Type-2 Fuzzy Adaptive Control Method of Ecological System

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(Received 29 October 2011 , accepted 16 April 2012)

Abstract: In this paper, a type-2 fuzzy adaptive controller of a Lotak-Volterra system is considered. For bettering the active system properly, the approximated error is introduced into the design of the controller. By using Lyapunov synthesis approach, we attain an adaptive law adjusting the free parameters in the controller such that the closed-loop system follows a desired trajectory. The nonlinear dynamic behavior and control of biomathematic system state are theoretically discussed, which are of practical significance. The scheme indicates that a type-2 fuzzy adaptive controller is a powerful tool to pursuit sustainable development of ecology.

Keywords: type-2 fuzzy sets; adaptive control systems; ecological system

1 Introduction

A sharp rise of interest in ecological problems has been in the last decade of the past century all over the world. Ecological models have been greatly introduced for people to study [1]. In order to make the established models suitable to the environment and reality, people take their best to construct models by considering more conditions [2-3]. So people have to propose more and more complex models. Lots of methods and results of mathematical ecology have been improved and progressed [4].

The Lotka-Volterra ecological system is probably the best known mathematical model of mathematical ecology. It is described by the following system

\[ \dot{x}_i = x_i(a_{i0} + \sum_{j=1}^{n} a_{ij}x_j) \quad i = 1, 2, ..., n \]  

where \( a_{i0} (i = 1, 2, ..., n) \) are the growth rate of the species, \( a_{ij} (i = 1, 2, ..., n) \) are the coefficients of species interaction.

With the development of science and technology, people are more and more concerned with the durative development of humans and their environment. But in certain circumstance, because of the limit of food and water, species which are of disadvantage in competition system will extinct. And the change of external environment can also lead this result. So the control problems are considered to the ecological. Since we do not have sufficient information about our phenomenon, it is difficult to use the conventional control.


But in practical ecosystem, due to the complexity of the structure uncertainties of the environment, the membership function may not be easily obtained and the rules were uncertain. And type-1 FLSs are unable to handle rule uncertainties directly, when the information that is used to construct the rules in a FLS is uncertain. On the other hand, type-2 fuzzy sets are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set. The concept of type-2 fuzzy sets was first introduced in [9] as an extension of the well-known ordinary fuzzy sets, the type-1 fuzzy sets. A T2FS is characterized by a fuzzy membership function; i.e., the membership grade for each element is also a fuzzy set in \([0,1]\) [10]. The membership function of T2FSs are three dimensional and include a footprint of uncertainty(FOU), which is a new third dimension of T2FSs, and the FOU provides an additional degree of freedom to make it possible to directly model and handle uncertainties [11]. The T2FS are useful especially when it is difficult to determine the exact and precise membership function [12]. In this paper, without the rigorous mathematical model, we
propose a type-2 adaptive fuzzy control approach to investigate Lotka-Volterra competitive system, then we can control the balance of ecological system [13-15].

This paper is organized as follows. A brief of interval type-2 fuzzy logic system is presented in section 2. Section 3 presents the design of an adaptive type-2 fuzzy controller. In section 4, we will apply the fuzzy controller to an ecological system and simulation can be worked out by computer with the membership function of the fuzzy sets of the universe of discourse. And conclusion is given in section 5.

2 Interval type-2 fuzzy logic system

Consider a type-2 FLS having 2 inputs \( x_1, x_2 \) and one output \( y \). The type-2 fuzzy rule base consists of a collection of IF-THEN rules as in the type-1 case. We assume there are \( n \) rules and the rule of a type-2 relation between the input space and output space can be expressed as:

\[
R^i : \text{if } x_1 \text{ is } F^i_1 \text{ and } x_2 \text{ is } F^i_2, \text{ then } y \text{ is } G^i, \quad i = 1, 2, ..., n,
\]

where \( F^i_1, F^i_2 \) are antecedent type-2 sets and \( G^i \) are consequent type-2 sets.

There are many kinds of type reduction, such as centroid, height, modified weight and center-of-sets[16]. The center-of-sets type reduction will be used in this paper and can be expressed as

\[
Y_{\text{cos}} = [y_l, y_r] = \left[ \int_{y_l}^{y_r} \int_{y_l}^{y_r} \int_{x_1}^{x_2} \int_{x_1}^{x_2} 1 / \sum_{i=1}^{n} f^i_y y^i \right] (2)
\]

where \( Y_{\text{cos}} \) is the interval set determined by two end points \( y_l \) and \( y_r \), and \( f^i \) is the interior type-2 FLS with singleton fuzzification and meet under minimum or product t-norm \( \mu \) and \( \nu \) can be obtained as:

\[
f^i = \mu_{F^i_1}(x_1) \ast \mu_{F^i_2}(x_2)
\]

(3)

\[
\hat{f}^i = \bar{\mu}_{F^i_1}(x_1) \ast \bar{\mu}_{F^i_2}(x_2)
\]

(4)

Also, \( y^i \in Y^i \) and \( Y^i = [y_l^i, y_r^i] \) are the centroid of the type-2 interval consequent set \( \hat{G}^i \), the centroid of a type-2 fuzzy set. For any value \( y \in Y_{\text{cos}} \), \( y \) can be expressed as:

\[
y = \frac{\sum_{i=1}^{n} f^i y^i}{\sum_{i=1}^{n} f^i}
\]

(5)

where \( y \) is a monotonic increasing function with respect to \( f^i \), also, \( y_l \) is the minimum associated only with \( f^i_l \), and \( y_r \) is the maximum associated only with \( f^i_r \). Note that \( y_l \) and \( y_r \) depend only on mixture of \( f^i \) or \( \hat{f}^i \) values. Therefore, the left-most point \( y_l \) and the right-most point \( y_r \) can be expressed as a fuzzy basis function expansion, i.e.

\[
y_l = \frac{\sum_{i=1}^{n} f^i_l y^i_l}{\sum_{i=1}^{n} f^i_l} = \sum_{i=1}^{n} y^i_l \xi^i_l
\]

(6)

\[
y_r = \frac{\sum_{i=1}^{n} f^i_r y^i_r}{\sum_{i=1}^{n} f^i_r} = \sum_{i=1}^{n} y^i_r \xi^i_r
\]

(7)

respectively, where \( \xi^i_l = f^i_l / \sum_{i=1}^{n} f^i_l \) and \( \xi^i_r = f^i_r / \sum_{i=1}^{n} f^i_r \).

Let \( \xi = [\xi^1_l, \xi^2_l, ..., \xi^n_l] \), \( \xi = [\xi^1_r, \xi^2_r, ..., \xi^n_r] \), \( y_l = [y^1_l, y^2_l, ..., y^n_l]^T \), \( y_r = [y^1_r, y^2_r, ..., y^n_r]^T \), then \( y_l \) and \( y_r \) can be rewritten

\[
y_l = \frac{\sum_{i=1}^{n} f^i_l y^i_l}{\sum_{i=1}^{n} f^i_l} = \sum_{i=1}^{n} y^i_l \xi^i_l = y_l^T \xi_l
\]

(8)

and

\[
y_r = \frac{\sum_{i=1}^{n} f^i_r y^i_r}{\sum_{i=1}^{n} f^i_r} = \sum_{i=1}^{n} y^i_r \xi^i_r = y_r^T \xi_r.
\]

(9)

For illustrative purposes, we briefly provide the computation procedure for \( y_r \). Without loosing of generality, assume the \( y^i_r \) are arranged in ascending order, i.e. \( y^1_r \leq y^2_r \leq ... \leq y^n_r \).

IJNS homepage: http://www.nonlinearscience.org.uk/
Step 1: Compute $y_r$ in (9) by initially setting $f_i^1 = (\bar{f}_i + f_i^1)/2$ for $i = 1, 2, \ldots, n$, where $\bar{f}_i$ and $f_i^1$ have been pre-computed by (3) and (4) and let $y'_r = y_r$.

Step 2: Find $R$ ($1 \leq R \leq n - 1$) such that $y_R^R \leq y'_r \leq y_R^{R+1}$.

Step 3: Compute $y_r$ in (9) with $f_i^1 = f_i^1$ for $i \leq R$ and $f_i^2 = f_i^1$ for $i > R$ and let $y''_r = y_r$.

Step 4: If $y''_r \neq y'_r$, then go to Step 5; if $y''_r = y'_r$, then stop and set $y_r = y''_r$.

Step 5: Set $y_r$ equal to $y''_r$ and return to step 2.

The point to separate two sides by number $R$ can be decided from the above algorithm, one side using lower firing strengths $f_i^1$'s and another side using upper firing strengths $f_i^2$'s. Therefore, the $y_r$ in (9) can be rewritten as

$$y_r = \sum_{i=1}^{R} \frac{f_i^1 y_i^1}{L} + \sum_{i=R+1}^{n} \frac{f_i^1 y_i^2}{L} = \sum_{i=1}^{R} \frac{f_i^1 y_i^1}{L} + \sum_{i=R+1}^{n} \frac{f_i^1 y_i^2}{L} = \xi^T \Theta_r$$

where $P^1_r = \frac{f_i^1}{D_r}, P^1_r = \frac{f_i^1}{D_r}$ and $D_r = \sum_{i=1}^{R} f_i^1 + \sum_{i=R+1}^{n} f_i^2$.

In the meantime, we have $P^1_r = \frac{f_i^1}{D_r}, P^1_r = \frac{f_i^1}{D_r}$.

Let us define tracking errors as $e = y_{1m} - x_1$. So we have the following ideal controller:

$$\mu^*(X) = -x_1 f(X) + \dot{x}_{1m} + ke$$

$$\dot{x}_i = x_i (a_i + b_i x_i + c_i x_2) + u_i (x_1, x_2)$$

$$\dot{x}_2 = x_2 (a_2 + b_2 x_2 + c_2 x_1)$$

(13)
where $k$ is positive constants and $f (X) = a_1 + b_1 x_1 + c_1 x_2$. It is easy for us to verify
\[ e \to 0, \quad t \to \infty. \]

The optimal parameters can be expressed as
\[
\theta^* = \arg \min_{\theta \in \mathbb{R}^n} \left[ \sup_{X \in \mathbb{R}^2} |\mu (X | \theta) - \tilde{\mu}^* (X)| \right]
\]

We denote minimum approximation error
\[ \omega = \tilde{\mu} (X | \theta^*) - \tilde{\mu}^* (X) \]

Through simple calculation, we have
\[
\dot{e} = -ke + \tilde{\mu} (X | \theta^*) - \tilde{\mu} (X | \theta) + \tilde{\mu}^* (X) - \tilde{\mu} (X | \theta^*) = -ke + \tilde{\mu} (X | \theta^*) - \tilde{\mu} (X | \theta) - \omega
\]
\[
\dot{e} = -ke - \omega + \theta^T \xi (X) - \theta^T \xi (X) = -ke + \omega + \theta^T \xi (X)
\]
\[
\dot{e} = -ke - \omega + \frac{\partial^T \xi_t (X) + \partial^T \xi_r (X)}{2} = -ke - \omega + \frac{1}{2} \theta^T \xi_t (X) + \frac{1}{2} \theta^T \xi_r (X)
\]

where $\theta_t = \theta_t^* - \theta_t$ and $\theta_r = \theta_r^* - \theta_r$.

Next, the Lyapunov function is designed as:
\[
V (t) = \frac{1}{2} \left( e^2 + \frac{1}{2r_1} \theta^T \theta_t + \frac{1}{2r_2} \theta^T \theta_r \right)
\]

where $\lambda$ is positive constant. Differentiating $V$, we have
\[
\dot{V} (t) = e \dot{e} + \frac{1}{2r_1} \theta^T \dot{\theta}_t + \frac{1}{2r_2} \theta^T \dot{\theta}_r
\]
\[
\dot{V} (t) = e \left( -ke - \omega + \frac{1}{2} \theta^T \xi_t (X) + \frac{1}{2} \theta^T \xi_r (X) \right) + \frac{1}{2r_1} \theta^T \dot{\theta}_t + \frac{1}{2r_2} \theta^T \dot{\theta}_r
\]
\[
\dot{V} (t) = -ke^2 - e \omega + \frac{1}{2r_1} \theta^T \left[ \theta_t + r_1 e \xi_t (X) \right] + \frac{1}{2r_2} \theta^T \left[ \theta_r + r_2 e \xi_r (X) \right]
\]

where $\dot{\theta}_t = -\dot{\theta}_r = r_2 e \xi_r (x)$ and $\dot{\theta}_t = -\dot{\theta}_t = r_1 e \xi_t (x)$, substitute(15) and (16) into (22) and we obtain
\[
\dot{V} = -ke^2 - e \omega < 0.
\]

## 4 Application for an ecological system

The realistic ecological model is given as follows:

\[
\begin{align*}
\frac{dx_1}{dt} &= x_1 (t) (5 \sin 2\pi t - 2x_1 (t) - 3x_2 (t)) \\
\frac{dx_2}{dt} &= x_2 (t) (4 + 2 \cos 20t - 4x_2 (t) - x_1 (t))
\end{align*}
\]

From the dynamic behavior of system (24) in Figure.1 (given initial condition $x_1 (0) = x_2 (0) = 1, x_1 \in [0, 1], x_2 \in [0.5, 1.5]$), we can see the system is not persistent.
Under the ecological system, the trajectory \( x_{1m} \) is desired as \( x_{1m}(t) = 0.3 + 0.5 \sin t \).

For simplification, we take the following fuzzy rules:

\[ R^i: \text{If } x_1 \text{ is } \tilde{A}_1^i \text{ and } x_2 \text{ is } \tilde{A}_2^i, \text{ Then } \tilde{C}^i, \quad i = 1, 2, 3 \]

where \( \tilde{A}_1^i, \tilde{A}_2^i, \tilde{C}^i \) are fuzzy set, and the corresponding membership function as follows:

\[
\mu_{\tilde{A}_j^i}(x_i) = \exp \left\{ -\frac{1}{2} \left( \frac{x_i - m_j^i}{\delta_j^i} \right)^2 \right\} \quad i = 1, 2, \quad j = 1, 2, 3
\]

By using the singleton fuzzifier, product engine and center-average defuzzifier, we have the fuzzy mapping function

\[
\mu(X | \theta) = \frac{1}{2} \left[ \xi_\theta^{T} \xi_\theta \right] \theta_r = \xi^{T} \theta.
\]

So the corresponding fuzzy control model is the following:

\[
\begin{align*}
\frac{dx_1}{dt} &= x_1(t) (5 \sin 2\pi t - 2x_1(t) - 3x_2(t)) + \mu(X | \theta) \\
\frac{dx_2}{dt} &= x_2(t) (4 + 2 \cos 20t - 4x_2(t) - x_1(t))
\end{align*}
\]

The design algorithm for type-2 adaptive fuzzy control is proposed as follows.

Step 1: Define the type-2 membership functions as:

\[
\begin{align*}
\mu_{\tilde{A}_1^1}(x_1) &= e^{-\left(\frac{x_1 - 0.3(1\pm 0.1)}{10}\right)^2} \\
\mu_{\tilde{A}_1^2}(x_1) &= e^{-\left(\frac{x_1 - 0.7(1\pm 0.1)}{10}\right)^2} \\
\mu_{\tilde{C}_1}(x_1) &= e^{-\left(\frac{x_1 - 1.5(1\pm 0.1)}{10}\right)^2} \\
\mu_{\tilde{A}_2^1}(x_2) &= e^{-\left(\frac{x_2 - 0.5(1\pm 0.1)}{10}\right)^2} \\
\mu_{\tilde{A}_2^2}(x_2) &= e^{-\left(\frac{x_2 - 1.5(1\pm 0.1)}{10}\right)^2} \\
\mu_{\tilde{C}_2}(x_2) &= e^{-\left(\frac{x_2 - 1.5(1\pm 0.1)}{10}\right)^2}
\end{align*}
\]

\[
\xi_l = \left[ \frac{\mu_{\tilde{A}_1^1}}{A}, \frac{\mu_{\tilde{A}_1^2}}{A}, \frac{\hat{\mu}_{\tilde{A}_2^1}}{A}, \frac{\hat{\mu}_{\tilde{A}_2^2}}{A}, \frac{\mu_{\tilde{A}_1^1} * \mu_{\tilde{A}_2^1}}{A}, \frac{\mu_{\tilde{A}_1^2} * \mu_{\tilde{A}_2^2}}{A}, \frac{\mu_{\tilde{A}_1^1} * \hat{\mu}_{\tilde{A}_2^1}}{A}, \frac{\mu_{\tilde{A}_1^2} * \hat{\mu}_{\tilde{A}_2^2}}{A} \right].
\]
\[
A = \left[ \frac{\bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2}}{A}, \frac{\bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2}}{A}, \frac{\bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2}}{A} \right]
\]

\[
B = \bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2} + \bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2} + \bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2} + \bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2} + \bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2} + \bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2} + \bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2} + \bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2}
\]

\[
\xi_r = \left[ \frac{\bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2}}{B}, \frac{\bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2}}{B}, \frac{\bar{\mu}_{A_1} \cdot \bar{\mu}_{A_2}}{B} \right]
\]

\[
\xi_l = [\xi_l^1, \xi_l^2, ..., \xi_l^9]^T \quad \xi_r = [\xi_r^1, \xi_r^2, ..., \xi_r^9]^T
\]

Step 2: specify positive design parameters as follows:

\[
k = 0.5, \quad r_1 = r_2 = 1
\]

The adaptive laws are designed as:

\[
\dot{\theta}_l = e \xi_l (X)
\]

\[
\dot{\theta}_r = e \xi_r (X)
\]

Linearized feedback control law can be written as:

\[
\hat{\mu} (X | \theta) = \frac{1}{2} [\hat{\mu}_l (X | \theta_l) + \hat{\mu}_r (X | \theta_r)]
\]

\[
\hat{\mu}_l = \xi_l^T \theta_l \quad \hat{\mu}_r = \xi_r^T \theta_r
\]

Figure 2 shows the state variable output with type-2 adaptive fuzzy control. We can obtain that the Lotak-Volterra system attains sustainable balance under the type-2 adaptive fuzzy control.

5 Conclusion

In this paper, we design a type-2 adaptive fuzzy controller to guarantee persistence of an unstable L-V competitive system. Simulation shows that our fuzzy controller is a powerful tool to attain sustainable balance of ecology. A type-2 adaptive fuzzy controller would be very useful in sustainable development of the world’s population and ecology.

Acknowledgements

The research was supported by the Natural Science Foundation of China (No. 11072090).
Figure 2: The extinction of Lotak-Volterra system with type-2 fuzzy control

References


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