Finite-time Chaos Control of Unified Hyperchaotic Systems with Single Parameter

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Abstract: This paper investigates into the finite-time chaos control problem of unified hyperchaotic systems with single parameter. In terms of the finite-time stability theory of hyperchaotic systems, nonlinear control laws are presented to realize finite-time chaos control. The controllers are simple, effective and easy to be constructed. Simulation results for hyperchaotic Lorenz system, hyperchaotic Lü system, and hyperchaotic Chen system are provided to demonstrate the effectiveness of the proposed controlling scheme.

Keywords: finite-time chaos control; unified hyperchaotic system; cascade-connected system

1 Introduction

As is well-known, chaos is an interesting phenomenon which may lead to irregularity and unpredictability in dynamic systems, and it has been intensively studied in the last several decades. Chaos synchronization has been a hot and interesting topic since the pioneering work of Pecora and Carroll [1]. It can be applied in various fields such as chemical reactors, power converters, biological systems, information processing, secure communication[2–4], etc. Until now, a wide variety of approaches have been proposed for the synchronization of chaotic systems which include adaptive control [5–8], observer-based control [9], sliding mode control [10], back-stepping control [11], active control [12], nonlinear control [13], Lyapunov function method control [14], and so on.

Notice that the above mentioned literatures mainly investigate the asymptotic synchronization of chaotic systems. However, in the view of practical application, optimizing the synchronization time is more important than achieving synchronization asymptotically [15–23]. To obtain fast convergence speed, many effective methods, such as finite-time control, have been proposed. Finite-time synchronization means the optimality in convergence time. Moreover, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties [24]. Recently, based on the step-by-step control method. Wang et al. realized the finite-time synchronization of two chaotic systems by designing a proper controller [15, 25]. The method has the ability to achieve global stability in finite time. In addition, the step-by-step method has the advantage of reducing controller complexity.

This paper mainly present continuous controller schemes to realize finite-time control of the unified hyperchaotic systems with single parameter. The controllers are simple, effective and easy to be constructed.

The rest of the paper is organized as follows. After giving some preliminaries in Section 2, according to the finite-time stability theory and the step-by-step controller design method, Section 3 addresses the chaos control of unified hyperchaotic system and a suitable controller is derived. Section 4 present numerical examples to demonstrate the theoretical results. Finally, conclusion is given in Section 5.
2 Preliminaries

Finite-time stability means that the dynamic system state converges to a desired target within a finite time. Firstly, we list some lemmas that are necessary for the demonstration of the main results.

Lemma 1 (See [15]). For system
\[
\dot{x} = f(x),
\]
assume that a continuous, positive-definite function \( V(t) \) satisfies the following differential inequality:
\[
\dot{V}(t) \leq -c V^n(t), \quad \forall t \geq t_0, V(t_0) \geq 0,
\]
where \( c > 0 \) and \( 0 < \eta < 1 \) are constants. Then, for any initial time \( t_0 \), \( V(t) \) satisfies following inequality:
\[
V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0), \quad t_0 \leq t \leq t_1,
\]
and
\[
V(t) \equiv 0, \quad \forall t \geq t_1,
\]
with \( t_1 \) given by
\[
t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}.
\]
Thus, for any initial value \( V(t_0) \), the system (1) has \( V(t) = 0 \) in \( t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)} \), that is, the system can achieve global stability in finite time.

Lemma 2 (See [15, 26]). Let \( 0 < c < 1 \), for positive real numbers \( a \) and \( b \), the following inequality holds:
\[
(a + b)^c \leq a^c + b^c.
\]
This result is quite straightforward and the proof is omitted here.

3 Main results

It is well known that chaotic systems are extremely sensitive to their initial conditions. Before 1990, most of scientists thought that chaotic systems cannot be controlled to a desired target. Nevertheless, in 1990, the pioneering work of Ott, Grebogi and Yorke denied this viewpoint completely [25, 27]. In this section, our target is to design control schemes that realize finite-time chaos control for the unified hyperchaotic systems with single parameter.

Unified hyperchaotic systems have been presented and investigated in [28]. The new systems represent the continued transition from the hyperchaotic Lorenz system to the hyperchaotic Lü system, and then to the hyperchaotic Chen system, they are chaotic over the entire spectrum of the key system parameter \( \alpha \in [0, 1] \).

In this paper, we consider the unified hyperchaotic systems presented as follows:
\[
\begin{cases}
\dot{x}_1 = (10\alpha + 35)(x_2 - x_1) + (10\alpha + 35)x_2x_3, \\
\dot{x}_2 = (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1x_3 - x_4, \\
\dot{x}_3 = -\frac{8}{3}x_1 + x_1x_2, \\
\dot{x}_4 = x_1 + (24 + \alpha)x_2,
\end{cases}
\]
where \( x = [x_1, x_2, x_3, x_4]^T \) and \( \alpha \) are the state vectors and the parameter of the unified hyperchaotic system (2), respectively. When \( 0 \leq \alpha < 0.8 \), system (2) is called as the hyperchaotic Lorenz system. When \( \alpha = 0.8 \), it is called hyperchaotic Lü system. And it is called hyperchaotic Chen system when \( 0.8 < \alpha \leq 1 \). Choose the initial conditions of the hyperchaotic Lorenz system as \( x(0) = [-3, 2, -1, 3]^T \), and of the hyperchaotic Lü system as \( x(0) = [-2, -3, 2, 1]^T \), and choose the hyperchaotic Chen system initial value as \( x(0) = [3, -3, -4, 3]^T \). The chaotic trajectories and attractors are shown in Figures 1-6.

In this section, consider the unified hyperchaotic systems with a certain parameter and design controllers to globally stabilize the unstable equilibrium to \( x_i(0) = [0, 0, 0, 0]^T \) in a finite time. As to other chaotic systems, we can also adopt the controller design method presented following to realize the chaos convergence.
Figure 1: Trajectories of hyperchaotic Lorenz system ($\alpha = 0$).

Figure 2: Attractors of hyperchaotic Lorenz system ($\alpha = 0$).
Figure 3: Trajectories of hyperchaotic Lü system ($\alpha = 0.8$).

Figure 4: Attractors of hyperchaotic Lü system ($\alpha = 0.8$).

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Figure 5: Trajectories of hyperchaotic Chen system ($\alpha = 1$).

Figure 6: Attractors of hyperchaotic Chen system ($\alpha = 1$).
The controlled unified hyperchaotic systems can be described as
\[
\begin{cases}
\dot{x}_1 = (10\alpha + 35)(x_2 - x_1) + (10\alpha + 35)x_2x_3 + u_1, \\
\dot{x}_2 = (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1x_3 - x_4 + u_2, \\
\dot{x}_3 = -\frac{8+\alpha}{2}x_1x_2 + u_3, \\
\dot{x}_4 = x_1 + (24 + \alpha)x_2 + u_4.
\end{cases}
\] (3)

where \(u_1, u_2, u_3\) and \(u_4\) are the control inputs. The controller design procedure can be divided into three steps as follows.

**Step 1:** Let \(u_1 = -(10\alpha + 35)x_2 - (10\alpha + 35)x_2x_3 - x_1^\beta\), where \(\beta = \frac{q}{p}\) and \(p > q\) are positive integers. By the controller \(u_1\), the first equation of (3) is
\[
\dot{x}_1 = -(10\alpha + 35)x_1 - x_1^\beta.
\] (4)

Consider the candidate Lyapunov function
\[V_1 = \frac{1}{2}x_1^2.\] The time derivative of \(V_1\) along the trajectory of (4) is
\[
\dot{V}_1 = -x_1 \left(-(10\alpha + 35)x_1 - x_1^\beta\right)
= -(10\alpha + 35)x_1^2 - x_1^{\beta+1}
\leq -x_1^{\beta+1}
= -\left(\frac{1}{2}\right) - \frac{\alpha+1}{2} \left(\frac{1}{2}x_1^{\beta}ight)^{\frac{\beta+1}{2}}
= -\left(\frac{1}{2}\right) - \frac{\alpha+1}{2} V_1^{\frac{\beta+1}{2}}.
\] (5)

Since \(0 < \beta < 1\), then \(0 < \frac{\beta+1}{2} < 1\). From Lemma 1, the state \(x_1\) will reach \(x_1 = 0\) at a finite time \(T_1 = \frac{x_1(0)}{1-x_1^\beta}\).

**Step 2:** Let \(u_4 = -x_1 - (24 + \alpha)x_2 - x_4^\beta\), by the controller \(u_4\), the fourth equation of (3) is
\[
\dot{x}_4 = -x_4^\beta.
\] (6)

Choose Lyapunov function for (6) as
\[V_2 = \frac{1}{2}x_4^2.\]

The derivative of \(V_2\) along the trajectories of (6) as follows.
\[
\dot{V}_2 = x_4 \left(-x_4^\beta\right)
= -x_4^{\beta+1}
= -\left(\frac{1}{2}\right) - \frac{\alpha+1}{2} \left(\frac{1}{2}x_4^{\beta}ight)^{\frac{\beta+1}{2}}
= -\left(\frac{1}{2}\right) - \frac{\alpha+1}{2} V_2^{\frac{\beta+1}{2}}.
\] (7)

From Lemma 1, the state \(x_4\) will reach \(x_4 = 0\) at a finite time \(T_2 = \frac{x_4(0)}{1-x_4^\beta}\).

**Step 3:** Let \(u_2 = -L_1x_2 - x_2^\beta\), where \(L_1 \geq 29\alpha - 1 + u_3 = -x_3^\beta\). If \(t > \max\{T_1, T_2\}\), then \(x_1 \equiv x_4 \equiv 0\). Substituting \(x_1 = x_4 = 0\) into the second and the third equations of system (3), it yields
\[
\begin{cases}
\dot{x}_2 = (29\alpha - 1)x_2 - L_1x_2 - x_2^\beta, \\
\dot{x}_3 = -\frac{8+\alpha}{2}x_3 - x_3^\beta.
\end{cases}
\] (8)

Choose Lyapunov function for (8) as follows
\[V_2 = \frac{1}{2}x_2^2 + \frac{1}{2}x_3^2.\] (9)
The derivative of $V_2$ along the trajectories of (8) is
\[
\dot{V}_2 = x_2 \left[ (29\alpha - 1)x_2 - L_1 x_2 - x_2^\beta \right] - x_3 \left( \frac{8 + \alpha}{3} x_3 + x_3^\beta \right)
\]
\[
= - (L_1 - (29\alpha - 1)) x_2^2 - x_2^\beta + x_3 \frac{8 + 2\alpha}{3} x_3^\beta + x_3^\beta
\]
\[
\leq - \frac{1}{2} \dot{V}_2^\beta \left[ \left( \frac{1}{2} x_2^2 \right)^\beta + \left( \frac{1}{2} x_3^2 \right)^\beta \right] - \frac{1}{2} \dot{V}_2^\beta
\]
\[
\leq - \frac{1}{2} \frac{1}{2} \dot{V}_2^\beta
\]
(10)

It should be noticed that Lemma 2 have been used to deduce the above result. Because of $0 < \beta < 1$, $0 < \frac{\beta + 1}{2} < 1$.

From Lemma 1, the state $x_2$ and $x_3$ will converge to $x_2 = 0$, $x_3 = 0$ in a finite time $T_3 = \max\{T_1, T_2\} + \max\{x_2(0), x_3(0)\}$. Then after the time $T_3$, the system state (3) will converge to $x_1 = x_2 = x_3 = x_4 = 0$. This means the controlled unified hyperchaotic systems (3) can be stabilized in finite-time by the controllers as follows.

\[
\begin{align*}
\{ & u_1 = -(10\alpha + 35)x_2 - (10\alpha + 35)x_2 x_3 - x_1^\beta, \\
& u_2 = -L_2 x_2 - x_2^\beta, \\
& u_3 = -x_3^\beta, \\
& u_4 = -x_1 - (\alpha + 24)x_2 - x_4^\beta. 
\end{align*}
\]

4 Numerical simulations

In this section, to illustrate the performance and effectiveness of the proposed method, simulation results for the unified hyperchaotic systems are presented. In the numerical simulations, fourth-order Runge-Kutta (ode45) algorithm is used to achieve the numerical solutions of the hyperchaotic systems. Figures 7-9 show the simulating response curves of the controlled hyperchaotic Lorenz system, the hyperchaotic Lü system and the hyperchaotic Chen system, respectively. The controller parameters in the simulations are chosen as $L_1 = 29\alpha + 1$, $\beta = 2/3$. The figures show that the state variables converge to $x_1 = x_2 = x_3 = x_4 = 0$ in a finite time.

5 Conclusion

In this paper, the problem of finite-time chaos control of the unified hyperchaotic systems is investigated. On the basis of the finite time stability theory, the step-by-step control and nonlinear control approach, suitable controllers are proposed and introduced. The simulation results demonstrated that the proposed controllers can stabilizing the hyperchaotic systems to zeros in a finite time. This controller design method can also be applied on the other chaotic systems.

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Figure 7: Controlled response curves of hyperchaotic Lorenz system ($\alpha = 0$) with initial condition $x(0) = [1, 2, -1, 3]^T$.

Figure 8: Controlled response curves of hyperchaotic L"u system ($\alpha = 0.8$) with initial condition $x(0) = [2, -2, 4, -2]^T$. 

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