Convective Heat Transfer in a Dusty Fluid over a Vertical Permeable Surface with Thermal Radiation

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Abstract: This paper focuses on the radiation effects on a steady free convective boundary layer flow of a dusty fluid past a vertical permeable stretching surface. The system of governing partial differential equations are reduced to a system of ordinary differential equations using similarity transformations. The resultant system of ordinary differential equations are then solved numerically using Runge-Kutta Fehlberg fourth-fifth method. The effects of physical parameters like fluid-particle interaction parameter, local Grashof number, suction parameter, Prandtl number, radiation parameter and Eckert number on the flow and heat transfer characteristics are computed and presented graphically. Further, the rate of heat transfer at the surface is also discussed. The presents results are compared with the previous study and found there is a good agreement.

Keywords: Dusty fluid; free convection; fluid-particle interaction parameter; radiation parameter; suction parameter

1 Introduction

The study of laminar flow and heat transfer over a stretching sheet has considerable interest because of its ever increasing industrial applications and important bearings on several technological processes. It plays an important role in many industries such as chemical industry, power and cooling industry for drying, chemical vapour deposition on surfaces and cooling of nuclear reactors etc. Such processes occur when the effect of buoyancy forces in free convection becomes significant. The problem of free convection under the influence of magnetic field has attracted numerous researchers in view of its applications in geophysics and astrophysics. A wide variety of problems dealing with heat and fluid flow over a stretching sheet have been studied with both Newtonian and non-Newtonian fluids under the influence of imposed electric and magnetic fields, under different thermal boundary conditions have been reported in the literature. Crane [1] was the first to examine the problem of steady two-dimensional boundary layer flow of an incompressible and viscous fluid caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point. Grubka and Bobba [2] analyzed heat transfer effects by considering the power-law variation of surface temperature.

Chakrabarti [3] analyzed the boundary layer flow of a dusty gas. Datta and Mishra [4] have investigated boundary layer flow of a dusty fluid over a semi-infinite flat plate. Further, many investigators such as Agranat [5], Vajravelu et al. [6], Evgeny et al. [7], XIE Ming-liang et al. [8] and Palani et al. [9]. Recently Gireesha et al. [10] obtained the numerical solution for boundary layer flow and heat transfer of a dusty fluid over a stretching sheet with non-uniform heat source/sink. The above investigations deal with the flow and heat transfer induced by a horizontal stretching sheet, but there arise some situations where the stretching sheet moves vertically in a cooling liquid. Under such circumstances the fluid flow and the heat transfer characteristics are determined by two mechanisms, namely motion of the stretching sheet and the buoyant force. The thermal buoyancy resulting from heating/cooling of a vertically moving stretching sheet has a large impact on the flow and heat transfer characteristics. Also, in the above studies, the radiation effect is ignored. But, in the systems operating at high temperatures, the thermal radiation heat transfer becomes very important and its effects cannot be neglected. Thermal radiation effect play a significant role in controlling heat transfer processes in polymer industry.

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The flow and heat transfer over a vertical stretching sheet under various physical situations have been reported by many researchers like Daskalakis [11], Chen [12], Chamkha [13], Ishak et al. [14], Saleh et al. [15], Gireesha et al. [16]. Kumari et al. [17] have studied the unsteady free convection flow over a continuous moving vertical surface in an ambient fluid by taking two different heating processes like constant surface temperature (CST) and constant surface heat flux (CHF). Chen [18] analyzed MHD mixed convection of a power law fluid past a stretching surface in the presence of thermal radiation and internal heat generation/absorption. Mukhopadhyay and Layek [19] studied free convective flow and radiative heat transfer of a viscous incompressible fluid with variable viscosity over a stretching porous vertical plate.

Since no attempt has been made to analyze the effects of thermal radiation on a steady boundary layer flow of an incompressible viscous dusty fluid and heat transfer over a permeable vertical stretching surface, therefore, this problem is examined. The resulting coupled non-linear ordinary differential equations are solved by RKF-45 method with the help of algebraic software Maple. Flow and heat transfer characteristics are presented in tables and figures, and are discussed in detail. The present results are compared with the existing results for the case of steady state flow reported by Grubka and Bobba [2], Abel and Mahesha [20] and Chen [18] for various values of Prandtl number, and it is found that there is a good agreement.

2 Flow analysis of the problem

Consider a steady two dimensional laminar boundary layer flow of an incompressible viscous dusty fluid over a vertical stretching sheet. The flow is generated by the action of two equal and opposite forces along the $x-$ axis and $y-$ axis being normal to the flow. The sheet being stretched with the velocity $U_w(x)$ along the $x-$axis, keeping the origin fixed in the fluid of ambient temperature $T$ as shown in figure 1.

![Figure 1: Schematic diagram of the flow.](image)

The dust particles are assumed to be spherical in shape and uniform in size and number density of the dust particle is taken as a constant throughout the flow. Under these assumption along with the boundary layer approximations, the governing basic boundary layer equations for momentum equations take the following form [6]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho} (u_p - u) + g\beta^* (T - T_\infty), \tag{2}
\]

\[
u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = \frac{K}{m} (u - u_p), \tag{3}
\]

\[
u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = \frac{K}{m} (v - v_p), \tag{4}
\]

\[rac{\partial}{\partial x} (\rho_p u_p) + \frac{\partial}{\partial y} (\rho_p v_p) = 0, \tag{5}
\]

where $(u, v)$ and $(u_p, v_p)$ are the velocity components of the fluid and dust particle phases along $x$ and $y$ directions respectively. $\mu, \rho, \rho_p$ and $N$ are the co-efficient of viscosity of the fluid, density of the fluid, density of the particle phase,
number density of the particle phase respectively. $K$ is the stiffness resistance (drag co-efficient). $T$ and $T_{\infty}$ are the fluid temperature with in the boundary layer and in the free stream respectively. $g$ is the acceleration due to gravity, $\beta^*$ is the volumetric coefficient of thermal expansion, $m$ is the mass of the dust particle respectively. In deriving these equations, the drag force is considered for the interaction between the fluid and particle phases.

The boundary conditions for the flow problem are given by

$$u = U_w(x), \quad v = -V_w(x) \quad \text{at} \quad y = 0, \quad u \to 0, \quad u_p \to 0, \quad v_p \to v, \quad \rho_p \to \omega \rho \quad \text{as} \quad y \to \infty,$$

where $U_w(x) = cx$ is a stretching sheet velocity, $c > 0$ is the stretching rate, $\omega$ is the density ratio.

To convert the governing equations into a set of similarity equations, we now introduce the following transformation as,

$$u = cx f'(\eta), \quad v = -\sqrt{c} \eta f(\eta), \quad \eta = \sqrt{c \gamma y}, \quad u_p = cx F(\eta), \quad v_p = \sqrt{c} \gamma G(\eta), \quad \rho_p = H(\eta),$$

which are identically satisfies (1). Substituting (7) into (2) to (5), we obtain the following non-linear ordinary differential equations,

$$f'''(\eta) + f(\eta)f''(\eta) - \left[f'(\eta)\right]^2 + Gr \theta(\eta) + u^* \beta H(\eta) [F(\eta) - f'(\eta)] = 0, \quad (8)$$

$$G(\eta) F''(\eta) + [F'(\eta)]^2 + \beta [F(\eta) - f'(\eta)] = 0, \quad (9)$$

$$G(\eta) G''(\eta) + \beta [f(\eta) + G(\eta)] = 0, \quad (10)$$

$$H(\eta) F'(\eta) + H'(\eta) G'(\eta) + G(\eta) H'(\eta) = 0, \quad (11)$$

where a prime denotes differentiation with respect to $\eta$ and $l^* = \frac{m N_c}{c^* c_{\infty}}$, $\tau = \frac{g^*}{K}$ is the relaxation time of the particle phase, $\beta = \frac{1}{c^*}$ is the fluid particle interaction parameter, $Gr = \frac{g^* (T_w - T_{\infty})}{c_{\infty}^3}$ is the local Grashof number and $\rho_p = \frac{\rho_d}{\rho}$ is the relative density.

The boundary conditions defined as in (6) will becomes,

$$f(\eta) = f_0, \quad f'(\eta) = 1 \quad \text{at} \quad \eta = 0, \quad (12)$$

$$f'(\eta) = 0, \quad F(\eta) = 0, \quad G(\eta) = -f(\eta), \quad H(\eta) = \omega \quad \text{as} \quad \eta \to \infty,$$

where $f_0 = \frac{V_w}{(\omega \gamma)^{1/2}}$ is the suction parameter.

### 3 Heat transfer analysis

The governing dusty boundary layer heat transport equations in the presence of radiation for two dimensional flow is ([22]):

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{k}{\gamma_T} \frac{\partial^2 T}{\partial y^2} + \frac{N c_p (T_p - T)}{\tau_T} + \frac{N}{\tau_v} (u_p - u)^2 - \frac{\partial \eta_r}{\partial y}, \quad (13)$$

$$u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} = \frac{c_p}{c_{m T}} (T_p - T), \quad (14)$$

where $T$ and $T_{\infty}$ is the temperature of the fluid and temperature of the dust particle, $\tau_T$ is the thermal equilibrium time and is the time required by the dust cloud to adjust its temperature to the fluid, $\tau_v$ is the relaxation time of the dust particle i.e., the time required by a dust particle to adjust its velocity relative to the fluid, $k$ is the thermal conductivity.

Using the Rosseland approximation for radiation [12], radiation heat flux is simplified as

$$q_r = \frac{4 \sigma^* \partial T^4}{3 k^*}, \quad (15)$$

where $\sigma^*$ and $k^*$ are the Stefan-Boltzmann constant and the mean absorption coefficient respectively. Assuming that the temperature differences within the flow such that the term $T^4$ may be expressed as a linear function of the temperature,
we expand $T^4$ in a Taylor series about $T_\infty$ and neglecting the higher order terms beyond the first degree in $(T - T_\infty)$ we get

$$T^4 \approx 4T_\infty^3 T - 3T^4_\infty. \tag{16}$$

Substituting equations (15) and (16) in (13) reduces to

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \left( k + \frac{16\pi^2 T^3_\infty}{3k^*} \right) \frac{\partial^2 T}{\partial y^2} + \frac{Ne_p}{\tau_p}(T_p - T) + \frac{N}{T_v}(u_p - u)^2 \tag{17}$$

In order to solve the (17) and (14), we consider non dimensional temperature boundary condition as

$$T = T_w = T_\infty + A \left( \frac{x}{l} \right)^2 \text{ at } y = 0,$$

$$T \rightarrow T_\infty, \quad T_p \rightarrow T_\infty \quad \text{as } y \rightarrow \infty, \tag{18}$$

where $T_w$ and $T_\infty$ denote the temperature at the wall and at large distance from the wall respectively. $A$ is a positive constant, $l = \sqrt{\frac{\pi}{2}}$ is a characteristic length.

Now define the non-dimensional fluid phase temperature $\theta(\eta)$ and dust phase temperature $\theta_p(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, \tag{19}$$

where $T - T_\infty = A \left( \frac{x}{l} \right)^2 \theta(\eta)$.

Using (18) and (19) into (17) to (14), we obtain the following non-linear ordinary differential equations

$$(1 + Nr)\theta''(\eta) + Pr[f(\eta)\theta'(\eta) - 2f'(\eta)\theta(\eta)] + \frac{NP_r}{\rho c_T} \left[ \theta_p(\eta) - \theta(\eta) \right] + \frac{NPrEc}{\rho Tr_v}[F(\eta) - f'(\eta)]^2 = 0, \tag{20}$$

$$2F(\eta)\theta_p(\eta) + G(\eta)\theta_p'(\eta) + \frac{c_p}{ccm_{TT}} \left[ \theta_p(\eta) - \theta(\eta) \right] = 0, \tag{21}$$

where $Pr = \frac{\nu c_p}{k^*}$ is the Prandtl number, $Ec = \frac{c f^2}{A f p}$ is the Eckert number, $Nr = \frac{16\pi^2 T^3_\infty}{3k^*}$ is the Radiation parameter.

The corresponding boundary conditions for $\theta(\eta)$ and $\theta_p(\eta)$ will becomes

$$\theta(\eta) = 1 \quad \text{at} \quad \eta = 0,$$

$$\theta(\eta) \rightarrow 0, \quad \theta_p(\eta) \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty. \tag{22}$$

**Table 1:** Comparison results for the wall temperature gradient $-\theta'(0)$ in the case of $\beta = 0$, $Gr = 0$, $Nr = 0$, $f_0 = 0$, $Ec = 0$ and $N = 0$.

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**4 Results and discussion**

The equations (8) to (11) and (20) to (21) with the boundary conditions (12) and (22) are solved numerically using RKF45 methods with the help of symbolic algebra software Maple [21]. It is very efficient in using the well known Runge Kutta Fehlberg fourth-fifth method (RKF45 Method) to obtain the numerical solutions of a boundary value problem. The RKF45 algorithm in Maple has been well tested for its accuracy and robustness. In order to check the accuracy of our present numerical solution procedure used, a comparison of wall temperature gradient $-\theta'(0)$ is made with those reported by
Grubka and Bobba [2], Abel and Mahesha [20] and Chen [12] for various values of Prandtl number as given in Table 1. It can be seen from this table that there is a very good agreement between the present results. Hence this confirms that the numerical method adopted in the present study gives very accurate results. Further, the impact of some important physical parameters on wall temperature gradient $\theta'(0)$ may be analyzed from Table 2. It is to noted that the effect of increasing the fluid-particle interaction parameter $\beta$ is to decrease the wall temperature gradient. Also observed that the effect of suction parameter $f_0$ is to decrease the wall temperature gradient.

Figure 2: Velocity profiles for different values of $\beta$.

Figure 2, is a plot of velocity distribution with $\eta$ for various values of fluid particle interaction parameter ($\beta$). It is clearly observed from this figure that if $\beta$ increases we can find the decrease in the fluid phase velocity and increase in the dust phase velocity. Also it reveals that for the large values of $\beta$ i.e., the relaxation time of the dust particle decreases ultimately as it tends to zero then the velocities of both fluid and dust particle will be the same.

The figure 3, illustrates the variation of velocity profiles with $\eta$ for various values of local Grashof number ($Gr$). From this plot it is observed that the effect of increasing values of local Grashof number is to increase the velocity distribution of both the fluid and dust phases. Physically $Gr > 0$ means heating of the fluid or cooling of the boundary surface, $Gr < 0$ means cooling of the fluid or heating of the boundary surface and $Gr = 0$ corresponds to the absence of free convection current. Figure 4, depict the effect of local Grashof number ($Gr$) on temperature profiles versus $\eta$. It is evident from these plots that increasing value of $Gr$ results in thinning of the thermal boundary layer associated with an increase in the wall temperature gradient and hence produces an increase in the heat transfer rate.

Figure 3: Velocity profiles for different values of $Gr$.  
Figure 4: Temperature profiles for different values of $Gr$.

The evolution of the dimensionless temperature profiles $\theta(\eta)$ and $\theta_p(\eta)$ are plotted for different values of radiation
parameter $Nr$ in figure 5. It is observed that the increase in the thermal radiation parameter $Nr$ produces a significant increase in the thickness of the thermal boundary layer, so the temperature distribution increases with increasing the value of $Nr$. Thus the radiation should be at its minimum in order to facilitate the cooling process.

Figure 5: Temperature profiles for different values of $Nr$.

Figure 6, show the effect of suction parameter on the velocity profiles versus $\eta$. It is observed that both fluid and dust phase velocity profiles tends asymptotically to the horizontal axis, i.e., the non-dimensional velocities absorbs maximum at the wall. So that velocity profiles decreases as suction parameter $f_0$ increases. It is also interesting to note that there is a significant enhancement of temperature at the wall, when it is porous. The fluid and dust phase temperature profiles start to decrease monotonically from the very beginning which can be seen from the figure 7.

Figure 6: Velocity profiles for different values of $f_0$. Figure 7: Temperature profiles for different values of $f_0$.

Figure 8, depicts the graph of temperature profile $\theta(\eta)$ for various values of Prandtl number $Pr$. It can be seen that the fluid phase temperature and dust phase temperature decreases with increase of Prandtl number. Which implies that the momentum boundary layer is thicker than the thermal boundary layer and consequently the temperature gradient decreases with the increase in Prandtl number. Temperature profiles for the selected values of Eckert number is plotted in figure 9. One can observe that the effect of increasing values of Eckert number enhances the temperature at a point which is true for both the fluid phase as well as dust phase. It is noted from all the graphs that the fluid phase is parallel to that of dust phase and also the fluid phase is higher than the dust phase.

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5 Conclusions

In this study, a numerical analysis is presented to investigate the free convective heat transfer of a dusty fluid over a vertical permeable stretching sheet. Thermal radiation terms have been included in the energy equation. Velocity and temperature profiles are presented graphically and analyzed. Influence of physical parameters found to effect the problem under consideration are the fluid particle interaction parameter, local Grashof number, suction parameter, radiation parameter, Prandtl number and Eckert number. Numerical computations shows that the present values of wall temperature gradient is in close agreement with those obtained by previous investigators in the absence of $\beta$, $f_0$, $Gr$, $Nr$ and $Ec$. On the basis of the above study we have the following observations:

1. Velocity of fluid phase decreases and dust phase increases as $\beta$ increases.
2. Velocity of fluid and dust phase increases and temperature of fluid and dust phase decreases as $Gr$ increases.
3. Velocity and temperature of both fluid and dust phase decreases as $f_0$ increases.
4. Temperature decreases with increasing $Pr$, while it increases with increasing in the value of $Nr$ in the thermal boundary layer.
5. Wall temperature gradient increases with increasing $Nr$ and $Ec$. While it decreases as the other parameter $\beta$, $f_0$, $Gr$ and $Pr$ increases.
6. Radiation should be at its minimum in order to facilitate the cooling process.

References


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Table 2: Values of wall temperature gradient $-\theta'(0)$ for different values of $\beta$, $Gr$, $Pr$, $Nr$, $f_0$ and $Ec$.

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