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Chaos Control of a Non-smooth Generalized BVP Circuit System Using Dichotomy Method

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Abstract: Chaos control of the non-smooth generalized Bohoffer -Van der Pol (BVP) system is researched in this paper. Firstly, the dichotomy method is introduced to design the chaos controller on the basis of the frequency-domain method. And the theorem 1 is obtained to guarantee every solution of the system is either convergent or unbounded. When the controller, which is obtained by solving a linear matrix inequality (LMI), is injected to the system, the chaotic oscillations are disappeared. But in real circuits it may be harmful to the circuit components when the voltage or current tends to infinity. Furthermore, the feasibility of the controller must be considered in real circuits. So the theorem 2 is then built to improve the controller. Numerical simulations are used to demonstrate the effciency of the method.

Keywords: dichotomy method;linear matrix inequality;non-smooth extended BVP circuit system

1 Introduction

Over the past decades, chaos control has become one of the more interesting issues in nonlinear systems since the pioneering work of Ott et al[1]. On one hand, chaos is beneficial because it maybe enhance reaction kinetics mechanism in transporting heat/mass transfer. On the other hand, chaos is undesirable because it causes irregular behavior in nonlinear dynamical systems. Furthermore, chaotic behavior is usually unpredictable in detail and may cause detrimental effects on some occasions. Therefore, the ability to control chaos (either promote or eliminate it) is practically important. Based on different control purpose, lots of methods on chaos controls have been designed. Ott et al. gave the famous OGY control method in 1990, which has been found very effective for controlling a large number of chaotic systems. Shinbrot et al.[2], Ditto et al.[3], Grebogi and Lai[4] discussed concepts of chaos and its control presenting discrete chaos control techniques based on OGY method. Pyragas[5] presented an overview of continuous chaos control methods based on time-delayed feedback and mentioned several numerical and experimental applications. Many other kinds of ways are also found to control chaos, such as the method of adaptive control[6-9], the method of chaos control based on sampled data[10,11], the method of pulse feedback of systematic variable[12,13], the method of the active control[14,15]and linear error feedback control[16,17] et al.

Chuas system, the simplest autonomous circuit which can generate chaos, has been extensively studied as a classic model in chaos control. Gamez-Guzman et al.[18] studied the synchronization of multi-scroll Chuas systems on the basis of generalized Hamilton system. While Agiza et al.[19] researched the adaptive synchronization of Chuas system under the unknown parameter conditions. Most of the above papers focused on smooth dynamical systems. In this paper, we consider the control of chaos of the non-smooth generalized BVP oscillator[20], which belongs to the family of Chuas circuit[21].

It is known that dichotomy is one of the most important properties of nonlinear systems, which means limit cycles or strange attractors do not exist in the system. And the method of dichotomy based on frequency-domain theory can be applied to control chaotic solutions[22]. Especially for the family of piece-wise linear circuit, the method of dichotomy is very useful to restrain the chaotic oscillations. So we control the chaotic oscillations based on the method of dichotomy in this paper. The description of the system is given in section 2. In section 3, the chaos controller is designed based on the method of dichotomy to restrain the chaotic oscillations. An improved the controller to restrain the chaotic oscillations in section 4, which is different from the controllers given in papers. Conclusions are given in the end.

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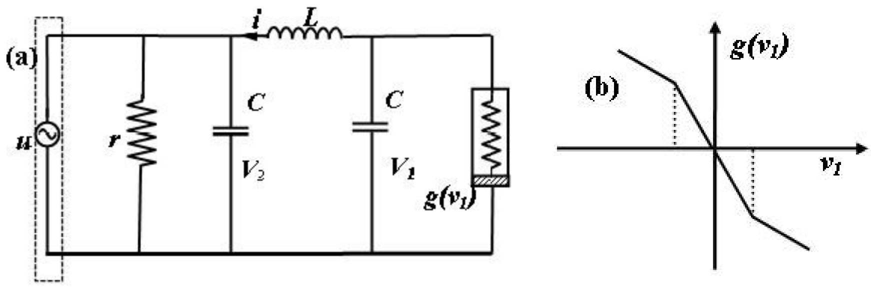

Figure 1: (a) The circuit of the BVP oscillator with non-smooth characteristics is consisted of two capacitors, an inductor, a linear resister and a piecewise-linear resister. A voltage is introduced to be the controller.(b) The $v-i$ characteristics of the nonlinear resistor

2 The non-smooth generalized BVP circuit system

The circuit of the non-smooth generalized BVP oscillator, which is composed of two capacitors, an inductor, a linear resister and a piecewise-linear resister, belongs to the family of Chuas circuit. It can generate chaos on some parameter conditions. Fig.1(a) shows the circuit diagram of the system, while Fig. 1(b) shows the characteristics of the nonlinear resistor.

Applying Kirchhoff laws in the circuit, the state equation of the circuit can be built as following:

$$C\frac{dv_1}{dt} = -i - g(v_1), C\frac{dv_2}{dt} = i - \frac{v_2}{r}, L\frac{di}{dt} = v_1 - v_2 \tag{1}$$

The piecewise-linear resistor is written in the form: $g(v_1) = -av_1 - b(|1+Bx| - |1-Bx|)/2$
By introducing $f(x) = Ax + (|1+Bx| - |1-Bx|)/2, A = a\sqrt{L/C}, B = bc\sqrt{L/C}, x = v_1\sqrt{C/L}/b, \cdot = d/d\tau, \tau = t/\sqrt{LC}, \delta = \sqrt{L/C}/r, y = v_2\sqrt{C/L}/b, z = i/b,$
the normalized equation of eq. (1) can be rewritten in the form:

$$\dot{x} = -z + f(x), \dot{y} = z - \delta y, \dot{z} = x - y \tag{2}$$

The system (2) will perform different dynamical behaviors in different conditions. For example, we find that it presents periodic oscillation, quasi-periodic oscillation and chaotic oscillation in paper [20].

3 Controller design and numerical simulation

Firstly, some notions should be standard. For one $n \times m$ matrix *M*, M^T and M^* denote the transpose and complex conjugate transpose of M respectively.$M > 0$(respectively $M \geq 0$) means matrix *M* is positive definite (positive semi-definite) and $M < 0$ means matrix *M* is negative definite (positive semi-negative). And *I* indicates identity matrix. Then, to a nonlinear system of differential equations

$$\dot{x} = f(t, x) \quad (f : R_+ \times R^n \to R^n) \tag{3}$$

we have:

Definition 1 *A solution $x(t)$ of (3) is said to be convergent if $x(t) \to c$ as $t \to +\infty$ where* c *is an equilibrium point of* (3)*; Eq.*(3) *is said to be dichotomous if its every bounded solution is convergent.*

Remark 1 *From definition* 1*, if system* (3) *is dichotomous, then one of the following two possibilities must be true for any solution $x(t)$ of* (3)*:*
1) $x(t)$ *is convergent.*
2) $x(t)$ *is unbounded.*

Consider the following nonlinear system:

$$d\sigma/dt = c^T z + \rho\varphi(\sigma), dz/dt = Mz + N\varphi(\sigma) \tag{4}$$

where $M \in R^{n\times n}$ is a constant real matrix, $N \in R^n$,and $c \in R^n$ are real vectors and ρ is a number. $z \in R^n, \sigma \in R$. we suppose that the nonlinear function $\varphi : R \to R$ is piece-wise continuously differentiable on R. We also suppose that for all $\sigma \in R, \varphi(\sigma)$ satisfies

$$+\infty > \mu_2 \geq d\varphi/d\sigma \geq \mu_1 > -\infty \tag{5}$$

We characterize the system (4) by transfer function of its linear part from the input φ to the output$-d\sigma/dt$.

$$P(s) = c^T(M - sI)^{-1}N - \rho \tag{6}$$

Lemma 2 *Let the matrix$M$has no pure imaginary eigenvalues. Suppose that (M, N) is controllable, (M, c) is observable and $P(0) \neq 0$. Suppose also that there exist real numbers $\gamma, \varepsilon > 0$ and $\zeta \geq 0$ such that the following frequency-domain inequality is true:*

$$Re\{\gamma P(i\omega) + \zeta[\mu_(1)P(i\omega) + i\omega]^*[\mu_2 P(i\omega) + i\omega]\} - \varepsilon|P(i\omega)|^2 \geq 0 \quad (\forall\omega \in R) \tag{7}$$

Then the system (4), (5) is dichotomous.

Lemma 3 *Given $M \in R^{n\times n}, N \in R^{n\times m}$ and symmetric $\Omega \in R^{(n\times m)\times(n\times m)})$, with $det(i\omega I - M) \neq 0$ all $\omega \in R$ and(M, N) controllable, the following two statements are equivalent:*
(i)$[-N^T((M - i\omega)^{-1})^* \quad I]\Omega[-N^T((M - i\omega)^{-1})^* \quad I]^T \leq 0$
(ii) There is one matrix $P = P^T$,which satisfies

$$\begin{pmatrix} M^TP + PM & PN \\ N^TP & 0 \end{pmatrix} + \Omega \leq 0 \tag{8}$$

Eq. (2) with an injected control signal is rewritten as the following form

$$d(\sigma)/dt = c^T\bar{x} + \rho\varphi(\sigma), d\bar{x}/dt = M\bar{x} + N\varphi(\sigma) + u_1 \tag{9}$$

where

$$M = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -\delta & 1 \\ 1 & -1 & 0 \end{pmatrix} \quad , N = (1\ \ 0\ \ 0)^T,\ \bar{x} = (x\ \ y\ \ z)^T, b_1 = (0\ \ 0\ \ 1)^T, c = (0\ \ 0\ \ -1)^T,$$

$$\rho = 1, \sigma = x, u_1 = b_1K\bar{x}, \varphi(\sigma) = f(x) = -Ax - (|1 + Bx| - |1 - Bx|)/2 \tag{10}$$

Obviously $\mu_1 = \min\{A, B\} \leq f'(x) \leq \max\{A, B\} = \mu_2$. *For all $\omega \in R$, the frequency-domain inequality of system (9) can be transformed into*

$$\gamma\frac{[c^T(M + b_1K - i\omega I)^{-1}N - \rho] + [c^T(M + b_1K - i\omega I)^{-1}N - \rho]^*}{2} + \zeta\omega^2 + (\zeta\mu_1\mu_2 - \varepsilon)$$

$$[c^T(M+b_1K-i\omega I)^{-1}N-\rho)][c^T(M+b_1K-i\omega I)^{-1}N-\rho]^* + \frac{\zeta(\mu_1 + \mu_2)}{2}\{c^T(M+b_1K)(M+b_1K-i\omega I)^{-1}N-c^TN$$

$$+[c^T(M + b_1)K(M + b_1K - i\omega I)^{-1}N - c^TN]^*\} \geq 0$$

Further more, the inequality (11) can be expressed by

$$[-N^T((M + b_1K - i\omega)^{-1})^* \quad I]\begin{pmatrix} -\alpha c^Tc & -R \\ -R^T & \beta \end{pmatrix}[-N^T((M + b_1K - i\omega)^{-1})^* \quad I]^T \leq 0 \quad (\forall\omega \in R) \tag{11}$$

where $\alpha = \zeta\mu_1\mu_2 - \varepsilon, R = -\alpha\rho c - 0.5\gamma c + 0.5\zeta(\mu_1 + \mu_2)(M + b_1K)^Tc, \beta = -\alpha\rho^2 + \gamma\rho - \zeta((\mu_1 + \mu_2)c^TN)$
Applying lemma $1and2$ to the system (9), the following theorem can be obtained based on LMI.

Table 1: Parameters and initial values of the chaos solutions

Fig	A	B	δ	Initial value
Fig.2(a)	0.96	1.0	1.2	$[1.0, 1.0, -1.0]^T$
Fig.3(a)	0.96	1.0	1.2	$[1.0, 1.0, 0.1]^T$
Fig.4(a)	1.01	0.8	1.2	$[1.0, 0.0, -1.0]^T$

Table 2: The control matrixes and the necessary parameters for the chaos controllers

Fig	ρ	η	r_1	r_2	Control matrix K
Fig.2(b)	1.0	-2.0	1.9	0.5	[1.0,1.0,-11.4077]
Fig.3(b)	1.0	-2.0	1.9	0.5	[1.0,1.0,-11.4077]
Fig.4(b)	1.0	-15000	15000	9999999.5	[-1.0,1.0,-0.0009]

Theorem 4 *Suppose that there exist numbers $\eta, \varepsilon > 0, \zeta \geq 0$ such that*

$$\begin{pmatrix} MQ + QM^T + b_1Y + Y^Tb_1^T & QJ - 0.5r_2Y^Tb_1^Tc + N & Qc \\ J^TQ - 0.5r_2c^Tb_1Y + N^T & \rho^2 r_1 c - r_2 c^T b + \eta\rho & 0 \\ c^TQ & 0 & -1/r_1 \end{pmatrix} \leq 0 \tag{12}$$

where $r_1 = \varepsilon - \zeta\mu_1\mu_2, r_2 = \zeta(\mu_1 + \mu_2)$ and $J = 0.5\eta c + \rho r_1 c - 0.5r_2M^Tc$. Then there is a control matrix $K = YQ^{-1}$. Furthermore,if

(I) $(M + b_1K, b)$ is controllable and $(M + b_1K, c)$ is observable,

(II) $\varphi(\sigma)$ has a finite number of isolated zeros, or the matrix $Q > 0$, then system (9) is dichotomous.

The controller based on the dichotomy method is obtained when the feasible control matrix $K[K_1 \;\; K_2 \;\; K_3]$ is given via solving the LMI. Now, we choose three typical chaotic oscillations of system (2) (shown in Fig. 2(a), Fig. 3(a) and Fig. 4(a)), and use the controller we have designed to restrain the chaotic oscillations. The initial values and parameters conditions of the chaotic oscillations of the system (2) are given in Table 1, meanwhile, the control matrixes and some necessary parameters for the chaos controllers are given in Table 2.

Obviously, the chaotic oscillations disappear when the chaos controllers are injected to the system. From the time series of the system (9), the state variables $y(\tau)$ and $z(\tau)$ are convergent, but the state variables $x(\tau)$ is unbounded all the time. Now, $z(\tau)$ is supposed to be convergent to $\beta \in R$, and then $x(\tau)$ is expressed by

$$x(\tau) = \begin{cases} e^{(A+B)\tau} - \beta\tau - \frac{e^{K_3\tau}}{k_3} + C_1 & |x| \leq 1, C_1 \in R \\ \\ e^{A\tau} \pm \tau - \beta\tau - \frac{e^{K_3\tau}}{k_3} + C_1 & |x| > 1, C_1 \in R. \end{cases} \tag{13}$$

where $A > 0, B > 0, 0 > k_3, \tau \in R_+$. It is easy to see the state variable $x(\tau)$ is unbounded when $\tau \to +\infty$. According to the Kirchhoff voltage and current laws, the state variables of system (2) reflect the voltage and current in the non-smooth generalized BVP circuit. In real circuits, it may be harmful to the circuit components when the voltage or current tends to

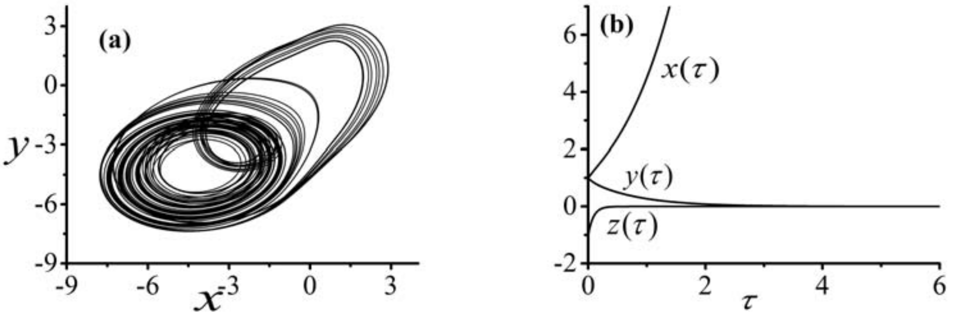

Figure 2: (a) Chaos solution of the system (2),(b) The state curves of the controlled system (9).

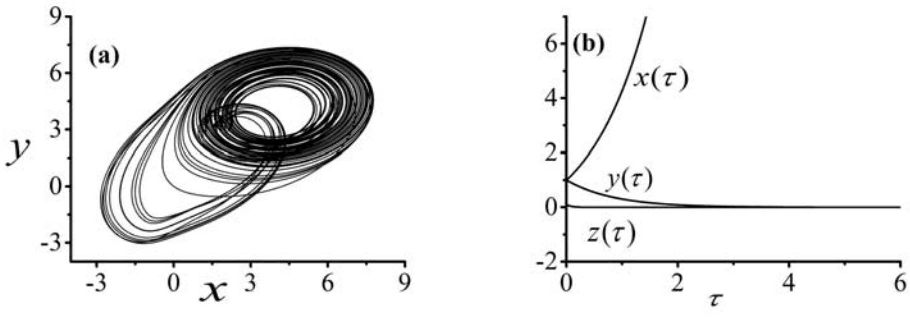

Figure 3: (a) Chaos solution of the system (2);(b) The state curves of the controlled system (9).

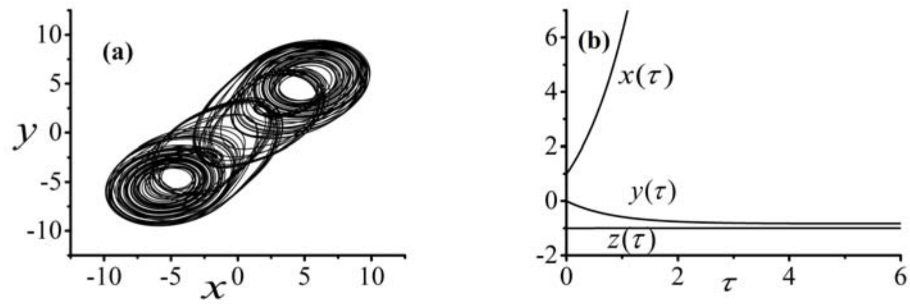

Figure 4: (a) Chaos solution of the system (2);(b) The state curves of the controlled system (9).

infinity. So when the chaos controller is injected, we need to design an improved controller based on dichotomy method in order to prevent the circuit components from being destroyed.

4 The improved controller and numerical simulation

We introduce a new controller u_2 instead of u_1 to system (9), and then the system is expressed by

$$\frac{d\sigma}{dt} = c^T\overline{x} + \rho\varphi(\sigma), \frac{d\overline{x}}{dt} = M\overline{x} + N\varphi(\sigma) + u_2 \tag{14}$$

where $u_2 = b_2K\bar{x}$, $u_2 = (0 \quad 0 \quad 1)^T \ [k_1 \quad k_2 \quad k_3]\ (x \quad y \quad z)^T$, $b_2 = (1\ 0\ 0)^T$ and

$$\varphi(\sigma) = f(x) = -Ax - (|1 + Bx| - |1 - Bx|)/2. \tag{15}$$

The equilibriums of the system (15) can be found by solving the intersection points of two following equations

$$f_1 = f(x), f_2 = (\delta - k_1 - k_2 - \delta k_3)x \tag{16}$$

Lemma 5 *For such a system*

$$\dot{x} = k\alpha[y - x - F(x)], \dot{y} = k[x - y + z], \dot{z} = k[-\beta y - \gamma z] \quad (x, y, z \in R) \tag{17}$$

where $F(x) = b_0x + (a_0 - b_0)[|x+1| - |x-1|]/2$ *and* $\alpha, \beta, a_0, b_0, k$ *are real numbers ,k being equal either to 1 or -1.If for all any* $m \in (\min\{a_0, b_0\}, \max\{a_0, b_0\})$*the inequalities*

$$\alpha\gamma(1+m)+\alpha m+\gamma+\beta > 0, k^3[\alpha\gamma(1+m)+\alpha m+\gamma+\beta][1+\gamma+\alpha(1+m)] > 0, k^3[\alpha\beta(1+m)+\alpha m\gamma] > 0 \tag{18}$$

are true then system(15) ,(16) *is globally asymptotically stable.*

According to the Lemma 3, we can obtain the following result.

Table 3: The control matrixes and the necessary parameters for the chaos controller

Fig	ρ	η	r_1	r_2	Control matrix K
Fig.6(b)	1.0	-10.0	5.0	1.0	[-5.4738,6.6204,-23.2367]
Fig.7(b)	1.0	-10.0	5.0	1.0	[-5.4738,6.6204,-23.2367]
Fig.8(b)	1.0	-10.0	5.0	1.0	[-5.4738,6.6204,-23.2367]

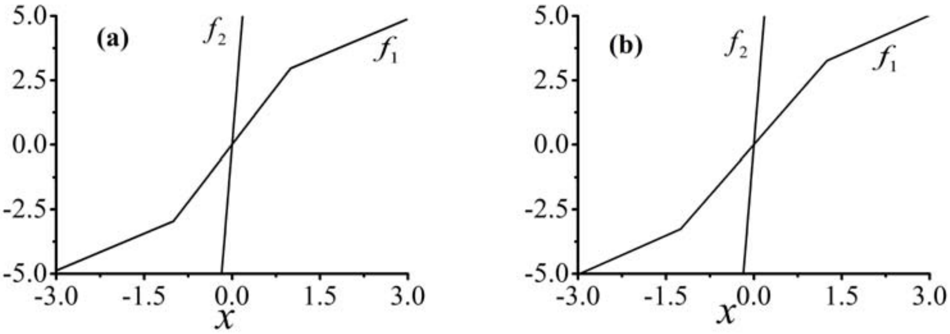

Figure 5: (a) The curves of f_1 and f_2 under the single-scroll chaos oscillations, (b) The curves of f_1 and f_2 under the double-scroll chaos oscillation.

Theorem 6 *Suppose that the dichotomous system*(15)*just has a single equilibrium. If for any* $m \in (\min\{A, B\}, \max\{A, B\})$ *, the inequalities*

$$2 - k_3 - (k_1 - m)\delta > 0, [2 - k_3 - (k_1 - m)\delta](\delta - k_1 - m) > (1 - k_3)\delta - k_2 - m - k_1 > 0 \tag{19}$$

are true, then all solutions of system (15) *are convergent to the single equilibrium.*

Now we use theorem 1 *and theorem* 6 *to design new controller for restraining three chaotic oscillations, which have been shown in Fig.*$2(a)$, $Fig.3(a)$ *and Fig.* $4(a)$. *The new control matrixes and some necessary parameters for the new chaos controller are given in Table* 3.

The only difference between the single-scroll chaotic attractors and the double-scroll chaotic attractor is the values of parameters A *and* B. *When* $A = 0.96$ *and* $B = 1.0$, *there are two symmetric single-scroll chaotic attractors, but When* $A = 1.01$ *and* $B = 0.8$, *there is one double-scroll chaotic attractor. When the new controller is injected, the system* (15) *has only an equilibrium which is shown in Fig.* 5. *And for any* $m \in (\min\{A, B\} = 0.8, \max\{A, B\} = 1.01)$ *the inequalities* (20) *are true.*

Applying the theorem 6, *it is easy to find that every solution of the dichotomous system* (15) *is globally asymptotically stable, i.e. all the state values of system* (15) *are convergent to original point.*

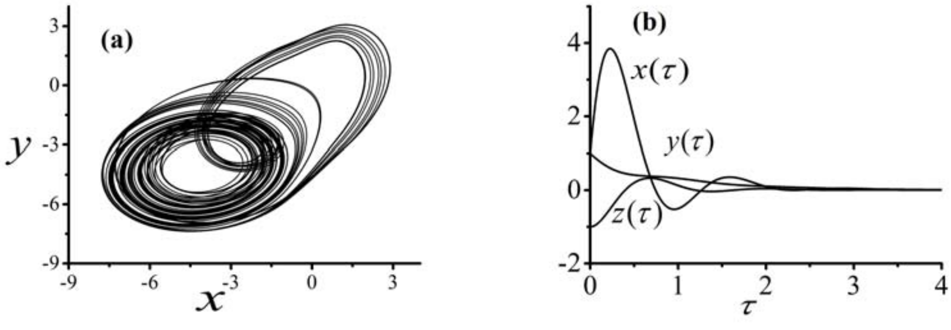

Figure 6: (a) Chaos solution of the system (2);(b) The state curves of the controlled system (15).

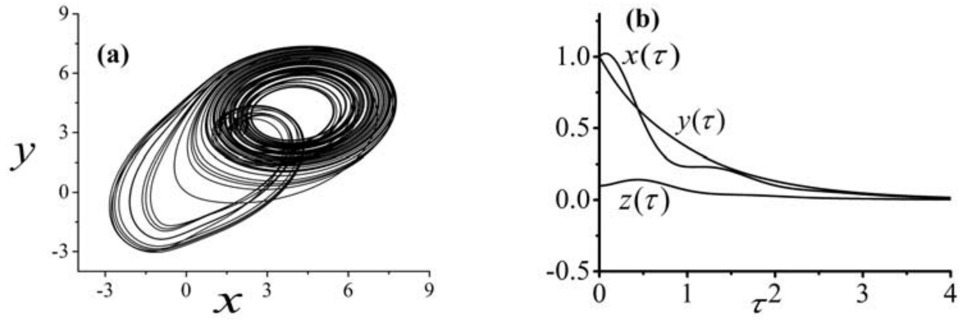

Figure 7: (a) Chaos solution of the system (2);(b) The state curves of the controlled system (15).

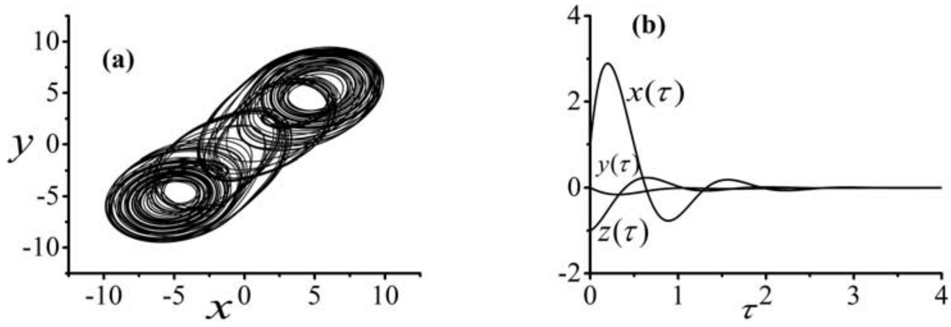

Figure 8: (a) Chaos solution of the system (2);(b) The state curves of the controlled system (15).

5 Conclusions

In this paper, we introduce the dichotomy method to restrain the chaotic oscillations of the non-smooth generalized BVP system. By this method, theorem 4 is built to guarantee every solution of the system is ether convergent or unbounded. The chaos controller can be obtained easily via solving the LMI in theorem 1. Although the chaotic oscillations disappear when the controllers are injected to the system, it may be harmful to the circuit components when the voltage or current tends to infinity in real circuits. According to the Routh-Hurwitz criterion, theorem 6 is built to ensure that the solution of the dichotomous system is globally asymptotically stable. So an improved controller is designed by applying theorem 4 and theorem 6. The chaotic oscillations are disappeared when the improved controller is injected to the system. Meanwhile, all the state variables of the system are convergent. The effciency of the improved method is demonstrated by Numerical simulations.

Acknowledgments

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