ v_i,\ w_i\ \ i \geq 0$ are computed from the recursion

$$\theta_{i+1} = \frac{v_i}{i+1}, \quad \theta_0 = a \tag{10}$$
$$v_{i+1} = \frac{\psi^2(\theta w)_i - \beta(wv^2)_i}{i+1}, \quad v_0 = 0, \tag{11}$$
$$w_{i+1} = -\beta\frac{(w^2v)_i}{i+1}, \quad w_0 = \frac{1}{1+\beta a}. \tag{12}$$

The coefficients $(\theta w)_i$ for the series expansion of the product θw is obtained using Cauchy product [10]

$$(\theta w)_i = \sum_{j=0}^{i} \theta_j w_{i-j}, \ \ i \geq 0. \tag{13}$$

For the coefficients $(wv^2)_i$ and $(w^2v)_i$ above, repeated use of the Cauchy product (13) yields

$$(wv^2)_i = \sum_{j=0}^{i}\left(\sum_{k=0}^{j} v_k v_{j-k}\right) w_{i-j}, \quad (w^2v)_i = \sum_{j=0}^{i}\left(\sum_{k=0}^{j} w_k w_{j-k}\right) v_{i-j} \tag{14}$$

which are then used in the recursive relations (10)-(12). From equations (10)-(12), we obtain

$$\theta_1 = 0, \quad v_1 = \frac{\psi^2 a}{1+\beta a}, \quad w_1 = 0; \tag{15}$$

$$\theta_2 = \frac{1}{2}\,\frac{\psi^2 a}{(1+\beta a)}, \quad v_2 = 0, \quad w_2 = -\frac{1}{2}\,\frac{\beta\,\psi^2 a}{(1+\beta a)^3}; \tag{16}$$

$$\theta_3 = 0, \quad v_3 = -\frac{1}{6}\,\frac{\psi^4 a\,(2\,\beta\,a-1)}{(1+\beta a)^3}, \quad w_3 = 0; \tag{17}$$

$$\theta_4 = -\frac{1}{24}\,\frac{\psi^4 a\,(2\,\beta\,a-1)}{(1+\beta a)^3}, \quad v_4 = 0, \quad w_4 = \frac{1}{24}\,\frac{\psi^4 a\,(8\,\beta\,a-1)}{(1+\beta a)^5}; \tag{18}$$

$$\theta_5 = 0, \quad v_5 = \frac{1}{120}\,\frac{\psi^6 a\,(2\,\beta\,a-1)\,(14\,\beta\,a-1)}{(1+\beta a)^5}, \quad w_5 = 0; \tag{19}$$

$$\theta_6 = \frac{1}{720}\,\frac{\psi^6 a\,(2\,\beta\,a-1)\,(14\,\beta\,a-1)}{(1+\beta a)^5}, \quad v_6 = 0, \quad w_6 = -\frac{1}{720}\,\frac{\beta\psi^6 a\left(1-46\,\beta\,a+178\beta^2\,a^2\right)}{(1+\beta a)^7}; \tag{20}$$

$$\theta_7 = 0, \quad v_7 = -\frac{1}{5040}\,\frac{\beta\psi^8 a\,(2\,\beta\,a-1)\left(1-76\,\beta\,a+448\beta^2\,a^2\right)}{(1+\beta a)^7}, \quad w_7 = 0; \tag{21}$$

$$\theta_8 = -\frac{1}{40320}\,\frac{\psi^8\,a(2\,\beta\,a-1)(448\,\beta^2\,a^2-76\,\beta\,a+1)}{(1+\beta\,a)^7}, \quad v_8 = 0, \tag{22}$$

$$w_8 = \frac{\beta\,\psi^8\,a(7784\,\beta^3\,a^3-3036\,\beta^2\,a^2+204\,\beta\,a-1)}{(1+\beta\,a)^9}, \tag{23}$$

For the sake of completeness, we report further computed coefficients of θ_i

$$\theta_9 = 0, \quad \theta_{10} = \frac{1}{3628800}\,\frac{\psi^{10} a\,(2\,\beta\,a-1)\left(22592\,\beta^3\,a^3-7152\,\beta^2\,a^2+303\,\beta\,a-1\right)}{(1+\beta a)^9}, \quad \theta_{11} = 0, \tag{24}$$

$$\theta_{12} = -\frac{1}{479001600}\,\frac{\psi^{12} a\,(2\,\beta\,a-1)\,\left(2288104\,\beta^4 a^4-903896\,\beta^3 a^3+81072\,\beta^2 a^2-1352\,\beta\,a+1\right)}{(1+\beta a)^{11}}\ldots$$

Substituting the coefficients θ_i in (7) we obtain the series solution

$$\begin{aligned}\theta(\xi) = {} & a+\frac{1}{2}\,\frac{\psi^2 a}{(1+\beta a)}\xi^2-\frac{1}{24}\,\frac{\psi^4 a\,(2\,\beta\,a-1)}{(1+\beta a)^3}\xi^4+\frac{1}{720}\,\frac{\psi^6 a\,(2\,\beta\,a-1)\,(14\,\beta\,a-1)}{(1+\beta a)^5}\xi^6 \\ & -\frac{1}{40320}\,\frac{\psi^8\,a(2\,\beta\,a-1)(448\,\beta^2\,a^2-76\,\beta\,a+1)}{(1+\beta\,a)^7}\xi^8 \\ & +\frac{1}{3628800}\,\frac{\psi^{10}\,a(2\beta\,a-1)(25592\,\beta^3\,a^3-7152\,\beta^2\,a^2+330\,\beta\,a-1)}{(1+\beta\,a)^9}\xi^{10} \\ & -\frac{1}{479001600}\,\frac{\psi^{12}\,a(2\beta\,a-1)(2288104\,\beta^4\,a^4-903896\,\beta^3\,a^3+81072\,\beta^2\,a^2-1352\,\beta\,a+1)}{(1+\beta\,a)^{11}}\xi^{12} \\ & +\ldots\end{aligned} \tag{25}$$

Remark 1 *The above computed coefficients using the Parker-Sochacki method are in agreement with those obtained by [9] via the method of repeated Taylor series expansion. However, in [6, eq. 17], using differential transformation method, the coefficient θ_8 was inaccurately reported. The present work therefore validates the results in [9].*

3.1 Fin efficiency

Based on Newton's law of cooling the fin efficiency of the convective straight fin is given in [6] as

$$\eta = \int_{\xi=0}^{1} \theta(\xi)d\xi.$$

Using (25), we obtain the fin efficiency straightforwardly as

$$\eta = a + \frac{1}{6}\,\frac{\psi^2 a}{(1+\beta\,a)} - \frac{1}{120}\,\frac{\psi^4 a\,(2\,\beta\,a-1)}{(1+\beta\,a)^3} + \frac{1}{5040}\,\frac{\psi^6 a\,(2\,\beta\,a-1)\,(14\,\beta\,a-1)}{(1+\beta\,a)^5}$$
$$- \frac{1}{362880}\,\frac{\psi^8\,a(2\,\beta\,a-1)(448\,\beta^2\,a^2-76\,\beta\,a+1)}{(1+\beta\,a)^7}$$
$$+ \frac{1}{39916800}\,\frac{\psi^{10}\,a(2\beta\,a-1)(25592\,\beta^3\,a^3-7152\,\beta^2\,a^2+330\,\beta\,a-1)}{(1+\beta\,a)^9}$$
$$- \frac{1}{6227020800}\,\frac{\psi^{12}\,a(2\beta\,a-1)(2288104\,\beta^4\,a^4-903896\,\beta^3\,a^3+81072\,\beta^2\,a^2-1352\,\beta\,a+1)}{(1+\beta\,a)^{11}}$$
$$+ \ldots \tag{26}$$

3.2 Fin effectiveness

The dimensionless fin effectiveness of convective straight fins with thickness t and length L is given by [7]

$$\eta^* = \int_{\xi=0}^{1} \tau\theta(\xi)d\xi$$

where $\tau = \frac{2L}{t}$. Using (25), the fin effectiveness is computed as

$$\eta^* = \tau\Big(a + \frac{1}{6}\,\frac{\psi^2 a}{(1+\beta\,a)} - \frac{1}{120}\,\frac{\psi^4 a\,(2\,\beta\,a-1)}{(1+\beta\,a)^3} + \frac{1}{5040}\,\frac{\psi^6 a\,(2\,\beta\,a-1)\,(14\,\beta\,a-1)}{(1+\beta\,a)^5}$$
$$- \frac{1}{362880}\,\frac{\psi^8\,a(2\,\beta\,a-1)(448\,\beta^2\,a^2-76\,\beta\,a+1)}{(1+\beta\,a)^7}$$
$$+ \frac{1}{39916800}\,\frac{\psi^{10}\,a(2\beta\,a-1)(25592\,\beta^3\,a^3-7152\,\beta^2\,a^2+330\,\beta\,a-1)}{(1+\beta\,a)^9}$$
$$- \frac{1}{6227020800}\,\frac{\psi^{12}\,a(2\beta\,a-1)(2288104\,\beta^4\,a^4-903896\,\beta^3\,a^3+81072\,\beta^2\,a^2-1352\,\beta\,a+1)}{(1+\beta\,a)^{11}}$$
$$+ \ldots\Big). \tag{27}$$

3.3 Computation of fin temperature at the tip $\theta(0) = a$

The common procedure for determining the constant a, as done in [6, 8], is to truncate the solution (25), impose the boundary condition at $\xi = 1$ and employ some root finding algorithm to obtain an approximation to the unknown initial data $\theta(0)$. We remark that this method suffers some form of errors e.g. truncation error. In this article, we adopt shooting strategy with Newton-Ralphson nonlinear solver to compute the missing data a. We iteratively choose different values for a and solve numerically (2) with initial conditions $\theta(0) = a,\ \theta(0) = 0$ until the boundary condition $\theta(1) = 1$ is fulfilled. In section 4, the computed values of a for different choices of the parameters β and ψ are reported.

4 Numerical results and comparison with existing results

In line with Section 3.3, for $\beta = 0$ and $\psi = 0.5$, we computed the constant $a = 0.886818883936038$. Whereas for $\beta = 0$ and $\psi = 1$, we obtained $a = 0.648054272220277$. Similarly, for $\beta = 0.4, \psi = 1, a = 0.716046465486149$ and for $\beta = 0.2, \psi = 0.5, a = 0.903447179258743$.

For $\beta = 0$, the exact solution to (2) is given by

$$\theta(\xi) = \frac{\cosh(\psi\xi)}{\cosh(\xi)}.$$

When $\beta \neq 0$, the temperature distribution is computed according to (25) and the results are presented in Table 5 and Table 6. The results are compared with those of the exact solution (in the linear case), those computed via the homotopy analysis

Table 1: Here our results for $\theta(\xi)$ are compared with the exact solution for $\beta = 0$ and $\psi = 0.5$.

ξ	Exact	10th order PSM	Absolute error
0.0	0.8868188838	0.8868188839	1.0×10^{-10}
0.1	0.8879276382	0.8879276384	2.0×10^{-10}
0.2	0.8912566745	0.8912566746	1.0×10^{-10}
0.3	0.8968143168	0.8968143166	2.0×10^{-10}
0.4	0.9046144616	0.9046144618	2.0×10^{-10}
0.5	0.9146766141	0.9146766140	1.0×10^{-10}
0.6	0.9270259345	0.9270259344	1.0×10^{-10}
0.7	0.9416933025	0.9416933025	0
0.8	0.9587153943	0.9587153941	2.0×10^{-10}
0.9	0.9781347735	0.9781347739	4.0×10^{-10}
1.0	0.9999999999	0.9999999999	0

Table 2: Here our results for $\theta(\xi)$ are compared with the exact solution for $\beta = 0$ and $\psi = 1$.

ξ	Exact	10th order PSM	Absolute error
0.0	0.6480542738	0.6480542722	1.6×10^{-9}
0.1	0.6512972462	0.6512972447	1.5×10^{-9}
0.2	0.6610586205	0.6610586188	1.7×10^{-9}
0.3	0.6774360918	0.6774360900	1.8×10^{-9}
0.4	0.7005935709	0.7005935691	1.8×10^{-9}
0.5	0.7307628261	0.7307628242	1.9×10^{-9}
0.6	0.7682458010	0.7682457993	1.7×10^{-9}
0.7	0.8134176383	0.8134176364	1.9×10^{-9}
0.8	0.8667304328	0.8667304307	2.1×10^{-9}
0.9	0.9287177568	0.9287177542	2.6×10^{-9}
1.0	1.00000000000	0.9999999964	3.6×10^{-9}

method (HAM) [8], homotopy perturbation method (HPM) [7] and differential transformation method (DTM) [6]. It is easily seen that our method performs just as good as those obtained via other mentioned famous methods. In Table 1-4, accuracy of the proposed method is further depicted. The temperature distribution is illustrated graphically for different values of the parameters in Figure 1 and Figure 2. Also in Figure 3 and 4 the fin efficiency and fin effectiveness (26) and (27) are plotted against the thermo-geometric fin parameter ψ.

5 Conclusion

We have successfully derived, using the Parker-Sochacki method, an analytical solution of the nonlinear boundary value problem arising from efficiency and effectiveness problem in convective straight fins with temperature dependent thermal conductivity. Involving only elementary operations, the explicit representation of the temperature distribution, fin efficiency as well as fin effectiveness were computed. Effects of different parameters on the temperature distribution, fin efficiency and fin effectiveness were also investigated and results have been presented both graphically and in tables. The computed results validated the result in [9]. When compared with existing results in the literature, Parker-Sochacki method competes favorably well and always produce highly accurate series solution. Furthermore, banking on the ease of computation and application of the method, it remains a viable alternative to other famous power series solution of nonlinear boundary value problems.

The author would like to thank referees for their valuable comments.

Table 3: Here our results for $\theta(\xi)$ are compared with the numerical solution for $\beta = 0.2$ and $\psi = 0.5$.

ξ	Numerical	10th order PSM	Absolute error
0.0	0.9034471816	0.9034471813	3.0×10^{-10}
0.1	0.9044037555	0.9044037553	3.0×10^{-10}
0.2	0.9072745722	0.9072745721	1.0×10^{-10}
0.3	0.9120629135	0.9120629134	1.8×10^{-9}
0.4	0.9187742391	0.9187742391	0
0.5	0.9274161715	0.9274161718	3.0×10^{-10}
0.6	0.9379984754	0.9379984759	5.0×10^{-10}
0.7	0.9505330304	0.9505330311	7.0×10^{-9}
0.8	0.9650337986	0.9650337999	1.3×10^{-9}
0.9	0.9815167864	0.9815167878	1.4×10^{-9}
1.0	1.00000000000	1.0000000020	2.0×10^{-9}

Table 4: Here our results for $\theta(\xi)$ are compared with the numerical solution for $\beta = 0.4$ and $\psi = 1$.

ξ	Numerical	10th order PSM	Absolute error
0.0	0.7160464718	0.7160464655	6.3×10^{-9}
0.1	0.7188301796	0.7188301644	1.52×10^{-8}
0.2	0.7271884325	0.7271884233	9.2×10^{-9}
0.3	0.7411426042	0.7411425986	5.6×10^{-9}
0.4	0.7607278638	0.7607278575	6.3×10^{-9}
0.5	0.7859925542	0.7859925489	5.3×10^{-9}
0.6	0.8169973564	0.8169973537	2.7×10^{-9}
0.7	0.8538142400	0.8538142374	2.6×10^{-9}
0.8	0.8965252366	0.8965252385	1.9×10^{-9}
0.9	0.9452211231	0.9452211293	6.2×10^{-9}
1.0	1.00000000000	1.0000000020	2.0×10^{-9}

Table 5: Our results on the dimensionless temperature distribution $\theta(\xi)$ within the fin for the constant thermal conductivity $\beta = 0$ and $\psi = 0.5$ are compared against other methods in the literature.

ξ	Exact	PSM (this work)	HPM[7]	HAM[8]	DTM[6]
0.0	0.88681	0.88681	0.8868	0.88681	0.88681
0.1	0.88792	0.88792	0.8879	0.88792	0.88792
0.2	0.89125	0.89125	0.8913	0.89125	0.89125
0.3	0.89681	0.89681	0.8968	0.89681	0.89681
0.4	0.90461	0.90461	0.9046	0.90461	0.90461
0.5	0.91467	0.91467	0.9147	0.91467	0.91467
0.6	0.92702	0.92702	0.9270	0.92702	0.92702
0.7	0.94169	0.94169	0.9417	0.94169	0.94169
0.8	0.95871	0.95871	0.9587	0.95871	0.95871
0.9	0.97813	0.97813	0.9781	0.97813	0.97813
1.0	0.99999	0.99999	1.00000	1.00000	1.00000

Table 6: Our results on the dimensionless temperature distribution $\theta(\xi)$ within the fin for thermal conductivity $\beta = 0.2$ and $\psi = 0.5$. are compared against other methods in the literature.

ξ	Numerical	PSM (this work)	HAM[8]	DTM[6]
0.0	0.90344	0.90344	0.90344	0.90344
0.1	0.90440	0.90440	0.90440	0.90440
0.2	0.90727	0.90727	0.90727	0.90727
0.3	0.91206	0.91206	0.91206	0.91206
0.4	0.91877	0.91877	0.91877	0.91877
0.5	0.92741	0.92741	0.92741	0.92741
0.6	0.93799	0.93799	0.93799	0.93799
0.7	0.95053	0.95053	0.95053	0.95053
0.8	0.96503	0.96503	0.96503	0.96503
0.9	0.98151	0.98151	0.98151	0.98151
1.0	0.99999	0.99999	0.99999	0.99999

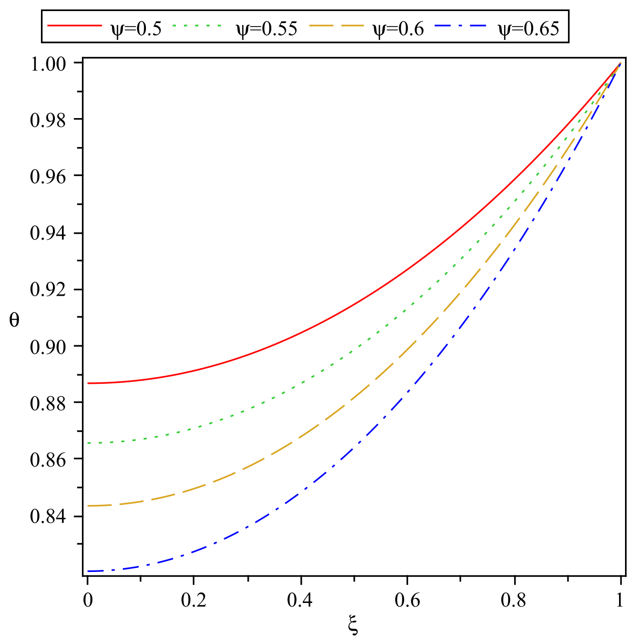

Figure 1: Temperature distribution in convective fins for varying thermo-geometric fin parameter ψ for $\beta = 0$.

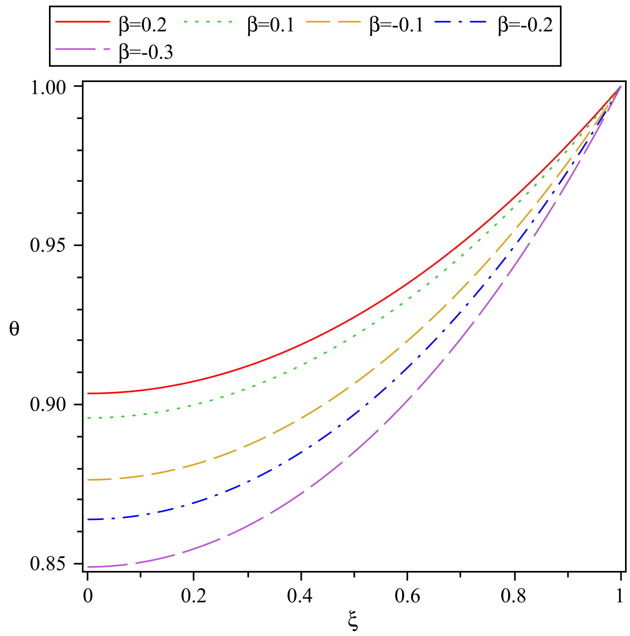

Figure 2: Temperature distribution in convective fins with variable thermal conductivity parameter β for $\psi = 0.5$.

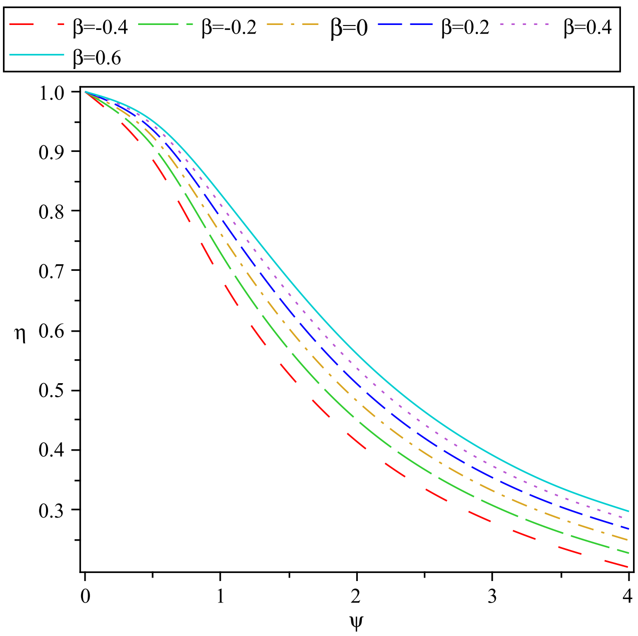

Figure 3: Variation of fin efficiency with the thermo-geometric fin parameter ψ for varying values of thermal conductivity parameter β.

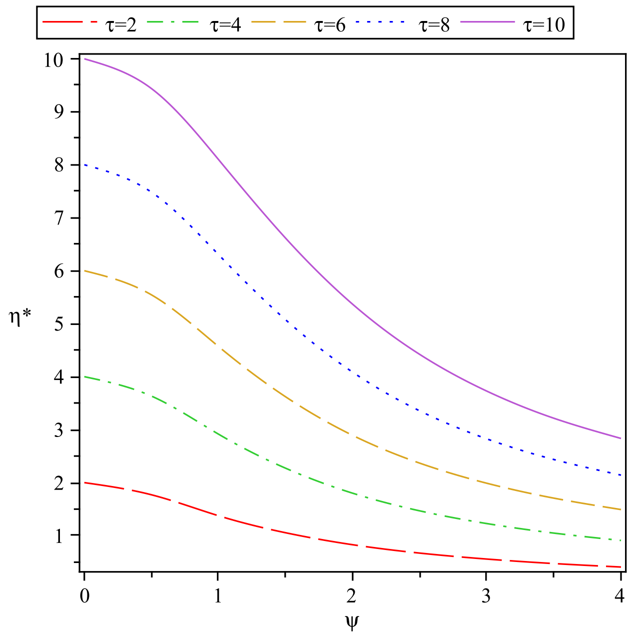

Figure 4: Variation of fin effectiveness with the thermo-geometric fin parameter ψ for varying values of τ the length/thickness ratio of the fin.

References

[1] G. Edgar Parker and James S. Sochacki. Implementing the picard iteration. *Neural, Parallel Sci. Comput.*, 4(1)(1996):97–112.

[2] Saheed Ojo Akindeinde and Eric Okyere. New analytic technique for the solution of nth order nonlinear two-point boundary value problem. *British Journal of Mathematics and Computer Science*, 15(2)(2016):1–11.

[3] A. Aziz A. Kraus and J. Welty. *Extended Surface Heat Transfer*, 2001

[4] D.Q. Kern and D.A. Kraus. *Extended Surface Heat Transfer*, 1972

[5] Sin Kim and Cheng-Hung Huang. A series solution of the fin problem with a temperature-dependent thermal conductivity. *Journal of Physics D: Applied Physics*, 39(22)(2006):4895.

[6] A.A. Joneidi, D.D. Ganji, and M. Babaelahi. Differential transformation method to determine fin efficiency of convective straight fins with temperature dependent thermal conductivity. *International Communications in Heat and Mass Transfer*, 36(7)(2009):757 – 762.

[7] Erdem Cuce and Pinar Mert Cuce. Homotopy perturbation method for temperature distribution, fin efficiency and fin effectiveness of convective straight fins. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 227(8)(2013):1754–1760.

[8] M. Subha L. Rajendran K. Saravanakumar, V. Ananthaswamy. Analytical solution of nonlinear boundary value problem for fin efficiency of convective straight fins with temperature-dependent thermal conductivity. *ISRN Thermodynamics*, 2013(3), 2013.

[9] Rafael Cortell. A numerical analysis to the non-linear fin problem. *Journal of Zhejiang University SCIENCE A*, 9(5)(2008):648–653.

[10] Donald E. Knuth. Addison Wesley Longman Publishing Co., Inc., Redwood City, CA, USA. *The Art of Computer Programming*, 1(1997)(3).