Unsteady MHD Micropolar Fluid Flow Generated Due to Longitudinal and Torsional Oscillations of Circular Cylinder

G. Nagaraju *
Department of Mathematics, GITAM University, Hyderabad campus, Rudraram, Medak Dist, Andhara Pradesh.502329, India
(Received 9 January 2013, accepted 7 April 2014)

Abstract: The flow generated by a circular cylinder, performing longitudinal and torsional oscillations, in an infinite expanse of a micropolar fluid is studied under the influence of uniform transverse magnetic field. Analytical expressions for the velocity components and micro rotation components are obtained by using no slip and hyper stick conditions on the boundary of the cylinder. The effects of Magnetic parameter, coupling number, Reynolds number and couple stress parameter on the transverse and axial velocity components are discussed. Explicit expression for the drag force acting on the wall of the cylinder is derived and the effects of micropolar parameters on the drag are shown graphically.

Keywords: micropolar fluid; transverse magnetic field; drag; longitudinal and torsional oscillations

1 Introduction

The theory of Micropolar fluid was introduced by Eringen [5]. A micropolar fluid is a non-Newtonian fluid with local microstructure exhibiting micro rotations. The theory of micropolar fluids can be applied in many situations where fluids exhibit couple stresses. These fluids are lubricating fluids, dusty fluids and additives, certain polymer solutions, colloidal suspensions, complex biological fluid structures, etc. Recently, Chen et al [1] presented the numerical simulation for unsteady compressible Micropolar fluid flow. Conjugate heat transfer within a concentric annulus filled with micropolar fluid has been considered by Intiaz and Mahfouz[2].

Recently, it has been observed that the flow problems of electrically conducting fluids have great attention in magnetohydrodynamics. Magnetohydrodynamics plays a great role in solving problems of astrophysics, geophysics and physical applications in petroleum industry, cooling systems with liquid metals industry, etc. Recently, Abbas et al [3] presented the thermal radiation effects on MHD flow over a stretching cylinder in a porous medium. Most recently, Sinha and Misra [4] considered the MHD flow of blood through a dually stenosed artery with that presented the effects of viscosity variation, variable hematocrit and velocity-slip.

The flow of a fluid due to a cylindrical rod oscillating with longitudinal and torsional motion has received considerable attention because of its relevance in many technical problems of practical importance such as mixing, Oil Drilling and towing operations. For Example, in offshore engineering, bluff bodies in the form of cylinders which are used in construction and they form base developments in oil extraction from underneath the Sea. Using various types of fluids, the flow generated due to longitudinal and torsional oscillations of a circular cylinder has been examined by few authors. Ramkissoon and Mazumdar [6] have derived an analytical expression for shear stresses, drag on the cylinder and velocity which were depicted graphically. Ramkissoon et al [7] has examined a polar fluid by using transform methods and they have presented the effect of micropolar parameters on the microrotation and velocity fields graphically. Pontrelli [8] has studied the axi-symmetric flow of a homogeneous Oldroyd-B fluid with suction or injection velocity applied at the surface. Camlet-Elulu and Majumdar [9] have investigated longitudinal and torsional oscillations of a micropolar fluid and examined the effect of micropolar fluid on two components of the velocity field through graphical curves by using Mathematica Software. Owen and Rahman [10] have studied the same type of flow with an Oldroyd-B liquid. Fatac and Corina Fatac [11] have obtained the starting solutions corresponding to the motion of a second grade fluid by means of
the finite Hankel transforms. Karim Rahaman et al [12] has studied the motion of viscoelastic incompressible flow of the upper convected Maxwell fluid at different frequencies of oscillations of the cylinder along its axis and he has presented velocity components graphically for definite values of the flow parameters. Mehrdad Massoudi and Tran X.Phouc [13] have obtained a solution numerically for the flow of a second grade fluid and they have presented the results graphically for the shear stresses at the wall. Bano et al [14] has studied the motion of a second grade fluid for an oscillating rod in the presence of a uniform magnetic field. Ramana Murthy et al [15] has found a numerical solution for MHD flow of micropolar fluid between two concentric rotating cylinders with porous lining. Most recently, on the characteristic torsional oscillations of a hollow shaft with a longitudinal radial through cut has been described by Khakimov [16].

In this paper, an attempt has been made to study the flow of a micropolar fluid generated by a circular cylinder subjected to longitudinal and torsional oscillations under the influence of transverse magnetic field. Theory of Modified Bessel’s functions is employed to obtain an exact solution for the velocity and micro-rotation components. The effects of parameters mentioned earlier on velocity, micro-rotation components and drag are discussed and presented through graphs.

2 Formation and solution of the problem

When a circular cylinder is considered with radius \( a \) within an incompressible micropolar fluid, the cylinder will be subjected to torsional oscillations \( e^{i\omega_1 t} \) and longitudinal oscillations \( e^{i\omega_2 t} \) with different amplitudes \( q_0 \sin \beta_0, q_0 \cos \beta_0 \) along the respective directions with \( \omega_1 \) as the frequency of longitudinal oscillations, \( \omega_2 \) as the frequency of torsional oscillations, \( q_0 \) as the magnitude of oscillations and \( \beta_0 \) is the angle between the direction of torsional oscillation and the base vector \( e_0 \). i.e., The cylinder oscillates with velocity as given by the expression

\[
Q = q_0 (\sin \beta_0 e^{i\omega_1 t} e_1 + \cos \beta_0 e^{i\omega_2 t} e_z)
\]

The flow of the micropolar fluid is being generated due to these oscillations of the cylinder by choosing the cylindrical polar coordinate system \((R, \theta, Z)\) with an origin at the center of the cylinder and Z-axis along the axis of the cylinder. The physical model is shown in figure 1.

![Figure 1: Geometry of the problem-non dimensional form](http://www.nonlinearscience.org.uk/)

When body forces and body couples are neglected, we get the field equations governing the incompressible micropolar fluid dynamics are:

\[
\nabla_1 \cdot Q = 0
\]

\[
\rho \left( \frac{\partial Q}{\partial \tau} + Q \cdot \nabla_1 Q \right) = -\nabla_1 P + \kappa \nabla_1 \times l - (\mu + \kappa) \nabla_1 \times \nabla_1 \times Q + J \times B
\]

\[
\rho j \left( \frac{\partial l}{\partial \tau} + Q \cdot \nabla_1 l \right) = -2\kappa l + \kappa \nabla_1 Q - \gamma \nabla_1 \times \nabla_1 \times l + (\alpha + \beta + \gamma) \nabla_1 (\nabla_1 \cdot l)
\]

where \( Q \) is velocity vector, \( l \) is micro rotation vector, \( P \) is the fluid pressure, \( \rho \) is fluid density, \( j \) micro-inertia coefficient, \( \tau \) is time, \( \alpha, \beta, \gamma, \kappa \) and \( \mu \) are material coefficients, \( J \) the electric current density and \( B \) the total magnetic field, where...
\[ B = B_0 + b, \] where \( B_0 \) represents the imposed magnetic field and \( b \) denotes the induced magnetic field. The current density, magnetic field \( B \) and electric field \( E \) are related by Maxwell’s equations.

\[ \nabla \times E = \frac{\partial B}{\partial t}, \nabla \cdot B = 0, \nabla \times B = \mu' \nabla \cdot J, \nabla \cdot J = 0, J = \sigma_e (E + Q \times B) \]

In the above equation \( \nabla_1 \) is the dimensional gradient, \( \sigma_e \) is electrical conductivity and \( \mu' \) is the magnetic permeability. The following assumptions are made for the discussion.

(i) The density \( \rho \), magnetic permeability \( \mu' \) and electric conductivity \( \sigma_e \) are constant throughout the flow region.

(ii) The electric conductivity \( \sigma_e \) of the fluid is finite.

(iii) Total magnetic field \( B \) is perpendicular to the velocity field \( Q \) and the induced magnetic field \( b \) is negligible compared with the applied magnetic field \( B_0 \) so that the magnetic Reynolds number is small Rossow [17], Hunt and Moreau [18].

(iv) There is no energy added or extracted from the fluid by the electric field, which implies that there is no electric field present in the fluid flow region.

Under these assumptions, the MHD force involved in (2) can be put into the form:

\[ \mathbf{J} \times \mathbf{B} = -\sigma_e B_0^2 \mathbf{Q}. \]

It is understood that, the flow is generated due to the oscillations of the cylinder in transverse and axial directions. As there is no radial flow in the radial direction, the velocity and micro-rotation components are assumed to be axially symmetric, i.e., independent of \( \theta \). Since the cylinder is of infinite length, the flow at any cross section of the cylinder is same and hence the flow variables are independent of axial variable \( Z \) (See in Laglosis[19]). Hence the velocity and micro-rotations depend only on radial distance and time.

By introducing the following non-dimensional scheme

\[ q = \frac{Q}{q_0}, v = \frac{a}{q_0}, p = \frac{P}{\rho q_0}, t = \frac{q_0 \tau}{a}, r = \frac{R}{a} \]

and

\[ z = \frac{Z}{a} \]

The equations in (1), (2) and (3) for the flow variables, the following non-dimensional forms can be taken as

\[ \nabla \cdot q = 0 \] (4)

\[ Re \left( \frac{\partial q}{\partial t} + q \cdot \nabla q \right) = -Re \nabla p + c \nabla \times v - \nabla \times \nabla \times q - M^2 q \] (5)

\[ \varepsilon \left( \frac{\partial v}{\partial t} + q \cdot \nabla v \right) = -2sv + s \nabla \times q - \nabla \times \nabla \times v, \] (6)

where cross viscosity parameter or coupling number \( c \), couple stress parameter \( \varepsilon \), Reynolds number \( Re \), gyro parameter \( \varepsilon \) and Magnetic parameter \( M \) are represented by

\[ c = \frac{\kappa}{\mu + \kappa}, s = \frac{\kappa a^2}{\mu}, Re = \frac{\rho_0 q_0 a}{\mu}, \varepsilon = \frac{\rho_0 q_0 a}{\mu}, \text{ and } M = \sqrt{\frac{\mu B_0^2 a^2}{\mu + \kappa}}. \]

The velocity vector \( q \), micro-rotation vector \( \varepsilon \) and pressure \( p \) are to be represented in these equations

\[ q = v(r)e^{\sigma_1 t} e_\theta + w(r)e^{\sigma_2 t} e_z \] (7)

\[ v = B(r)e^{\sigma_1 t} e_\theta + C(r)e^{\sigma_2 t} e_z \] (8)

\[ p = p_1(r)e^{2\sigma_1 t}, \] (9)

where \( \sigma_1 = \frac{\omega_1 a}{q_0} \) and \( \sigma_2 = \frac{\omega_2 a}{q_0} \). Using (7), (8) and (9) in (5) and comparing the coefficients of \( v_r, v_\theta, e_z \) we get

\[ \frac{dp}{dr} = \frac{v^2}{r} \] (10)

\[ Re \sigma_1 v = -c \frac{dC}{dr} + \frac{d^2 v}{dr^2} + \frac{1}{r} \frac{dv}{dr} - \frac{v}{r^2} = M^2 v \] (11)
Similarly, the equation (6) yields the following equations

\[

\varepsilon B v = e^{i(\sigma_1 + \sigma_2)t} = 0
\]

(13)

\[

\varepsilon i \sigma_2 B = -2sB - s\frac{dw}{dr} + \frac{d^2 B}{dr^2} + \frac{1}{r} \frac{dB}{dr} - \frac{B}{r^2}
\]

(14)

\[

\varepsilon i \sigma_1 C = -2sC + s \left( \frac{dv}{dr} + \frac{v}{r} \right) + \left( \frac{d^2 C}{dr^2} + \frac{1}{r} \frac{dC}{dr} \right)
\]

(15)

Eliminating \( \frac{dC}{dr} \) from (11) and (15) we get

\[

[D^4 - (M^2 + i\sigma_1 (Re + \varepsilon) + s(2 - c)) D^2 + (iRe\sigma_1 + M^2) (i\varepsilon \sigma_1 + 2s)] v = 0,
\]

(16)

where \( D^2 = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} \)

This can be written as

\[

(D^2 - \lambda_1^2)(D^2 - \lambda_2^2)v = 0,
\]

(17)

where \( \lambda_1^2 + \lambda_2^2 = M^2 + i\sigma_1 (Re + \varepsilon) + s(2 - c) \) and \( \lambda_1^2 \lambda_2^2 = (iRe\sigma_1 + M^2)(i\varepsilon \sigma_1 + 2s) \) Similarly equation (12) and (14) reduces to

\[

[D^4 - (M^2 + i\sigma_2 (Re + \varepsilon) + s(2 - c)) D^2 + (iRe\sigma_2 + M^2) (i\varepsilon \sigma_2 + 2s)] B = 0
\]

(18)

This can be written as

\[

(D^2 - \alpha_1^2)(D^2 - \alpha_2^2)B = 0,
\]

(19)

where \( \alpha_1^2 + \alpha_2^2 = M^2 + i\sigma_2 (Re + \varepsilon) + s(2 - c) \) and \( \alpha_1^2 \alpha_2^2 = (iRe\sigma_2 + M^2)(i\varepsilon \sigma_2 + 2s) \) From equation (11) we find,

\[

\frac{dC}{dr} = \frac{1}{c} \left( D^2 - M^2 - Rei\sigma_1 \right) v
\]

(20)

Substituting the expression (20) for \( \frac{dC}{dr} \) in (15) we obtain the equation for C as

\[

c(i\sigma_1 \varepsilon + 2s)C = v''' + \frac{2}{r} v'' + \left( b_1 - \frac{1}{r^2} \right) v' + \left( b_1 \frac{1}{r} + \frac{1}{r^3} \right) v,
\]

(21)

where \( b_1 = sc - Rei\sigma_1 - M^2 \) Similarly, from equation (14), we have

\[

\frac{dw}{dr} = \frac{1}{s} \left( D^2 - 2s - i\sigma_2 \varepsilon \right) B
\]

(22)

Substituting the equation (22) for \( \frac{dw}{dr} \) in (12) we obtain the equation for w as

\[

s(M^2 + iRe\sigma_2)w = B''' + \frac{2}{r} B'' + \left( b_2 - \frac{1}{r^2} \right) B' + \left( \frac{b_2}{r} + \frac{1}{r^3} \right) B.
\]

(23)

Now the equations (17) and (19) are solved for v and B under the no slip and hyper stick conditions.

\[

V(1) = \cos(\beta_0) \quad and \quad w(1) = \sin(\beta_0) \quad (\text{Noslipcondition}).
\]

(24)

\[

B(1) = 0 \quad and \quad C(1) = \sigma_1 \quad (\text{Hyperstickcondition}).
\]

(25)

As q and \( \epsilon \) are at rest as \( r \to \infty \) the solutions of (17) and (19) can be written as

\[

v = a_1 K_1(\lambda_1 r) + a_2 K_1(\lambda_2 r)
\]

(26)

\[

B = a_3 K_1(\alpha_1 r) + a_4 K_1(\alpha_2 r).
\]

(27)

Now the constants \( a_1, a_2, a_3, \) and \( a_4 \) can be found out numerically for different values of micropolar parameters by using the boundary conditions (24) and (25) in (21),(23),(26) and (27).
3 Calculation for drag

The drag D acting on the cylinder of length L is given by:

\[ D = aL \int_0^{2\pi} (T_{21} \cos \beta_0 + T_{31} \sin \beta_0) \, d\theta \]  

(28)

The stress components in (28) are defined by the following constitutive equation for micropolar fluids

\[ T_{ij} = (-P \delta_{ij} + (2\mu + \kappa) \varepsilon_{ij} + \kappa \varepsilon_{ijm}(\omega_m - I_m)), \]  

(29)

where \( \omega_m = \frac{1}{2} (\nabla_1 \times Q)_m \) the subscript m represents component in m'th direction, \( \varepsilon_{ij} \) is strain rate tensor, \( \delta_{ij} \) is Kronecker delta and \( \varepsilon_{ijm} \) is the alternating symbol.

Now the stress components \( T_{31} \) and \( T_{21} \) on the cylinder (at \( r = 1 \)) can be calculated as

\[ T_{31} = \frac{g_0(\mu + \kappa)}{as} [a_3 K_1(\alpha_1)((1-c)\alpha_1^2 + s(c-2) - (1-c)\varepsilon_2) + a_4 K_2(\alpha_2)((1-c)\alpha_1^2 + s(c-2) - (1-c)\varepsilon_2)] e^{\sigma_2 t} \]  

(30)

and

\[ T_{21} = \frac{g_0(\mu+\kappa)}{a} [a_1 \left\{ K_1(\lambda_1) \left( 2(\beta_1 + \lambda_1^2) Re \varepsilon_2 - c \right) - \lambda_1 K_2(\lambda_1) \left( \frac{b_1 + \lambda_1^2}{\lambda_1^2} Re \varepsilon_2 + (1-c) \right) \right\} 
+ a_2 \left\{ K_1(\lambda_2) \left( 2(\beta_1 + \lambda_2^2) Re \varepsilon_2 - c \right) - \lambda_2 K_2(\lambda_2) \left( \frac{b_2 + \lambda_2^2}{\lambda_2^2} Re \varepsilon_2 + (1-c) \right) \right\}] \]  

(31)

Now finally the non-dimensional drag \( D' \) is given by

\[ D' = (T_{21} \cos \beta_0 + T_{31} \sin \beta_0) \text{on} \ r = 1, \]  

(32)

where \( D' = \frac{D}{\pi \mu \varepsilon_0} \).

4 Numerical calculation and results

The analytical expressions for the non-dimensional velocity components \( \nu \), \( w \) and micro rotation components \( \beta \), \( C \) and drag are given by (26), (23), (27), (21) and (32) respectively. For different values of the parameters like cross viscosity parameter or coupling number \( c \), Reynolds number \( Re \), couple stress parameter \( s \) and magnetic parameter \( M \) on velocity components \( \nu \), \( w \) and micro rotation components \( \beta, C \) are computed numerically and results are graphically presented in Figs 2-21. The drag is calculated numerically at different times for fixed \( \sigma_1 \) and \( \sigma_2 \).

When the angle \( \beta_0 = 0 \), the problem is reduced to rotatory oscillations about the axis of the cylinder. When \( \beta_0 = \frac{\pi}{2} \), the problem emerges itself as a special case of longitudinal oscillations along the axis of the cylinder. When \( \sigma_1 = \sigma_2 \) and oscillations are periodic, the results are found correlated with the results of Calmelet-Elhuu and Mazumdar[9]. The numerical results are presented in the form of graphs at \( s = 10, c = 0.4, M = 1, \varepsilon = 0.2, Re = 0.7, \sigma_1 = 1.5, \sigma_2 = 2.5, \beta_0 = 0.7, t = \pi \). It is understood from Figs 2-5 that while the Cross viscosity parameter \( \nu \) increases the axial velocity \( w \) and micro-rotation \( C \) decrease whereas transverse velocity \( w \) and micro rotation \( B \) are increasing at cylinder up to double the distance of radius of the cylinder. Here, it is observed that velocity and micro rotation components decrease as distance increases. It can be noticed from Figs 6-9 that the velocities \( \nu \), \( w \) and micro rotation \( C \) decrease when the Reynolds number \( Re \) increases and the maximum values of micro rotation \( B \) increase. From figs 10-13, it is observed that while couple stress parameter \( s \) increases, the transverse velocity \( w \) and micro rotation \( C \) decrease whereas maximum values of micro rotation \( B \) increase. Since the variations do not take place, the effect on \( w \) is found to be insignificant. Figures 14 to 17 show the effect of the magnetic parameter \( M \) on \( \nu \), \( w \), \( B \) and \( C \). These figures indicate that the velocities \( \nu \), \( w \) and \( C \) decrease and the amplitude of micro rotation \( B \) increases with an increase in the parameter \( M \). It occurs due to the imposing of a magnetic field normal to the flow direction. It raises the resistive force and controls the movement of the fluid.

The non-dimensional drag can be calculated numerically for different values of non-dimensional time in multiples of \( \pi/2a \) fixed values of \( \sigma_1, \sigma_2 \) and the results are shown in the Fig 18-19. It can be seen from fig 18 that the amplitude of drag decreases while \( c \) increases. From fig 19, it is observed that while \( s \) increases, drag decreases and all the curves assume the similar shape after \( s \) exceeds to a certain value. Drag in the case of viscous fluids is found less than that of the micropolar fluids.

Viscous fluids can be obtained from the micro polar fluids by applying the limit \( c \to 0 \) and \( s \to \infty \). These results are shown in figs 20-21 for velocities \( \nu \) and \( w \) and results obtained are found to be in accordance with the observations of Ramkissoon[6] in the case of viscous fluids.

IUNS email for contribution: editor@nonlinearscience.org.uk
G. Nagaraju: Unsteady MHD Micropolar Fluid Flow Generated Due to Longitudinal and Torsional Oscillations

Fig 2 Variation of $v$ with $r$

Fig 3 Variation of $w$ with $r$

Fig 4 Variation of $B$ with $r$

Fig 5 Variation of $w$ with $r$

Fig 6 Variation of $v$ with $r$

Fig 7 Variation of $w$ with $r$

Fig 8 Variation of $B$ with $r$

Fig 9 Variation of $w$ with $r$

Fig 10 Variation of $v$ with $r$

Fig 11 Variation of $w$ with $r$

Fig 12 Variation of $B$ with $r$

Fig 13 Variation of $w$ with $r$

IJNS homepage: http://www.nonlinearscience.org.uk/
Fig 14 Variation of $v$ with $r$

Fig 15 Variation of $w$ with $r$

Fig 16 Variation of $B$ with $r$

Fig 17 Variation of $w$ with $r$

Fig 18 Variation of $D'$ with $\sigma_2 t$

Fig 19 Variation of $D'$ with $c$

Fig 20 Variation of $ve^{i\sigma_1 t}$ with $r$

Fig 21 Variation of $we^{i\sigma_2 t}$ with $r$

**Acknowledgments**

The Author would like to express his sincere thanks to the Reviewers for their suggestions and comments to improve the quality of the paper.

*IJNS email for contribution: editor@nonlinearscience.org.uk*
References


