ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.23(2017) No.2, pp.67-74

Influences of Negative Ion on the Jeans Instability in a Dusty Plasma

Banamali Roy [a] *, Supriya Mukherjee[b]
[a] Deptpartment of Mathematics, Bangabasi Evening College,19 Rajkumar Chakraborty Sarani, Kolkata-700009, West Bengal, India
[b] Department of Mathematics, Gurudas College, 1/1, Suren Sarkar Road, Narkeldanga, Kolkata, Pin- 700054, West Bengal, India

(Received 29 September 2011, accepted 16 February 2017)

Abstract: We present an investigation of possible Jeans instability of self-gravitating dusty plasma consisting of Boltzmann distributed electrons, positive ions and negative ions. We have derived the dispersion relation by using continuity and momentum equations for the charged dust grains, together with Poisson equations for the electrostatic and self-gravitational potentials. It has been shown that in a negatively charged dusty plasma, negative ion number density enhances the Jeans instability whereas negative ion temperature suppress the Jeans instability.

Keywords: dusty plasmas, negative Ions, Jeans instability

1 Introduction

Ionized gases at very low temperature comprising of electrons, ions and highly charged massive grains of dust particles are commonly known as Dusty Plasma. With the growing importance of a such plasma in interstellar clouds, circumstellar clouds, interplanetary medium, cometary tails, planetary rings and the earth's magnetosphere [1–3], it has become a major area of research. In the laboratory, the dusty plasma occurs as a result of high Z impurities from the Tokamak walls during plasma etching and impurity generation in magneto hydrodynamic power generators.

The dust grains are charged due to their interaction with electrons, ions and background radiation and hence can change the plasma parameters which in turn affect the collective behaviour of the plasma. There are various mechanisms of charging of dust grains; viz; thermionic emission, field emission, radioactivity, impact ionization etc. Although, initially the dust grains are negatively charged, the presence of negative ions in the plasma plays an important role in the charging of dust grains positively under certain laboratory conditions. If the negative ion mass is more than the positive ion mass and if the negative ion density is comparable to the positive ion density, then the dust grains will be positively charged [4]. Experimentally, it has been shown that as the concentration of negative ions in the plasma is increased relatively, the magnitude of negative charge decreases and a transition to positively charged dust are observed [5]. Presence of negative ions along with the negatively charged dust grains can play a very important role in a complex plasma. Vladimirov et al. [6] pointed out that the equilibrium state of plasma as well as the ion acoustic wave propagation are significantly modified in a complex plasma in presence of negative ions due to relevant processes such as ionization, electron attachment, diffusion, positive-negative ion recombination, plasma particle collision, as well as elastic Coulomb and inelastic dust charging collisions. Negative ions are also very important for reactive laboratory and technological plasmas[8, 9].

At the beginning of the 20th century, Jeans [10] was the first to predict the instability of self gravitating large gas clouds. It is believed that this type of instability causes uncharged and extended mass clouds to collapse. However, the presence of plasma initiates the possibility of balancing the self gravitational, electromagnetic and pressure gradient forces and hence sets a limitation on the Jeans instability in self gravitating plasma. Several authors [11]-[14] have focused on the waves and instabilities in a self gravitating partially ionized magnetoplasma by ignoring the presence of the charged dust particles and the collisional effects. Jeans instabilities in unmagnetized dusty plasmas have been investigated by several

---

*Corresponding author. *E-mail address*: banamaliroy@yahoo.co.in (B. Roy), supriya_ju@yahoo.com (S. Mukherjee)

IJNS.2017.04.15/948

authors [15]-[19]. However in 2005, Jacobs and Shukla [20] reported that the magnetic field and ion neutral collisions are stabilizing factors for the Jeans instabilities in partially ionized astrophysical plasma. The role of nonthermal positive ions [21] and effect of secondary electrons [22] in a self-gravitating dusty plasma with variable charge dust grains have also been studied.

In the present paper, we present an investigation of the Jeans instability in an unmagnetized self-gravitating plasma composed of negatively charged dust grains with negative ions. We derive a new dispersion relation and examine numerically the influence of Jeans frequencies, negative ion number density and temperatures on the growth rates as well as on the real frequencies for our multi-component dusty plasma system.

The paper has been organized as follows. Section 2 provides the details of the formulation of the problem for the Jeans instability of self-gravitating dusty plasma consisting of Boltzmann distributed electrons, positive ions and negative ions. Section 3 contains the dispersion relation for dust acoustic waves in a self gravitating dusty plasma in presence of Boltzmannian species like positive ions, negative ions, electrons and warm negatively charged dust fluid. Section 4 contains the numerical results. The role and effect of negative ion number density and temperature on the Jeans instability are discussed and comparison with the absence of negative ion is provided in this section. We show that negative ion number density enhances the Jeans instability whereas negative ion temperature suppress the Jeans instability. Finally a conclusion is drawn in section 5.

2 Formulation of the problem

We consider a four component dusty plasma, comprising of negatively charged warm adiabatic dust grains, Boltzmann distributed electrons, positive ions and negative ions, characterized by the number density n_j, and the mass m_j of the j*th* species*(j=e for electrons, i for positive ions, n for negative ions and d for dust grains)* and n_{jo} *(j=e, i, n, d)* are the equilibrium number densities of j *th*species.

The quasi-neutrality condition at equilibrium is

$$n_{i0} = n_{e0} + n_{n0} + Z_{d0} n_{d0}, \tag{1}$$

where $-Z_{do}e$ is the equilibrium charge on the dust surface.

We are interested to study low frequency wave i.e $\omega << k_B v_{te}, k_B v_{ti}, k_B v_{tn}$, where v_{te}, v_{ti}, v_{tn} are thermal velocities of electrons, positive ions and negative ions, respectively. Electrons, positive ions and negative ions are considered to follow the Boltzmann relation. For electrostatic mode $(E = -grad\phi)$, the electron number density, positive ion number density and negative ion number density are connected with the electrostatic potential ϕ as,

$$n_e = n_{e0} \exp\left(\Phi\right), \tag{2}$$

$$n_i = n_{i0} \exp\left(\frac{-\Phi}{\sigma_i}\right), \tag{3}$$

$$n_n = n_{n0} \exp\left(\frac{\Phi}{\sigma_n}\right). \tag{4}$$

Here, $\sigma_i = \frac{T_i}{T_e}$, $\sigma_n = \frac{T_n}{T_e}$, T_e, T_i and T_n are the electron, positive ion and negative ion temperature, and $\Phi = \frac{e\phi}{k_B T_e}$, where k_B is the Boltzmann constant.

For an isolated dust grain, the dust charging in a gas discharge plasma, one of the most frequently used approach is the orbit motion limited (OML) theory [23]. The expressions for the positive ion current (I_i), negative ion current (I_n) and electron current (I_e) for spherical dust grain of radius a are as follows:

$$I_i = J_j \exp\left(\frac{-\Phi}{\sigma_i}\right)\left(1 - \frac{eq_d}{ak_B T_i}\right); \quad q_d < 0, \tag{5}$$

$$I_n = -J_n \exp\left(\frac{\Phi}{\sigma_n}\right) \exp\left(\frac{eq_d}{ak_B T_n}\right); \quad q_d < 0, \tag{6}$$

$$I_e = -J_e \exp(\Phi) \exp\left(\frac{eq_d}{ak_BT_e}\right); \quad q_d < 0, \tag{7}$$

where $J_j = \pi a^2 e \sqrt{\frac{8k_BT_j}{\pi m_j}} n_{j0}$ and q_d is the dust charge.

Considering electron, positive ion and negative ion currents due to collisions with plasma particles, the dust grain charging equation becomes,

$$\frac{\partial q_d}{\partial t} + v_d \frac{\partial q_d}{\partial x} = I_e + I_i + I_n. \tag{8}$$

where v_d denotes the dust velocity.

The dust charging frequency ν_{ch} is,

$$\nu_{ch} = \frac{a/\lambda_{Di}}{\sqrt{2\pi}} \omega_{pi} \left[1 + \left(\frac{z_0 + \sigma_i}{\alpha_n}\right)(1 + \beta_n)\right], \tag{9}$$

and using the current balance equation $I_{e0} + I_{i0} + I_{n0} = 0$ the ion-electron number density ratio is given by,

$$\frac{n_{i0}}{n_{e0}} = \sqrt{\frac{m_i}{m_e}} \frac{\sqrt{\sigma}_i}{\sigma_i + z_0} \alpha_n e^{-z_0}, \tag{10}$$

where

$$\alpha_n = 1 - \frac{n_{n0}}{n_{e0}} \sqrt{\frac{\sigma_n m_e}{m_n}} e^{\left(1 - \frac{1}{\sigma_n}\right) z_0}, \tag{11}$$

and

$$\beta_n = \frac{n_{n0}}{n_{e0}} \sqrt{\frac{m_e}{\sigma_n m_n}} e^{\left(1 - \frac{1}{\sigma_n}\right) z_0}. \tag{12}$$

It should be noted that, in absence of negative ion $i.e\ n_{n0} = 0$, the value of $\alpha_n = 1$ and $\beta_n = 0$.

We consider electronegative dusty plasma whose constituents are Boltzmann-distributed electrons, positive ions, negative ions (both singly charged), and charge fluctuating negatively charged warm dust grains. Therefore, the nonlinear dynamics of low phase velocity dust acoustic waves are governed by the dust continuity and momentum equations:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0, \tag{13}$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = -\frac{q_d}{m_d} \frac{\partial \phi}{\partial x} - \frac{k_B T_d}{m_d n_d} \frac{\partial n_d}{\partial x} - \frac{\partial \psi}{\partial x}. \tag{14}$$

Here, T_d represent dust temperature and the gravitational potential is represented by ψ.

The Poisson's equations for the overall charge balance,

$$\epsilon_0 \frac{\partial^2 \phi}{\partial x^2} = -\left(en_i - en_e - en_n + q_d n_d\right), \tag{15}$$

and mass densities

$$\frac{\partial^2 \psi}{\partial x^2} = 4\pi G m_d (n_d - n_{d0}), \tag{16}$$

where G is the universal gravitational constant and we have neglected the gravitational effects of positive ions, negative ions and electrons.

It should be noted that for a gravitating plasma, the assumption of an equilibrium value n_{d0} of the dust number density n_d is a consequence of what is known as Jeans Swindle.

3 Dispersion relation

Considering perturbation $(\delta n_e,\ \delta n_i,\ \delta n_n,\ \delta n_d,\ Z_{d0}eq_1) \sim e^{i(kx-wt)}$ about the equilibrium state $(n_{e0},\ n_{i0},\ n_{n0},\ n_{d0},\ -Z_{d0}e)$, equation (8) yields,

$$q_1 = -\frac{\beta_{ch}}{\nu_{ch} - i\omega}\Phi, \tag{17}$$

where

$$\beta_{ch} = \frac{a/\lambda_{Di}}{\sqrt{2\pi}}\omega_{pi}\left(\frac{z_0 + \sigma_i}{z_0}\right)\alpha_{2n}, \tag{18}$$

with

$$\alpha_{2n} = \frac{1}{\sigma_i} + \left(\frac{1+\beta_n}{\alpha_n}\right). \tag{19}$$

Using equations (13), (14) and (16), we derive the dust fluid density perturbation is related to Φ is given by,

$$\frac{\delta n_d}{n_{d0}} = -\frac{k^2 C_{da}^2}{\omega^2 + \omega_{Jd}^2 - k^2 V_{td}^2}\Phi, \tag{20}$$

where $C_{da} = \sqrt{\frac{Z_{d0}k_B T_e}{m_d}}$ is the dust acoustic speed, $\omega_{Jd}^2 = 4\pi G m_d n_{d0}$ is the squared Jeans frequency and $V_{td} = \sqrt{\frac{k_B T_d}{m_d}}$ is the dust thermal speed. The corresponding electron, positive ion and negative ion density fluctuations are given by

$$\frac{\delta n_e}{n_{e0}} = \Phi, \frac{\delta n_i}{n_{i0}} = -\frac{\Phi}{\sigma_i}, \frac{\delta n_n}{n_{n0}} = \frac{\Phi}{\sigma_n}. \tag{21}$$

Now substituting (17), (20) and (21) in the linearized Poisson equation and under the approximation $\omega << \nu_D$ and $k^2\lambda_{De}^2 << 1$, we obtain the dispersion relation for dust acoustic waves in a self gravitating dusty plasma in presence of Boltzmannian species like positive ions, negative ions and electrons,

$$\epsilon(\omega,k) = (1 + i\omega/\nu_{ch}) - A\frac{k^2 C_{da}^2}{\omega^2 + \omega_{Jd}^2 - k^2 V_{td}^2} = 0. \tag{22}$$

$$A = \frac{\frac{Z_{d0}n_{d0}}{n_{e0}}}{1 + \frac{1}{\sigma_i}\frac{n_{i0}}{n_{e0}} + \frac{1}{\sigma_n}\frac{n_{n0}}{n_{e0}} + \frac{Z_{d0}n_{d0}}{n_{e0}}\frac{\beta_{ch}}{\nu_{ch}}} \tag{23}$$

To study the Jeans instability, we turn to the determination of roots of this dispersion relation as follows:
The real part of the frequency ω_r satisfying $\Re\epsilon(\omega,k) = 0$ is given by

$$\omega_r^2 = \frac{k^2 Z_{d0}k_B T_e}{m_d}(\sigma_d + A) - \omega_{Jd}^2, \tag{24}$$

where, $\sigma_d = \frac{T_d}{Z_{d0}T_e} = \frac{V_{td}^2}{C_{da}^2}$ arises due to the presence of warm dust grains.

The decay/growth rate works out to be

$$\omega_i = -\frac{\Im\epsilon(\omega_r,k)}{\frac{\partial}{\partial\omega}\Re\epsilon(\omega_r,k)} = -\frac{1}{2}\frac{k^2 C_{da}^2\beta_{ch}}{\nu_{ch}^2}A^2, \tag{25}$$

which gives,

$$\frac{\omega_i}{k^2 C_{da}^2/\nu_{ch}} = -\frac{1}{2}\frac{\beta_{ch}}{\nu_{ch}}A^2, \tag{26}$$

which is always negative.

Equation (24) can be written in the form $\omega_r^2 = \omega_{Jn}^2 - \omega_{Jd}^2$, where

$$\omega_{jn}^2 = \frac{k^2 Z_{d0} k_B T_e}{m_d} \left(\sigma_d + A\right). \tag{27}$$

Both the expressions (24) and (26) show that the presence of negative ions and self-gravitating force modify the propagation characteristics of linear DAW which are shown graphically in Figs. 1-4 for different plasma parameters. Magnitude of ω_{Jn} increases with z_0 but after a certain value of z_0, ω_{Jn} decreases. Figure 1 shows that as negative ion number density increases, ω_{Jn} decreases which leads to an instability when $\frac{\omega_{Jn}}{\omega_{Jd}} < 1$.

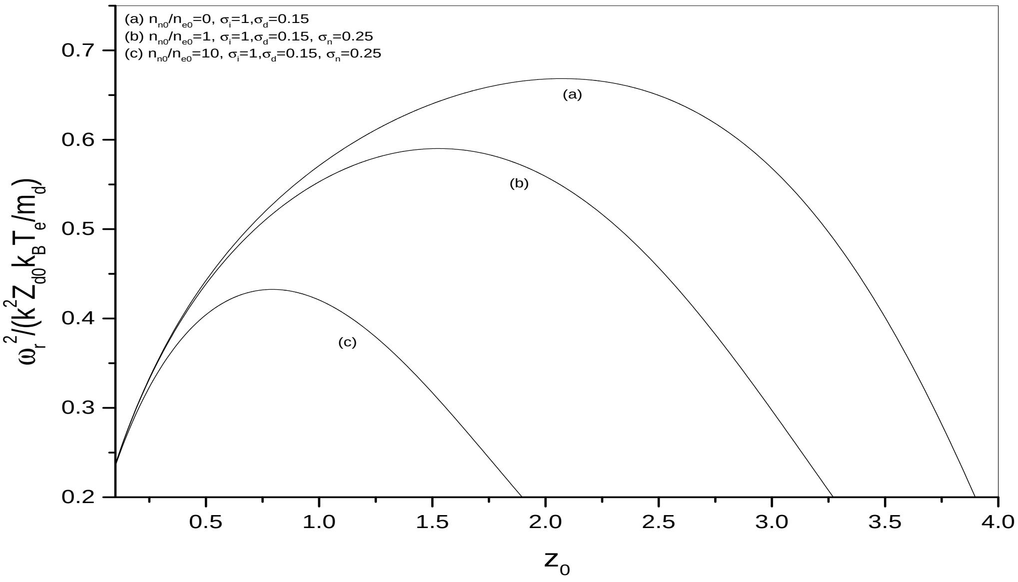

Figure 1: Plot of the square of the normalized real frequency $\omega_r^2/(k^2 Z_{d0} k_B T_e/m_d)$ against z_0 of DA wave $\omega_{Jd} = 0$ for different plasma parameters.

4 Numerical results

Equation (1) gives us the condition for $n_{i0}/n_{e0} > 1 + n_{n0}/n_{e0}$. We have chosen ratio n_{n0}/n_{e0} accordingly and obtain the value of n_{i0}/n_{e0} given by equation (11) which in turn depends on different values of $z_0 = Z_{d0}e^2/ak_BT_e$, $\sigma_i = T_i/T_e$, $\sigma_n = T_n/T_e$. For numerical computation, we use the negative ion data given in Ref. [5]. For negatively charged dust grains, we use the positive ion as potassium k^+ (mass = 39) and the negative ion as SF_6^- (mass = 146), where the positive ion is the lighter species. Further we have considered the case in which the positive ion temperature and electron temperature are same and we take $T_i = T_e = 0.2eV$. Also we take three values of negative ion temperature, $T_n = 0.025eV, 0.25eV$ and $0.5eV$, which explains the choice of the values of $\sigma_i = 1.0$ and $\sigma_n = 0.25, 0.5, 1$. As $V_{td} < C_{da}$ i.e., dust thermal speed < dust acoustic speed, the ratio $V_{td}^2/C_{da}^2 = \sigma_d < 1$. Figure 1 is plotted for ω_r^2 against z_0 for $n_{n0}/n_{e0} = 0, 1.0, 10; \sigma_i = 1.0; \sigma_d = 0.15; \sigma_n = 0.25$. Figure 1 shows that as negative ion number density increases, ω_r^2 increases with z_0 and attains peak value for certain value of z_0 and then decreases sharply, but ω_r^2 decreases with increase in negative ion number density. Hence, negative ion number density induces the increase in the Jeans instability for negatively charged dust grains, which is clear from the Figure 1.Figure 3 is plotted for ω_r^2 against z_0 for $\sigma_n = 0.25, 0.5, 1.0; \sigma_i = 1.0; \sigma_d = 0.15; n_{n0}/n_{e0} = 10$. It is found that as negative ion temperature increases, ω_r^2 increases with z_0 beyond $z_0 \approx 1.0$ and attains peak value around $z_0 \approx 1.0$ and then decreases sharply, but ω_r^2 increases with increasing negative ion temperature. This implies that, negative ion temperature suppresses the Jeans instability when equilibrium dust charge is negative, which is evident from the Figure 3. Figures 2 and 4 are plotted for ω_i against z_0 for the above set of plasma parameters. Figure 2 shows that magnitude of ω_i decreases as negative ion number density increases. However, Figure 4 shows that magnitude of ω_i increases as negative ion temperature increases. It is seen from the graphs (Figure 2 and Figure 4) and the equation namely, Eq. (26) that, ω_i is always negative, so that waves are purely damped for the negatively charged dust grains.

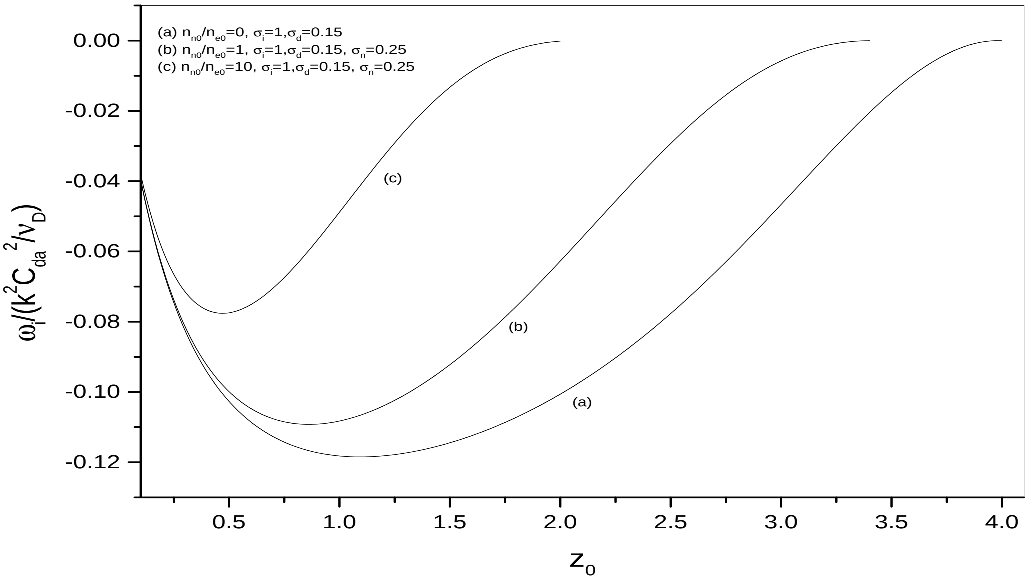

Figure 2: Plot of normalized imaginary frequency $\omega_i/(k^2C_{da}^2/\nu_{ch})$ as given by Eq. (26) against z_0 of DA wave for different plasma parameters.

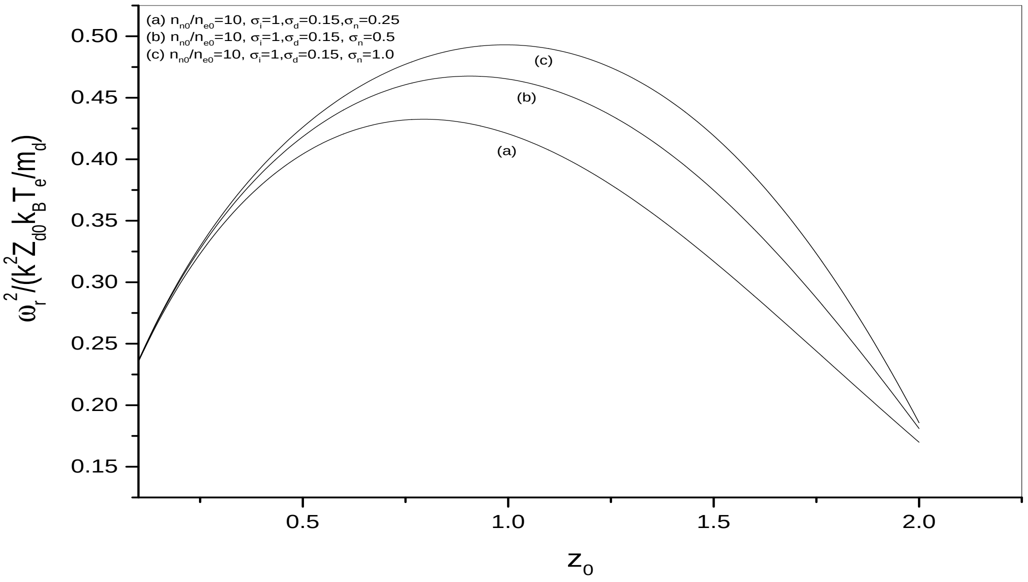

Figure 3: Plot of the square of the normalized real frequency $\omega_r^2/(k^2Z_{d0}k_BT_e/m_d)$ against z_0 of DA wave $\omega_{Jd} = 0$ for different plasma parameters.

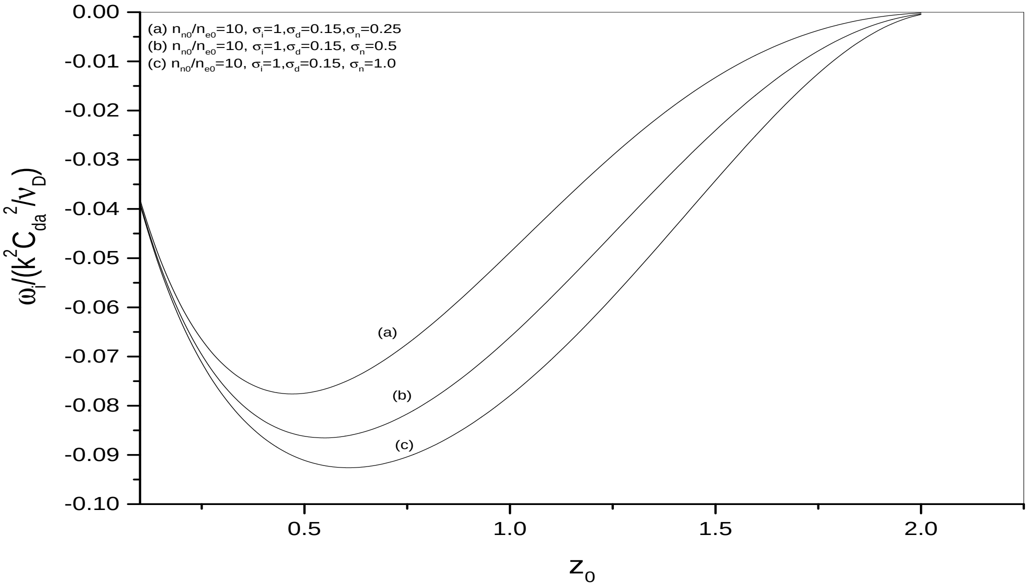

Figure 4: Plot of normalized imaginary frequency $\omega_i/(k^2C_{da}^2/\nu_{ch})$ as given by Eq. (26) against z_0 of DA wave for different plasma parameters.

5 Conclusion

From the results obtained in this paper and from the curves, it is seen that, negative ion number density enhances Jeans instability. Our result also shows that enhanced negative ion number density reduces the threshold of Jeans instability. In a physical situation, negatively charged dust grains repel lighter electron and attracts heavier positive ions which pronounces charge condensation and hence helps in gravitational collapse. These phenomena create favorable situations for the onset of Jeans instability.

On the other hand, our result also shows that higher temperature negative ion suppress Jeans instability *i.e*, enhanced negative ion temperature reduces the threshold of Jeans instability. Basically, in negative ion plasmas, charging due to negative ions together with positive ions and electrons, a significant fraction of the electrons are attached to the negative ions and thus the magnitude of the charge on the dust grains is reduced. It should be noted that, when $\frac{n_{i0}}{n_{e0}} >> 1$ and $\frac{m_i}{m_n} < 1$ *i.e* the positive ions are lighter than the negative ions, then the dust charge may be positive [5]. In this situation, dust grains attract lighter electrons and repel heavier positive ions which resist charge condensation by restricting dust mass enhancement. This phenomenon reduces gravitational effect and growth rate of Jeans instability. Our results can be useful in understanding the behaviour of DA waves in space and astrophysical plasmas [3], [24] and [25].

References

[1] C.K. Goertz, Dusty plasmas in the solar system,*Rev. Geophys,* 27,271 (1989).

[2] U. de Angelis,The physics of dusty plasmas,*Phys. Scr.,* 45,465 (1992).

[3] F. Verheest, *Waves in Dusty Space Plasmas*, (Kluwer, Dordrecht, 2000).

[4] A. A. Mamun & P. K. Shukla, Charging of dust grains in a plasma with negative ions *Phys. Plasmas,* 10, 1518, (2003).

[5] S. H. Kim & R. L. Merlino,Charging of dust grains in a plasma with negative ions *Phys. Plasmas,* 13, 052118, (2006).

[6] S. V. Vladimirov, K. Ostrikov, M. Y. Yu & G. E. Morfill,Ion-acoustic waves in a complex plasma with negative ions *Phys. Rev.,* E67, 036406, (2003).

[7] K. Ostrikov, I. B. Denysenko, S. V. Vladimirov, S. Xu, H. Sugai & M. Y. Yu,Low-pressure diffusion equilibrium of electronegative complex plasmas *Phy. Rev.,* E67,056408, (2003).

[8] K. Ostrikov,Colloquium: Reactive plasmas as a versatile nanofabrication tool *Rev. Mod. Phys.,* 77, 489 (2005)

[9] S. V. Vladimirov & K. Ostrikov,Dynamic self-organization phenomena in complex ionized gas systems: new paradigms and technological aspects *Phys. Reports,* 393, 175 (2004).

[10] J. H. Jeans, The distribution of molecular energy*Phil. Trans. R. Soc. (London)* 199, 1, (1901).

[11] D. S. Balsara,Wave propagation in molecular clouds Astrophys. *Astrophys. J,* 465, 775, (1996).

[12] A. A. Mamun,Effects of dust temperature and fast ions on gravitational instability in a self-gravitating magnetized dusty plasma, *Phys. Plasmas,* 5, 3542, (1998).

[13] F. Verheest, M. A. Hellberg & R. L. Mace,Self-gravitational magnetosonic modes in dusty plasmas with quasi inertialess plasma constituents, *Phys. Plasmas,* 6, 279 (1999).

[14] M. Salimullah & P. K. Shukla,On the stability of self-gravitating magnetized dusty plasmas, *Phys. Plasmas,* 6, 686, (1999).

[15] K. Avinash & P. K. Shukla,A purely growing instability in a gracitating dusty plasma, *Phys. Lett. A,* 189, 470 (1994).

[16] B. P. Pandey, K. Avinash, C. B. Dwivedi, Jeans instability of a dusty plasma,*Phys. Rev. E,* 49, 5599 (1994).

[17] F. Verheest & P. K. Shukla,Jeans instability of a dusty plasma, *Physica Scripta,* 55, 83 (1997).

[18] S. R. Pillay & F. Verheest,Effect of non-thermal ion distributions on the Jeans instability in dusty plasmas, *J. Plasma Phys.* 71, 177 (2005).

[19] P. K. Shukla & L. Stenflo,Jeans instability in a self-gravitating dusty plasma, *Proc. R. Soc. A,* 462, 403 (2006).

[20] G. Jacobs & P. K. Shukla,Stability of molecular clouds in partially ionized self-gravitating space plasmas, *J. Plasma. Phys.,* 71, 487 (2005).

[21] B. Roy, S. Sarkar, M. Khan & M. R. Gupta, The role of negative ions on the Jeans instability in a complex plasma in the presence of nonthermal positive ions, *Phys. Plasmas,* 13, 102904, (2006).

[22] S. Sarkar, B. Roy, S. Maity, M. Khan & M. R. Gupta,Effect of secondary electron emission on the Jeans instability in a dusty plasma, *Phys. Plasmas,* 14, 042106, (2007).

[23] J. E. Allen, Probe theory-the orbital motion approach,*Physica Scripta,* 45, 497 (1992).

[24] R. A. Cairns, A. A. Mamun & R. Bingham, Electrostatic solitary structures in non-thermal plasmas, *Geophys. Res. Lett.,* 22,2709(1995).

[25] R. A. Cairns, R. Bingham & R. Dendy, Ion sound solitary waves with density depressions,*J. Phys.,* 5,C6(1995).