Exact Traveling Wave Solutions of Some Nonlinear Equations Using \((G'/G)-\expansion\) Method methods

Fakir Chand 
Anand K Malik

Department of Physics, Kurukshetra University, Kurukshetra-136119, India

(Received 28 June 2009, accepted 26 September 2012)

Abstract: In this paper, we find the exact solutions of some nonlinear evolution equations by using \((G'/G)-\expansion\) expansion method. Four nonlinear models of physical significance i.e. the symmetric regularized long-wave equation, the Klein-Gordon-Zakharov equations, the Burgers-Kadomtsev-Petviashvili equation and the nonlinear Schrödinger equation with a cubic nonlinearity are considered and obtained their exact solutions. From the general solutions, other well known results are also derived.

Keywords: Nonlinear evolution equations; Solitons; Traveling wave solutions; Nonlinear Schrödinger equation

PACS No.: 02.30.Jr, 05.45.Yv.

1 Introduction

Nonlinear evolution equations are widely used in a variety of fields such as fluid mechanics, quantum mechanics, solid state physics, plasma physics, population dynamics, chemical kinetics, nonlinear optics etc. [1–4]. Exact analytic solutions of nonlinear equations are always in great demand to explain complex dynamics of the underlying physical systems. In past, considerable efforts have been made to obtain exact analytical solutions of nonlinear equations with varying degree of success. For this purpose, a number of methods have been developed for obtaining explicit traveling wave solutions of nonlinear evolution equations [5–19].

Recently, in a seminal paper, Wang et al. [17] introduced a new technique called \((G'/G)-\expansion\) method for an effective dealing of nonlinear wave equations. Thereafter, many researches reported a number of new applications of this method [21–30], and developed new variants of the method such as generalized and extended \((G'/G)-\expansion\) method [25, 26]. Here, in the present work, to expand the domain of applications of \((G'/G)-\expansion\) method, we solve four nonlinear equations namely, the symmetric regularized long-wave (SRLW) equation, the Klein-Gordon-Zakharov (KGZ) equations, the Burgers-Kadomtsev-Petviashvili (BKP) equation and the Schrödinger equation with a cubic nonlinearity.

The SRLW equation arises in several physical applications, including ion sound waves in a plasma [27]. Recently, Biswas and coworkers [8, 9] studied the RLW and its generalized version \(R(m, n)\) equations using solitary wave ansatz and obtained 1-soliton solutions and integrals of motions. The KGZ equations present a classical model for describing the interaction between the Langmuir wave and the ion acoustic wave in plasma [30]. Analytic and numerical solutions of (1+1) and (2+1) dimensional KGZ equation with power law nonlinearity are discussed in [12]. The BKP equation is used to model shallow-water waves with weakly non-linear restoring forces and governs the dust acoustic waves propagating in a collisionless un-magnetized plasma whose constituents are Boltzmann distributed electrons, ions, and massive high negatively charged warm adiabatic dust grains [31]. Similarly the nonlinear Schrödinger (NLS) equation appears in the study of nonlinear wave propagation in dispersive and inhomogeneous media such as plasma phenomena and nonuniform dielectric media [32]. The dynamics of soliton propagation through optical fibers with a variety of nonlinearities can readily be understood by solving the NLS equation [3, 4].

The organization of the paper is as follows: In section 2, a description of the main steps of the \((G'/G)-\expansion\) method for finding the traveling wave solutions of nonlinear equations are given. Four illustrative examples, solved by this method, are given in sections 3, 4, 5 and 6. Finally concluding remarks are presented in section 7.

*Corresponding author. E-mail address: fchand@kuk.ac.in (FC); kukmalik@gmail.com (AKM)

Copyright © World Academic Press, World Academic Union

IJNS.2012.12.30/686
2 The \((G'/G)^{\text{expansion}}\) method

Here we briefly describe the main working steps of the \((G'/G)^\text{-expansion method}\). For that, consider a nonlinear partial differential equation (PDE) of the form

\[ P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0, \]

where \(u = u(x, t)\) is an unknown function and \(P\) is polynomial in \(u(x, t)\) and its partial derivatives, and contains higher order derivatives and nonlinear terms.

**Step 1:** To find the traveling wave solution to Eq.(1), introduce the wave variable

\[ \xi = (x - ct), \]

which implies

\[ u(x, t) = u(\xi). \]

Based on this, we can easily derive

\[ \frac{\partial}{\partial t} = -c \frac{\partial}{\partial \xi}, \quad \frac{\partial^2}{\partial t^2} = c^2 \frac{\partial^2}{\partial \xi^2}, \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \quad \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2}, \]

and so on for other derivatives. Eq.(4) changes the PDE, Eq.(1), to an ODE as

\[ P(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, ...) = 0, \]

where \(u_\xi, u_{\xi\xi}\) etc. denote derivatives of \(u\) with respect to \(\xi\).

Next integrate Eq.(5) as many times as possible, and set the constants of integration to be zero.

**Step 2:** The solution of Eq.(5) can be expressed by a polynomial in \((G'/G)\)

\[ u(\xi) = \alpha_0 (G')^n + \alpha_{n-1} (G')^{n-1} + \ldots, \]

where \(G = G(\xi)\) satisfies the second order linear ODE of the form

\[ G'' + \lambda G' + \mu G = 0, \]

with \(\alpha_0, \alpha_{n-1}, \ldots, \alpha_0, \lambda\) and \(\mu\) are constants to be determined later and \(\alpha_0 \neq 0\). The positive integer \(n\) can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq.(5).

**Step 3:** Substituting Eq.(6) into Eq.(5) and using Eq.(7), collecting all terms with the same order of \((G'/G)\) together, and then equating each coefficient of the resulting polynomial to zero yields a set of algebraic equations for \(\alpha_0, \alpha_{n-1}, \ldots, \alpha_0, c, \lambda\) and \(\mu\).

**Step 4:** Since the general solutions of Eq.(7) have been well known, then substituting \(\alpha_0, \alpha_{n-1}, \ldots, \alpha_0\) and \(c\) and the general solutions of Eq.(7) into Eq.(6), we obtain more traveling wave solutions of Eq.(1).

Now in the following four sections, we employ the above recipe to develop analytic solutions of four nonlinear equations.

3 The SRLW equation

Now we start with the SRLW equation

\[ u_{tt} - u_{xx} + \left(\frac{u^2}{2}\right)_{xt} - u_{xxtt} = 0, \]

which, after using Eqs.(2)-(4), transforms into the following nonlinear ODE

\[ (c^2 - 1)u'' - c\left(\frac{u^2}{2}\right)' - c^2 u''' = 0. \]
On integrating Eq.(9) with respect to $\xi$ twice and setting constants of integration to zero, we obtain

\[(c^2 - 1)u - \frac{c}{2}u^2 - c^2u'' = 0,\]  

(10)

where primes on $u$ denote derivatives with respect to $\xi$. Further, for the solutions of Eq.(10), we make an ansatz

\[u(\xi) = \sum_{i=0}^{n} a_i \left(\frac{G'}{G}\right)^i,\]  

(11)

where $G = G(\xi)$ satisfies the second order linear ODE, Eq.(7). Here, $n$ is a positive integer which is determined by balancing the highest order derivative term with the highest order nonlinear term and comes out 2 for the present case. This suggests the choice of $u(\xi)$ in Eq.(11) as

\[u(\xi) = a_0 + a_1 \left(\frac{G'}{G}\right) + a_2 \left(\frac{G'}{G}\right)^2.\]  

(12)

The use of Eq.(12) in Eq.(10) and the rationalization of the resultant expression with respect to the powers of $\left(\frac{G'}{G}\right)$ yields the following set of algebraic equations

\[(c^2 - 1)a_0 - \frac{c}{2}a_0^2 - c^2(2a_2\mu^2 + a_1\lambda) = 0,\]  

(13a)

\[(c^2 - 1)a_1 - ca_1 a_0 - c^2(6a_2\mu \lambda + 2a_1\mu + a_1\lambda^2) = 0,\]  

(13b)

\[(c^2 - 1)a_2 - \frac{c}{2}(a_1^2 + 2a_2 a_0) - c^2(8a_2\mu + 3a_1\lambda + 4a_2\lambda^2) = 0,\]  

(13c)

\[-ca_1 a_2 - c^2(2a_2 + 10a_2\lambda) = 0,\]  

(13d)

\[-\frac{c}{2}a_2^2 - 6c^3a_2 = 0.\]  

(13e)

These equations can be easily solved for the constants $a_0$, $a_1$ and $a_2$ as

\[a_0 = -12c\mu, \quad 2c(\lambda^2 + 2\mu), \quad a_1 = -12c\lambda, \quad a_2 = -12c.\]  

(14)

By using Eq.(14) in Eq.(12), we get

\[u_1(\xi) = -12c\left(\frac{G'}{G}\right)^2 - 12c\lambda\left(\frac{G'}{G}\right) - 12c\mu,\]  

(15)

\[u_2(\xi) = -12c\left(\frac{G'}{G}\right)^2 - 2c\lambda\left(\frac{G'}{G}\right) - 2c(\lambda^2 + 2\mu).\]  

(16)

The general solution of Eq.(7) can be written as

\[G = c_1 e^{-\frac{1}{2}(\lambda - \sqrt{\lambda^2 - 4\mu})\xi} + c_2 e^{-\frac{1}{2}(\lambda + \sqrt{\lambda^2 - 4\mu})\xi}.\]  

(17)

Now, substitute this general solution in Eqs.(15) and (16), we obtain the traveling wave solutions of Eq.(8) as

\[u_1(\xi) = \frac{12cc_1c_2(\lambda^2 - 4\mu)}{c_1 e^{\frac{1}{2}(\lambda - \sqrt{\lambda^2 - 4\mu})\xi} + c_2 e^{-\frac{1}{2}(\lambda + \sqrt{\lambda^2 - 4\mu})\xi}^2},\]  

(18)

where $(\lambda^2 - 4\mu) = (\frac{c^2 - 1}{c})$, and

\[u_2(\xi) = (\lambda^2 - 4\mu) \left[ -2c + \frac{12cc_1c_2}{c_1 e^{\frac{1}{2}(\lambda - \sqrt{\lambda^2 - 4\mu})\xi} + c_2 e^{-\frac{1}{2}(\lambda + \sqrt{\lambda^2 - 4\mu})\xi}^2} \right],\]  

(19)

where $(\lambda^2 - 4\mu) = (\frac{1-c^2}{c})$.

**Special Cases:**

The traveling wave solutions in Eqs.(18) and (19) can further be analyzed under various conditions imposed on $c_1$ and $c_2$.

IJNS email for contribution: editor@nonlinearscience.org.uk
Thus, again using Eqs.(2)-(4), the above PDEs are transformed into ODEs as

\[ u_1(\xi) = 3\left(\frac{c^2 - 1}{c}\right) \text{sech}^2\left(\frac{\sqrt{c^2 - 1}}{2c}\right) \xi, \]  

(20)

which is same as given in [33]. However, Eq.(19) gives periodic solution in the form

\[ u_2(\xi) = \left(\frac{c^2 - 1}{c}\right) \left[2 - 3\text{sec}^2\left(\frac{\sqrt{c^2 - 1}}{2c}\right)\right] \xi. \]  

(21)

Case 2: For \( c_1 = c_2 \) and \( c^2 < 1 \). We obtain periodic solution from Eq.(18) as

\[ u_1(\xi) = 3\left(\frac{c^2 - 1}{c}\right) \text{sec}^2\left(\frac{1 - c^2}{2c}\right) \xi, \]  

(22)

This solution is same as obtained in [33]. However, from Eq.(19) we obtain soliton solution in the form

\[ u_2(\xi) = \left(\frac{c^2 - 1}{c}\right) \left[2 - 3\text{sech}^2\left(\frac{1 - c^2}{2c}\right)\right] \xi. \]  

(23)

Similarly by choosing \( c_1 = -c_2 \) with \( c^2 > 1 \) and \( c^2 < 1 \), one can recover the other results of [33] alongwith additional solutions.

4 The KGZ equations

Next we consider the KGZ equations

\[ u_{tt} - u_{xx} + u + uv + |u|^2 u = 0, \]
\[ v_{tt} - v_{xx} = (|u|^2)_{xx}. \]  

(24)

This system describes interaction between Langmuir waves and ion sound waves. These are apparently coupled equations by two functions \( u(x, t) \) and \( v(x, t) \) where the function \( u(x, t) \) is complex and denotes the fast time scale component of electric field raised by electrons and the function \( v(x, t) \) is real and denotes the deviation of ion density from its equilibrium.

Thus, again using Eqs.(2)-(4), the above PDEs are transformed into ODEs as

\[ (c^2 - 1)u'' + u + uv + uv + |u|^2 u = 0, \]
\[ (c^2 - 1)v'' = (|u|^2)'' . \]  

(25)

On integrating the second equation and for simplicity taking constant of integration to zero, we get

\[ v = \frac{|u|^2}{c^2 - 1}. \]  

(26)

Substitution of Eq.(26) in the first equation of Eq.(25) provides us

\[ (c^2 - 1)^2 u'' + (c^2 - 1)u + c^2 u^3 = 0. \]  

(27)

As before, using the balancing procedure one obtains \( n = 1 \) and thus the ansatz Eq.(11) for \( u(\xi) \) is written as

\[ u(\xi) = a_0 + a_1 \left(\frac{G'}{G}\right). \]  

(28)

Now on inserting Eq.(28) into Eq.(27) and the rationalization of the resultant expression with respect to the powers of \( \left(\frac{G'}{G}\right) \) provides us a set of algebraic equations

\[ 2a_1(c^2 - 1)^2 + c^2 a_3^3 = 0, \]  

(29a)

\[ 3a_1(c^2 - 1)^2 + 3c^2 a_1^2 a_0 = 0, \]  

(29b)

\[ a_1(2\mu + \lambda^2)(c^2 - 1)^2 + a_1(c^2 - 1) + 3c^2 a_1 a_0^2 = 0, \]  

(29c)

\[ a_1\mu + \lambda(c^2 - 1)^2 + a_0(c^2 - 1) + a_0^3 c^2 = 0. \]  

(29d)
On solving the above set of algebraic equations, we have
\[ a_1 = \pm \sqrt{-2} \left( \frac{c^2 - 1}{c} \right), \quad a_0 = \mp \frac{\lambda (c^2 - 1)}{\sqrt{-2c}}, \quad c = \pm \sqrt{\frac{\lambda^2 - 4\mu + 2}{\lambda^2 - 4\mu}}. \] (30)

So, with reference to the above solutions, Eq.(28) becomes
\[ u(\xi) = \pm \sqrt{-2} \left( \frac{c^2 - 1}{c} \right) \left( \frac{c'}{c} \right) \mp \frac{\lambda (c^2 - 1)}{\sqrt{-2c}}. \] (31)

Finally, the traveling wave solutions of Eq.(24) with the help of Eqs.(17), (26) and (31) are written as
\[ u(\xi) = \pm \left( \sqrt{1-c^2} \right) \left( c_1 e^{\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi} - c_2 e^{-\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi} \right), \] (32)
\[ v(\xi) = -\frac{1}{c^2} \left( c_1 e^{\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi} - c_2 e^{-\frac{1}{2} \sqrt{\lambda^2 - 4\mu} \xi} \right)^2. \] (33)

Now, we present some special cases of the general solutions given in Eqs.(32) and (33).

Case 1: When \( c_1 = c_2 \) and \( \lambda^2 - 4\mu > 0 \). For this particular choice, we have
\[ u_{\pm}(\xi) = \pm \left( \sqrt{1-c^2} \right) \tanh \left( \frac{1}{\sqrt{2(c^2-1)}} \right) \xi, \] (34)
\[ v(\xi) = -\frac{1}{c^2} \tanh^2 \left( \frac{1}{\sqrt{2(c^2-1)}} \right) \xi. \] (35)

Note that solution in Eq.(34) represent the kink shaped soliton solution while solution (35) represents solitary wave solution.

Case 2: If \( c_1 = c_2 \) and \( \lambda^2 - 4\mu < 0 \). For this case we get periodic solutions from Eqs.(32) and (33) as
\[ u(\xi) = \pm \left( \frac{c^2 - 1}{c} \right) \tan \left( \frac{1}{\sqrt{2(1-c^2)}} \right) \xi, \] (36)
\[ v(\xi) = \frac{1}{c^2} \tan^2 \left( \frac{1}{\sqrt{2(1-c^2)}} \right) \xi. \] (37)

Similarly, for different choices of \( c_1 \) and \( c_2 \), some other forms of traveling wave solutions of the KGZ equations can also be obtained.

5 The (2+1)-dimensional BKP equation

A nonlinear dust ion acoustic wave is governed by the KdV-Burger’s equation and such a wave is generated due to the dissipation caused by the non-adiabatic charge variation of the dust particles. But mostly investigations for the dust charge fluctuation effects are focused for one-dimensional cases. But, such one-dimensional studies are not sufficient to explain the observed wave phenomena in the low and higher altitude auroral regions. The wave structure and stability in higher dimensional systems are subjected to modification because of anisotropy. Therefore, for the study of such phenomena in higher dimensions the (2+1)-dimensional BKP equation is very helpful [31]. The BKP equation, obtained from the extension of the KP and the Burger’s equation, is given by [34]
\[ u_{tt} + uu_x + \beta u_{xx} + \alpha u_{yy} = 0. \] (38)

Following the recipe of the preceding section, we perform a traveling wave reduction of the BKP equation using \( u(x, y, t) = u(\xi) \) with the argument \( \xi = x + y - ct \), generating the nonlinear ODE
\[ \left( -cu' + \frac{1}{2} (u^2)' + \beta uu'' \right)' + \alpha u'' = 0. \] (39)
On integrating Eq.(39) with respect to $\xi$ twice and setting constants of integration to zero, we obtain

$$ (\alpha - c)u + \frac{1}{2}u^2 + \beta u' = 0. \quad (40) $$

Again, by balancing procedure, we get $n = 1$ and form of $u(\xi)$ is same as in Eq.(28). On substituting this ansatz in Eq.(40) and collecting all terms with the same powers of $(\frac{G'}{G})$ together, the left hand side of Eq.(40) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $a_0, a_1, \lambda, \mu$ and $c$ as

$$ 2(\alpha - c)a_0 + a_0^2 - 2\beta ma_1 = 0, \quad (41a) $$
$$ (\alpha - c)a_1 + a_1a_0 - \beta \lambda a_1 = 0, \quad (41b) $$
$$ a_1^2 - 2\beta a_1 = 0. \quad (41c) $$

The set of above algebraic equations yields

$$ a_0 = \beta \left( \lambda \pm \sqrt{\lambda^2 - 4\mu} \right), \quad a_1 = 2\beta, \quad c = \alpha \pm \beta \sqrt{\lambda^2 - 4\mu}. \quad (42) $$

So, using Eqs.(28) and (42), the traveling wave solutions of the BKP equation become

$$ u_1(\xi) = \frac{-2\beta c_1 \sqrt{(\lambda^2 - 4\mu)}}{(c_1 e^{\frac{1}{2} \sqrt{(\lambda^2 - 4\mu)} \xi} + c_2 e^{-\frac{1}{2} \sqrt{(\lambda^2 - 4\mu)} \xi})}, \quad (43) $$
$$ u_2(\xi) = \frac{-2\beta c_2 \sqrt{(\lambda^2 - 4\mu)}}{(c_1 e^{\frac{1}{2} \sqrt{(\lambda^2 - 4\mu)} \xi} + c_2 e^{-\frac{1}{2} \sqrt{(\lambda^2 - 4\mu)} \xi})}. \quad (44) $$

Here it is interesting to note that the above two general results reduce in some particular solutions which are obtained by some other methods. For example, if $c_1 = c_2$ and $\mu = 0$, one can obtain

$$ u_1(\xi) = \beta \lambda \left[ 1 + \tanh \left\{ \frac{\lambda}{2} (x + y - (\alpha + \beta \lambda) t) \right\} \right], \quad (45) $$
$$ u_2(\xi) = -\beta \lambda \left[ 1 - \tanh \left\{ \frac{\lambda}{2} (x + y - (\alpha - \beta \lambda) t) \right\} \right], \quad (46) $$

which represent kink shaped soliton solutions. Similarly, if $c_1 = -c_2$ and $\mu = 0$, we get

$$ u_1(\xi) = \beta \lambda \left[ 1 + \coth \left\{ \frac{\lambda}{2} (x + y - (\alpha + \beta \lambda) t) \right\} \right], \quad (47) $$
$$ u_2(\xi) = -\beta \lambda \left[ 1 - \coth \left\{ \frac{\lambda}{2} (x + y - (\alpha - \beta \lambda) t) \right\} \right]. \quad (48) $$

which represent traveling wave solutions. The above four solutions are same as derived in [34] using tanh-method.

### 6 The NLS equation

The NLS equation is one of the most important universal nonlinear model that naturally arises in many physical systems. It is a generic equation which appears when a quasi-monochromatic wave is propagating in a dispersive and weakly nonlinear medium, and was used to describe a variety of physical phenomena in hydrodynamics, nonlinear optics, especially in nonlinear optical fibers, quasi-one-dimensional nonlinear molecular systems, Bose-Einstein condensation etc [3, 4, 32, 35, 36]. The completely integrable NLS equation in one dimension is written as

$$ iw_t + w_{xx} + A|w|^2w = 0, \quad (49) $$

where $w$ is a complex valued function of the spatial coordinate $x$ and the time $t$, $A$ is a real parameter. In order to construct exact solutions of Eq.(49), we transform it into an ODE

$$ u'' - \alpha u + Au^3 = 0, \quad (50) $$
after using the transformation \( w(x, t) = u(x)e^{i\omega t} \), where \( \alpha \) is considered as a real parameter. Suppose the solution of the ODE (50) is expressed by a polynomial in \( (\frac{G'}{G}) \) as given in Eq.(11) and \( G = G(x) \) satisfies the second order linear ODE (7).

Considering the homogeneous balance between \( u'' \) and \( u^3 \) in Eq.(50), we get, \( n=1 \). Hence ansatz to Eq.(50) is same as given in Eq.(28). Now substituting Eq.(28) in Eq.(50) and equating the coefficients of various powers of \( (\frac{G'}{G}) \), we get the following algebraic equations

\[
\begin{align*}
2a_1 + Aa_1^3 &= 0, \\
3\lambda a_1 + 3Aa_1^2a_0 &= 0, \\
a_1(2\mu + \lambda^2) - a_1a_1 + 3Aa_1a_0^2 &= 0, \\
a_1\mu - a_1a_0 + Aa_0^3 &= 0.
\end{align*}
\]

These equations are solved for constants \( a_0, a_1 \) and \( \alpha \) in terms of parameters \( \lambda, \mu \) and \( \lambda \) as

\[
a_1 = \pm \sqrt{-2A}, \quad a_0 = \pm \frac{\lambda}{\sqrt{-2A}}, \quad \alpha = \frac{(4\mu - \lambda^2)}{2}.
\]

Now using Eqs.(17) and (52) in Eq.(28), we get

\[
u(x) = \pm \sqrt{\frac{4\mu - \lambda^2}{2A}} \left( c_1e^{\frac{i}{2}\sqrt{\lambda^2 - 4\mu}x} - c_2e^{\frac{-i}{2}\sqrt{\lambda^2 - 4\mu}x} \right).
\]

Finally the general solutions of the NLSE can be written as

\[
w(x, t) = \pm \sqrt{\frac{\alpha}{A}} \left( c_1e^{\frac{\alpha}{2\sqrt{-\alpha}}x} - c_2e^{\frac{\alpha}{2\sqrt{-\alpha}}x} \right) e^{i\omega t}.
\]

Next, we discuss some special cases of Eq.(54).

**Case 1:** When \( c_1 = c_2 \) and \( \lambda^2 - 4\mu > 0 \) or \( \alpha < 0 \). For this condition, Eq.(54) reduces to

\[
w(x, t) = \pm \sqrt{\frac{\alpha}{A}} \tanh\left( \sqrt{\frac{-\alpha}{2}}x \right) e^{i\omega t}.
\]

Note that the real and imaginary parts of Eq.(55) represents periodic wave solutions while modulus part represent kink shaped soliton solution.

**Case 2:** If \( c_1 = c_2 \) and \( \lambda^2 - 4\mu < 0 \) or \( \alpha > 0 \). For this case, we obtain

\[
w(x, t) = \pm i\sqrt{\frac{\alpha}{A}} \tanh\left( \sqrt{\frac{\alpha}{2}}x \right) e^{i\omega t},
\]

which again represents the periodic wave solution. Note that the above two results in Eqs.(55) and (56) are same as obtained by using tanh-method in [35].

Similarly by selecting different choices of \( c_1 \) and \( c_2 \), one can get other forms of exact solutions of the NLS equation with cubic nonlinearity.

### 7 Conclusion

In the present work, we successfully obtained the exact solutions of the symmetric regularized long wave equation, the Klein-Gordon-Zakharov equations, the Burgers-Kadomtsev-Petviashvili equation and the nonlinear Schrödinger wave equation with a cubic nonlinearity within the framework of \( (\frac{G'}{G}) \)-expansion method. The obtained exact and explicit analytic solutions are in general forms involving various arbitrary parameters. It is interesting to note that when parameters are taken as special values in general solutions, the solitary waves such as kink-antikink shaped, bell shaped soliton and periodic solutions can be derived. It is also interesting to note that from the general results, one can easily recover solutions which are obtained from others methods. We also conclude that the \( (\frac{G'}{G}) \)-expansion method is a direct, concise, powerful, effective and convenient technique can be used for all integrable and non-integrable nonlinear models. Performance of this method is reliable, simple and gives many new solutions. It is also a standard and computerizable method which allows in to solve complicated nonlinear evolution equations in diverse areas of science. Moreover, this method is capable of greatly minimizing the size of computational work compared to other existing techniques.
Acknowledgements

The authors are very thankful to the reviewer for his useful suggestions to improve the quality of presentation of this paper.

References


