The Laser Radiation Pressure as a Perturbing Force in the Space Research

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Abstract: Modeling the force of laser pressure on the dynamics of an artificial satellite is studied. The atmospheric effects are taken into consideration. Some important details of the model arising from the thermo-optical satellite surface properties are clarified. The model has been exemplified by computing the laser perturbing acceleration for a passive Earth’s satellite (e.g. the satellite Ajisai). To illustrate the role of the Earth’s atmosphere in the radiative force modeling, the model has been compared with that neglecting the atmospheric effects.

Keywords: laser radiation pressure; atmospheric effects; disturbing force

1 Introduction

The quantum description of light found that light consists of packets of energy called photons. Planck’s law stated that a photon of frequency $\omega$ will transport energy

$$E = h' \omega,$$

where $h'$ is Planck’s constant. In 1905, Einstein in his description of photoelectric effects concluded that each photon carries a momentum $E/c$, where $c$ is the speed of light. Quantum mechanics concluded that as the momentum transported by photons a radiation pressure results. The radiation pressure exerted by a flux of photons is

$$\tilde{P} = \frac{S}{c},$$

where $S$ is the radiation flux or intensity (the energy crossing a unit area in a unit time). Moreover, the electromagnetic theory found that photon pressure is just the energy density, $\tilde{U}$, of the photon wave, Maxwell, (1873)

$$\tilde{U} = \frac{S}{c}.$$

Therefore, the quantum and electromagnetic description of radiation pressure are equivalent. By Newton’s second law, changing the momentum of photons by reflecting surfaces results in an applied force and by his third law, a reactive force acts upon these surfaces.

Based on the previous concepts of photon pressure, an extensive studies were performed to explain the perturbations produced by the radiative forces on artificial satellites orbit, Milani, et al., (1987); Mignard, et al.,(1990); Vokrouhlicky, (1993); De Moraes, (1994); Vokrouhlicky (1994); El-Saftawy, et al., (1998), and El-Saftawy, (2005)

The purpose of this paper is to point out the effect of laser beam, used in satellite laser ranging stations, on the satellite dynamics.
2 The laser disturbing force

The surface orientation is defined by a unit vector \( \hat{n} \) normal to the surface and a transverse unit vector \( \hat{t} \) normal to \( \hat{n} \). Also, two unit vectors \( \hat{u} \) and \( \hat{v} \) define the direction of incident radiation and that of the specularly reflected radiation respectively (see Fig. 1).

![Figure 1 "Non perfect reflecting flat surface"](image)

For a flat non-perfectly reflecting surface, the total force \( \bar{f} \) is given by McInnes, (1999)

\[
\bar{f} = \bar{m},
\]

where \( \bar{m} \) is the unit vector in the direction of the total force and

\[
f = \frac{SA}{c} \left[ 4\beta \rho' \cos \eta + 2(1 + \beta \rho')(B_f(1 - \beta)\rho' + \alpha' \varepsilon_f B_f - \varepsilon_b B_f) \cos^3 \eta + \left( B_f(1 - \beta)\rho' + \alpha' \varepsilon_f B_f - \varepsilon_b B_f \right)^2 + (1 - \beta \rho')^2 \right]^{1/2},
\]

where \( A \) is the projected surface area, \( \beta \) is the surface specularity, \( \eta \) is the radiation incident angle, \( \rho' \) is the surface reflectivity, \( B_f \) and \( B_b \) are the non-Lambertian coefficient of the front and back surfaces of the spacecraft respectively, \( \alpha' \) is the absorption coefficient, \( \varepsilon_f \) and \( \varepsilon_b \) are the front and back surface emissivity respectively.

3 Atmospheric effects

The atmospheric components and weather conditions cause a laser beam attenuation. The beam attenuation is given by the improved version of Lambert’s law (Goody and Yung, (1989) and Nielsen, (1994))

\[
S(\rho) = S(0) e^{-\int_0^\rho \sigma(\rho) d\rho}
\]

where \( S(\rho) \) and \( S(0) \) are the beam intensities (in units of \( \text{Watt/m}^2 \)) at satellite range \( \rho \) and at the laser source respectively and \( \sigma \) is the total atmospheric attenuation or extinction coefficient (in units of \( \text{Km}^{-1} \))

\[
\sigma = \sigma_{\text{scat}} + \sigma_{\text{abs}} + \sigma_{\text{mol}} + \sigma_{\text{mol}}
\]

where, \( \sigma_{\text{scat}}, \sigma_{\text{abs}}, \sigma_{\text{mol}} \) and \( \sigma_{\text{mol}} \) are the attenuation coefficients due to aerosols scattering, aerosols absorption, molecular scattering and molecular absorption respectively. Moreover for long-range applications (e.g. satellite laser ranging), the beam spreads as it propagates through vacuum. So, the laser beam attenuation will be attenuated through El-Saftawy, et. al., (2006)

\[
S = \frac{P_o D^2}{\pi \rho^2 \lambda^2} e^{-\int_0^\rho \sigma(\rho) d\rho},
\]

where \( p_o \) is the power at the laser source \( D \) is the diameter of the laser transmitter and \( \lambda \) is the laser wavelength. Consequently, the magnitude of the total force exerted by a laser beam, fired from a ground station and attenuated by the Earth’s atmosphere, on the satellite surface will be:

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\[ f = \frac{P \cdot D^2}{\pi \varepsilon_0 \rho^3 \lambda^2} e^{-\sigma(\rho) \rho^3} \int_0^\rho A \left[ 4\beta \rho \cos^4 \eta + 2(1 + \beta \rho')(B_f(1 - \beta) \rho' + \alpha' \varepsilon \varepsilon_0 B_f - \varepsilon_0 B_s) \right. \]
\[ \cos^3 \eta + \left. \left( (B_f(1 - \beta) \rho' + \alpha' \varepsilon \varepsilon_0 B_f - \varepsilon_0 B_s)^2 + (1 - \beta \rho')^2 \right) \cos^2 \eta \right]^{1/2} \]

\[ \text{(8)} \]

4 The Acceleration components

The force vector is inclined by an angle, \( \vartheta \), to the incident direction. So as illustrated in Fig. 2, the force component in the incident direction will be:

\[ \bar{f}_\rho = f \hat{m} \cdot \hat{\rho} = f \cos \vartheta \frac{\bar{\rho}}{|ar{\rho}|} \]

(9)

The acceleration experienced by a satellite of mass \( M \) and cross-sectional area \( A \) in the incident direction will be:

\[ \vec{R} = \frac{f \cdot \bar{L}}{M} = \frac{P \cdot D^2}{\pi \varepsilon_0 \rho^3 \lambda^2} e^{-\sigma(\rho) \rho^3} \frac{A M}{M} \left[ (2 \beta \rho' + \frac{1}{2}(B_f(1 - \beta) \rho' + \alpha' \varepsilon \varepsilon_0 B_f - \varepsilon_0 B_s)^2 \right. \]
\[ + \frac{1}{2}(1 - \beta \rho')^2 + \frac{1}{2}(1 + \beta \rho')(B_f(1 - \beta) \rho' + \alpha' \varepsilon \varepsilon_0 B_f - \varepsilon_0 B_s) \cos \eta + (2 \beta \rho' \]
\[ + \frac{1}{2}(B_f(1 - \beta) \rho' + \alpha' \varepsilon \varepsilon_0 B_f - \varepsilon_0 B_s)^2 + \frac{1}{2}(1 - \beta \rho')^2 \right) \cos 2\eta + \frac{1}{2}(1 + \beta \rho'). \]
\[ \left. \right) \cos 3\eta + \frac{1}{2} \beta \rho' \cos 4\eta \right]^{1/2} \cos \vartheta \frac{\bar{\rho}}{|ar{\rho}|} \]

As shown in Fig. 2, the range vector, \( \bar{\rho} \) is related to the station coordinates vector, \( \vec{R} \), by the relation:

\[ \bar{\rho} = \hat{r} - \bar{R} \]

(10)

where \( \bar{R} \), the station coordinate vector, is given by Escobal, (1965)

\[ \bar{R} = \begin{pmatrix} G_1 \cos \varphi' \cos \theta' \\ G_1 \cos \varphi' \sin \theta' \\ G_2 \sin \varphi' \end{pmatrix}, \]

(11)

with

\[ G_1 = \frac{\alpha_e}{(1 - (2f_e - f_e^2) \sin^2 \varphi')} + h', \]

(12)

\[ G_2 = \frac{\alpha_f(1 - f_e^2)^{1/2}}{(1 - (2f_e - f_e^2) \sin^2 \varphi')} + h', \]

(13)

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where \( a_e \) is the Earth’s equatorial radius, \( f'_e \) is the Earth’s flattening, \( \varphi' \) is the geodetic latitude, \( h' \) is the station elevation above the sea level and \( \theta' \) is the sidereal time.

The components of the position vector \( \vec{r} \), in terms of Keplerian orbital element, can be written as Escobal, (1965)

\[
\vec{r} = \begin{pmatrix}
\cos \Omega \cos(\omega + v) - \sin \Omega \sin(\omega + v) \cos i \\
\sin \Omega \cos(\omega + v) + \cos \Omega \sin(\omega + v) \cos i \\
\sin(\omega + v) \sin i
\end{pmatrix}.
\]

The components of the disturbing accelerations \( \vec{S}, \vec{T} \) and \( \vec{W} \) in the direction of radius-vector, tangential to the orbit and perpendicular to it respectively will be described in the next subsections.

### 4.1 The radial component \( \vec{S} \)

The radial component of the acceleration, \( \vec{S} \), is defined as:

\[
\vec{S} = \vec{R} \cdot \frac{\vec{r}}{|r|} = \frac{P_p D^2}{4 \pi \mu r^3} e^{-\frac{1}{2} \sigma(\rho) d \rho} \frac{A}{\Delta} \left\{ \left[ \frac{3}{2} \beta \rho' + \frac{1}{2} B_f (1 - \beta) \rho' + \alpha' \frac{\epsilon J_f - \epsilon J_B}{\epsilon J_f + \epsilon J_B} \right] \cos \eta + (2 \rho') \right.
\]

\[
+ \left. \frac{1}{2} (1 - \beta \rho')^2 \right\} + \frac{3}{2} \left[ \frac{1}{2} \alpha' \frac{\epsilon J_f - \epsilon J_B}{\epsilon J_f + \epsilon J_B} \cos \eta + (2 \rho') \right.
\]

\[
+ \left. \frac{1}{2} (1 - \beta \rho')^2 \right\} \cos 2\eta 
\]

\[
\text{where } \eta = \int_0^\theta \cos^2 \theta' d\theta' + \int_0^\theta \sin^2 \theta' d\theta'.
\]

### 4.2 The transverse component \( \vec{T} \)

The transverse component, \( \vec{T} \), in the orbital plane can be written as:

\[
\vec{T} = \vec{R} \cdot \frac{\vec{r}}{|r|} \cos \theta = \frac{P_p D^2}{4 \pi \mu r^3} e^{-\frac{1}{2} \sigma(\rho) d \rho} \frac{A}{\Delta} \left\{ \left[ \frac{3}{2} \beta \rho' + \frac{1}{2} B_f (1 - \beta) \rho' + \alpha' \frac{\epsilon J_f - \epsilon J_B}{\epsilon J_f + \epsilon J_B} \right] \cos \eta + (2 \rho') \right.
\]

\[
+ \left. \frac{1}{2} (1 - \beta \rho')^2 \right\} + \frac{3}{2} \left[ \frac{1}{2} \alpha' \frac{\epsilon J_f - \epsilon J_B}{\epsilon J_f + \epsilon J_B} \cos \eta + (2 \rho') \right.
\]

\[
+ \left. \frac{1}{2} (1 - \beta \rho')^2 \right\} \cos 2\eta 
\]

\[
\text{where } \vec{H} \text{ is a unit vector in the direction of the angular momentum vector. It can be expressed as Escobal, (1965)}
\]

\[
\vec{H} = \sin i \sin \Omega \vec{I} - \sin i \cos \Omega \vec{J} + \cos i \vec{K}.
\]

### 4.3 The normal component \( \vec{W} \)

The normal to orbital plane component, \( \vec{W} \), can be written as:

\[
\vec{W} = \vec{R} \cdot \frac{\vec{r}}{|r|} = \frac{P_p D^2}{4 \pi \mu r^3} e^{-\frac{1}{2} \sigma(\rho) d \rho} \frac{A}{\Delta} \left\{ \left[ \frac{3}{2} \beta \rho' + \frac{1}{2} B_f (1 - \beta) \rho' + \alpha' \frac{\epsilon J_f - \epsilon J_B}{\epsilon J_f + \epsilon J_B} \right] \cos \eta + (2 \rho') \right.
\]

\[
+ \left. \frac{1}{2} (1 - \beta \rho')^2 \right\} + \frac{3}{2} \left[ \frac{1}{2} \alpha' \frac{\epsilon J_f - \epsilon J_B}{\epsilon J_f + \epsilon J_B} \cos \eta + (2 \rho') \right.
\]

\[
+ \left. \frac{1}{2} (1 - \beta \rho')^2 \right\} \cos 2\eta 
\]

\[
\text{where } \vec{G} \text{ is the geodetic axis of the spacecraft, and } \Theta \text{ is the sidereal time}.
\]

It is worth noted that the spacecraft is decomposed into a finite number of small elementary surfaces. So, the total force acting on the spacecraft is assumed to be the sum of the forces acting on each elementary surface Milani, et al., (1987); Antreasian and Rosborough, (1992); Kubo-oka and Sengoku, (1999) and Ping, et al., (2001). At a given time, the total radiant acceleration affecting the spacecraft is
\[ \tilde{R}(t) = \sum_{i=1}^{k} \tilde{R}_i(t), \]  

where \( k \) is the number of satellite surfaces and \( \tilde{R}_i(t) \), the acceleration experienced by each surface, is given by equation (11).

5 Numerical application

The model was applied on the Experimental Geodetic Satellite (EGS) known as AJISAI. It is a spherical passive satellite of 2.15 m diameter and a mass of 685.2 kg with \textit{COSPAR ID} 8606101 used for solid Earth studies; crustal movements and plate tectonics. It carries 318 mirrors and 120 laser retro reflector assemblies having 1436 corner cube reflectors. The mirror base made of an aluminum alloy and its surface has a protective coating of silicon oxide Sasaki and Hashimoto, (1987). The thermo-optical properties of the surface coating materials are given in table 1.

Our source of radiation is a ground based second harmonic Nd-YAG laser used for satellite laser ranging at Helwan SLR station. The parameters of laser system are described in (http://ilrs.gsfc.nasa.gov/cgi-bin/stations/select.cgi?order=by_site_name &site_code=HLWL&site_name=Helwan&pid_id=7831&tab_id=general).

In the present example, the following simplifying assumptions are adapted:

The absorbed light is not re-emitted, Milani, et.al., (1987).

The optical properties of each elementary surface can be expressed by a linear combination of black body, a perfect mirror and Lambert diffuse, Milani, et.al., (1987).

The retro-reflector specularity is assumed to be \( \beta_r = 1 \).

The metal mirror plates specularity is assumed to be that of TOPEX \( \beta_{mm} = .2 \) (P.G. Antreasian and Rosborough, (1992); Kubo-oka and Sengoku, (1999).

The dimensions of the metal mirror plates are chosen to be that of the LRE satellite (http://god.tksc.jaxa.jp/tr/top.html). The reflecting area of each corner cube is \( A_c = 2\sqrt{3} k^2 \), where \( k \) is the radius of the inscribed circle (\( k = 13.5 \text{mm} \)) (http://ilrs.gsfc.nasa.gov/docs/retromtg060406_slides.pdf #search=%22ILRS%20Meeting%20on%20Retroreflector%20Arrays%22).

The radiation assumed to fall normal to the satellite surfaces.

<table>
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<th>Table 1 The thermo-optical properties of AJISAI surface</th>
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<td>Properties</td>
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6 Discussion and interpretation

By applying the model on satellite AISAI, the relation between the acceleration components and satellite range is discussed and plotted in the following figures. Fig. 3. illustrates the variation of the radial component, \( \tilde{S} \), of the laser disturbing acceleration and the satellite range, \( \rho \), with time. For one satellite path, it can be seen that while the satellite range decreases the related radial component, \( \tilde{S} \), increases.
Similarly, the transverse and normal acceleration components $\tilde{T}$ and $\tilde{W}$ respectively increase as the satellite range, $\rho$, decreases as illustrated in Figure 4 and Figure 5 respectively.

Logically, the behavior of the acceleration components is accepted. This is because for a satellite (i.e. the same thermo-optical properties of the projected area), the radiation flux fall into the satellite surfaces depends on the distance traveled by the laser to the satellite (i.e. the satellite range). As the range increases the atmospheric attenuation on the laser beam will increase. Consequently, the laser flux reaching the satellite surface will decrease and produce a low radiation pressure on it. So, at the collimation point, which defined as the nearest point over the station or the point on the satellite orbit of the lowest range w.r.t. the station, the components of the disturbing acceleration reach their maximum values.

Moreover, the role of atmospheric attenuation of laser intensity is illustrated in the following figures. Fig. 6 represents a comparison of the radial component, $\tilde{S}$, of the laser disturbing acceleration in two cases; in presence of total the atmospheric attenuation and that taking into account only the laser beam spread.

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Another two comparisons of the acceleration components $\tilde{T}$ and $\tilde{W}$ in the two cases of the atmospheric effects are demonstrated in Fig. 7 and Fig. 8 respectively.

From Fig. 6 - Fig. 8 it can be noted that the behaviors of acceleration components in the two cases are nearly the same this is because the laser physical properties (e.g. its monochromatization and coherency). However, a slight difference occurs at the lowest values of the acceleration components, which related to the longest satellite ranges, as a result of the inverse relation between the satellite rang and the beam intensity (as given by eqn.(7)) which directly proportional to the acceleration components (as given by eqs.(14-16)). On the contrary for the same reason, the acceleration components are almost the same at the collimation point. The majority of the atmospheric effect is noticeable on the transverse component, $\tilde{T}$, as shown in Fig. 7.

7 Conclusion

The model is considered to be a general model describing the radiation pressure on flat non-perfectly reflecting surfaces. In its formulation, it depends not only on the thermo-optical properties of each satellite surfaces but also it concerned with
the beam attenuation through the propagating medium. The model takes into account the rotating earth through the station coordinates vector, $\vec{R}$, and the Keplerian orbital element through the position vector, $\vec{r}$.

The obtained results confirm that the laser pressure has a sensible effect on the dynamics of an artificial satellite especially at the collimation point. Moreover, the results verified that even though the atmospheric attenuation of the laser power, the laser pressure will plays a reasonable role in perturbing the satellite motion.

References