Projective Lag Synchronization of Delayed Chaotic Systems with Parameter Mismatch via Intermittent Control

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Abstract: In this paper, the projective lag synchronization of delayed chaotic systems with parameter mismatch is discussed. Because of parameter mismatch and projective factor, complete projective lag synchronization cannot be achieved. We shall show that the coupled systems can be in a state of projective lag synchronization with a small error level. Several criteria are derived and the error level is estimated by rigorously theoretical analysis. Numerical simulations are also given to verify the analytical approach.

Keywords: Delayed chaotic systems; Parameter mismatch; Intermittent control; Projective lag synchronization

1 Introduction

During the past few decades, synchronization of chaos systems has attracted considerable interest and received massive studies since the pioneering work of Pecora and Carroll [1]. By now several strategies for synchronization of chaotic systems have been presented, including generalized synchronization [2], phase synchronization [3], anticipating synchronization [4], lag synchronization [5,6,7], complete synchronization [8], projective synchronization [9,10], as well as exponential synchronization [11]. Among synchronization strategies, projective synchronization means that the state variables of the drive and response systems evolve in a constant scaling factor \( \lambda \). On the other side, lag synchronization is described as the accordance of the states of drive-response chaotic systems in which one of the systems is delayed by a finite time. In addition, many types of effective methods for synchronization of chaotic systems have been proposed, such as adaptive control [12], feedback control [13], intermittent control [14,15,21,22] and impulsive control [16]. Compared to continuous control of chaos, the discontinuous control methods including intermittent control and impulsive control may be more cost effective.

In this paper, we investigate the projective lag synchronization of delayed chaotic systems with parameter mismatch. There are related works in the field of synchronization with parameter mismatch. Chen and Cao in [17] studied projective synchronization of neural networks with mixed time-varying delays and parameter mismatch. The author in [18] investigated lag quasisynchronization of coupled delayed systems with parameter mismatch by periodically intermittent control. The author in [19] studied synchronization of delayed chaotic systems with parameter mismatches by using intermittent linear state feedback, which is contained in this paper. This work studies the projective lag synchronization of delayed chaotic systems, and proposed more complex criterions, so the proof process is quite more difficult, thus it will be more complex compared to the work of [19].

However, to the best of the authors’ knowledge, there are few works addressing the projective lag synchronization of delayed chaotic systems with parameter mismatch in the literature. Projective lag synchronization is defined if the state variables of the drive and the response systems, one of which is delayed by a finite time, synchronize up to a constant scaling factor. Projective lag synchronization includes the projective synchronization where the delayed-time \( \alpha = 0 \), the lag synchronization where the scaling factor \( \lambda = 1 \), and quasi-synchronization where \( \alpha = 0 \) and \( \lambda = 1 \) at the same time.

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Thus there will be many situations if we modify the parameters in our work, such as the paper [19]. In addition to, it makes more meaningful because it has a more ordinary form of synchronization and can suit for more practical situations. In this paper, we will explore the problem for projective lag synchronization of delayed couple systems with parameter mismatch. Some more complex criterions are proposed to realize projective lag synchronization of delayed couple systems with parameter mismatch up to a relatively small error bound by using Lyapunov stability theory and intermittent control method. In addition, numerical simulations are given to show the effectiveness of the theoretical results.

This paper is organized as follows. In Section 2, the problem of projective lag synchronization of coupled systems with parameter mismatch is formulated. Section 3 establishes some criteria for the projective lag synchronization by intermittent control method. In Section 4, numerical example is given to verify the effectiveness of this method. Finally, some conclusions are given in Section 5.

2 Problem formulation and preliminaries

Consider a class of delay systems defined by the following delay differential equations:

\[ \begin{align*}
\dot{x}(t) &= A_1x(t) + B_1f(x(t)) + C_1g(x(t - \tau)), \quad t > 0, \\
x(t) &= \varphi(t), \quad -\tau \leq t \leq 0,
\end{align*} \tag{1} \]

where \( x \in \mathbb{R}^n \) denotes the state vector, \( A_1, B_1, \) and \( C_1 \in \mathbb{R}^{n \times n} \) are constant matrices, \( \tau \) is non-negative which represents the time delay, \( f, g : \mathbb{R}^n \rightarrow \mathbb{R}^n \) are nonlinear functions satisfying the following Lipschitz condition: There exist positive constants \( L_f, L_g \) such that for all \( x, y \in \mathbb{R}^n, \)

\[ \begin{align*}
\| f(x) - f(y) \| &\leq L_f \| x - y \|, \\
\| g(x) - g(y) \| &\leq L_g \| x - y \|.
\end{align*} \]

In order to realize projective lag synchronization via periodically intermittent feedback control in the framework of the master-slave configuration with system (1), the slave(response) system is designed as

\[ \begin{align*}
\dot{y}(t) &= A_2y(t) + B_2f(y(t)) + C_2g(y(t - \tau)) + u(t), \quad t > 0, \\
y(t) &= \phi(t), \quad -\tau \leq t \leq 0,
\end{align*} \tag{2} \]

where \( y \in \mathbb{R}^n \) denotes the state vector, \( A_2, B_2, \) and \( C_2 \in \mathbb{R}^{n \times n} \) are constant matrices, and \( u(t) \) denotes the intermittent linear state feedback control input defined as follows:

\[ u(t) = \begin{cases} 
  k(\lambda x(t - \alpha) - y(t)), & n\omega \leq t \leq n\omega + \sigma \omega, \\
  0, & n\omega + \sigma \omega \leq t < (n + 1)\omega,
\end{cases} \tag{3} \]

where \( k \) denotes the control strength, \( \lambda \) is the projective factor, \( \sigma \) denotes the switching rate with \( 0 < \sigma < 1, \) \( \omega \) denotes the control period and \( \alpha \) is the transmittal delay. In this paper, we focus on the case of \( A_1 \neq A_2, B_1 \neq B_2, \) and \( C_1 \neq C_2, \) namely, there exists parameter mismatch in the coupled systems. Let \( \Delta A = A_2 - A_1, \Delta B = B_2 - B_1, \) and \( \Delta C = C_2 - C_1 \) denote the parameter mismatch errors. Leting the projective lag synchronization error signal be \( e(t) = y(t) - \lambda x(t - \alpha), \) and subtracting (1) from (2) yield the error dynamical system:

\[ \begin{align*}
\dot{e}(t) &= \dot{y}(t) - \lambda \dot{x}(t - \alpha) \\
&= A_2y(t) + B_2f(y(t)) + C_2g(y(t - \tau)) + u(t) - \lambda(A_1x(t - \alpha) \\
&\quad + B_1f(x(t - \alpha)) + C_1g(x(t - \tau - \alpha))) \tag{4}
\end{align*} \]

Namely,

\[ \begin{align*}
\dot{e}(t) &= (A_2 - KI)e(t) + \Delta A\lambda x(t - \alpha) + B_2(f(y(t)) - f(\lambda x(t - \alpha))) \\
&\quad + C_2(g(y(t - \tau)) - g(\lambda x(t - \tau - \alpha))) + (B_2f(\lambda x(t - \alpha)) \\
&\quad - B_1f(x(t - \alpha))) + (C_2g(\lambda x(t - \tau - \alpha)) \\
&\quad - C_1g(x(t - \tau - \alpha))), \quad n\omega \leq t \leq n\omega + \sigma \omega \\
\dot{e}(t) &= A_2e(t) + \Delta A\lambda x(t - \alpha) + B_2(f(y(t)) - f(\lambda x(t - \alpha))) \\
&\quad + C_2(g(y(t - \tau)) - g(\lambda x(t - \tau - \alpha))) + (B_2f(\lambda x(t - \alpha)) \\
&\quad - B_1f(x(t - \alpha))) + (C_2g(\lambda x(t - \tau - \alpha)) \\
&\quad - C_1g(x(t - \tau - \alpha))), \quad n\omega + \sigma \omega \leq t < (n + 1)\omega 
\end{align*} \tag{5} \]
It is clear that the origin $e = 0$ is not an equilibrium point of the error system (5) because of the presences of the parameter mismatch and the projective factor. However, it is possible to synchronize the master-slave systems up to a considerably small error bound. In this paper, we investigate the projective lag synchronization with error bound $\epsilon$ using the intermittent control.

To obtain the main result in this paper, the following preliminaries are necessary.

**Definition 1** The synchronization schemes (1) and (2) are said to projective lag synchronized with error bound $\epsilon > 0$ if there exist a compact set $\alpha$, $\sigma$, delay time $\tau$ and projective factor $\lambda$ such that, for any initial values $\phi(t), \Theta(t) \in \Omega$, $t \in [-\tau, 0]$, $\Omega$ denote a region of interest in the phase space that contains the chaotic attractor of system (1), the projective lag synchronization error satisfies

$$|| y(t) - \lambda x(t - \alpha) || \leq \epsilon.$$ 

**Lemma 1** For any vectors $x, y \in \mathbb{R}^n$ and a positive-definite matrix $Q \in \mathbb{R}^{n \times n}$, the following matrix inequality holds:

$$2x^T y \leq x^T Q x + y^T Q^{-1} y.$$ 

**3 Main Results**

This section investigates the projective lag synchronization problem of coupled delayed chaotic systems based on Lyapunov method and the intermittent control technique.

**Theorem 2** Let $\Omega = \{x \in \mathbb{R}^n \mid || x || \leq \gamma \}$ be a set containing the chaotic attractors of (1) and (2). Also, suppose that there exist positive scalars $\eta_1 > 0, \eta_2 > 0, \gamma_2 > 0$ and $\beta_i > 0 (i = 1, 2, \cdots, 10)$, such that

$$\begin{align*}
&(a) \ F_1 \leq 0, \\
&(b) \ F_2 \leq 0, \\
&(c) \ F_3 \leq 0, \\
&(d) \ F_4 \leq 0,
\end{align*}$$

(6)

where $r$ is the unique positive root of the equation: $-r = -\eta_1 + \beta_1 I L_2^2 e^{r\tau}, F_1 = A_1^T + A_2 - 2K I + \beta_1 B_1 B_1^T + \beta_1^{-1} L_1^2 I + \beta_2 C_2 C_2^T + \beta_4 I + \beta_5 I + \eta_1 I, F_2 = A_2^T + A_2 - \beta_4 I + \beta_5 B_2 B_2^T + \beta_6 I + \eta_2 I, F_3 = -3 \lambda^2 + \beta_1^{-1} (|| B_2 || + || B_1 ||)^2 || B_3 || + || C_2 || + \gamma_2 \lambda^2 - \gamma_2, F_4 = \beta_2^{-1} ||\Delta A ||^{2} \lambda^2 + \beta_1^{-1} (|| B_2 || + || B_1 ||)^2 || B_3 || + \gamma_2 \lambda^2 - \gamma_2$. Then the synchronization error system (5) converges exponentially to a small region $D$ containing the origin, where

$$D = \{e \in \mathbb{R}^n || || \leq \sqrt{\frac{\psi}{1 - r \psi}} + \frac{\gamma_2 \lambda^2}{r} \},$$

with

$$\psi = \left( \frac{2 \gamma_2^2}{r} + \frac{2 \gamma_2^2}{\eta_2 + \beta_1^{-1} L_2^2} \right) e^{(\eta_2 + \beta_1^{-1} L_2^2)(\omega - \sigma \omega)} - \frac{\gamma_2 \lambda^2}{\eta_2 + \beta_1^{-1} L_2^2},$$

Consequently, the master system (1) and the slave system (2) achieve projective lag synchronized with an error bound

$$\sqrt{\frac{\psi}{1 - r \psi}} + \frac{2 \gamma_2 \lambda^2}{r}.$$

**Proof.** Consider the following Lyapunov function:

$$V(t) = e(t)^T e(t).$$

(7)

When $n \omega \leq t \leq n \omega + \sigma \omega$, the derivative of $V$ with respect to $t$ along the solution of the first subsystem (5) is as follows:

$$\dot{V}(t) = 2e(t)^T \dot{e}(t)$$

$$= e(t)^T (A_1^T + A_2 - 2K I) e(t) + 2e(t)^T \Delta A \lambda x(t - \alpha) + 2e(t)^T B_2(f(y(t))$$

$$- f(\lambda x(t - \alpha))) + 2e(t)^T C_2(g(y(t) - \tau)) - g(\lambda x(t - \tau - \alpha))$$

$$+ 2e(t)^T (B_2 f(\lambda x(t - \alpha)) - B_1 \lambda f(x(t - \alpha)))$$

$$+ C_2 g(\lambda x(t - \tau - \alpha)) - C_1 \lambda g(x(t - \tau - \alpha))),$$

(8)

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Based on Lemma 1, we can obtain:

\[2e(t)^TB_2(f(y(t)) - f(\lambda x(t - \alpha))) \leq \beta_1e(t)^TB_2B_2^T e(t) + \beta_3^{-1}L_2^2 e(t)^T e(t) + e(t)^T(\beta_1B_2B_2^T + \beta_3^{-1}L_2^2 I)e(t).\]

\[2e(t)^T C_2(g(y(t) - \tau) - g(\lambda x(t - \tau - \alpha))) \leq \beta_2 e(t)^T C_2C_2^T e(t) + \beta_3^{-1}L_g^2 e(t - \tau)^T e(t - \tau).\]

\[2e(t)^T \Delta A \lambda x(t) \leq \beta_3 e(t)^T e(t) + \beta_3^{-1}(\| \Delta A \lambda x(t - \alpha) \|^2)(\| \Delta A \lambda x(t - \alpha) \|^2).\]

\[2e(t)^T[B_2, f(\lambda x(t - \alpha)) - B_1] \leq \beta_3 e(t)^T e(t) + \beta_3^{-1}L_2^2 V(t - \tau) + \gamma_2 \gamma_1^2.\]

Substituting these into (11), we are able to obtain following inequality:

\[
\tilde{V}(t) \leq e(t)^TF_1 e(t) - \eta_1 e(t)^T e(t) + \beta_2^{-1}L_2^2 e(t - \tau)^T e(t - \tau) \leq -\eta_1 V(t) + \eta_2 V(t) + \beta_1^{-1}L_2^2 V(t - \tau) + \gamma_2 \gamma_1^2.
\]

In the same way, when \(n\omega + \sigma \omega \leq t < (n + 1)\omega\), we have

\[
\tilde{V}(t) \leq \eta_2 V(t) + \beta_2^{-1}L_2^2 V(t - \tau) + \gamma_2 \gamma_1^2, \quad n\omega \leq t \leq n\omega + \sigma \omega
\]

\[
\tilde{V}(t) \leq \eta_2 V(t) + \beta_2^{-1}L_2^2 V(t - \tau) + \gamma_2 \gamma_1^2, \quad n\omega + \sigma \omega \leq t < (n + 1)\omega.
\]

Therefore,

\[
\begin{align*}
V(t) &\leq -\eta_1 V(t) + \beta_2^{-1}L_2^2 V(t - \tau) + \gamma_2 \gamma_1^2, \quad n\omega \leq t \leq n\omega + \sigma \omega \\
V(t) &\leq \eta_2 V(t) + \beta_2^{-1}L_2^2 V(t - \tau) + \gamma_2 \gamma_1^2, \quad n\omega + \sigma \omega \leq t < (n + 1)\omega
\end{align*}
\]

Using Lemma 3 in [19] and Lemma 4 in [19], we can obtain

\[
\begin{align*}
V(t) &\leq \| V(n\omega)\|_r e^{-r(t-n\omega)} + \frac{2\gamma_2^2}{\eta_2 + \beta_1^{-1}L_2^2}, \quad n\omega \leq t \leq n\omega + \sigma \omega \\
V(t) &\leq \| V(n\omega + \sigma \omega)\|_r e^{-r\sigma \omega} + \frac{2\gamma_2^2}{\eta_2 + \beta_1^{-1}L_2^2} e^{r(n\omega + \sigma \omega) - r(n\omega - \omega)}
\end{align*}
\]

where \(r\) is the unique root of the equation: \(-r = -\eta_1 + \beta_2^{-1}L_2^2 e^r\).

Based on Lemma 5 in [19], we have

\[
\| e(t) \|^2 = V(t) \leq \| V(0)\|_r e^{-r\tau} + \frac{\psi}{1-e^{-r}} + \frac{2\gamma_2^2}{\eta_2 + \beta_1^{-1}L_2^2}, \quad \text{for } t > 0,
\]

where \(\| V(0)\|_r = \max_{-\tau \leq t \leq 0} |V(t)| = \max_{-\tau \leq t \leq 0} (\psi(t) - \phi(t - \alpha))^2 (\psi(t) - \phi(t - \alpha)), \)

and

\[
p = r(\sigma \omega - \tau) - (\eta_2 + \beta_1^{-1}L_2^2) (\omega - \sigma \omega),
\]

\[
\psi = \left(\frac{\gamma_2^2}{r} + \frac{2\gamma_2^2}{\eta_2 + \beta_1^{-1}L_2^2}\right) e^{r(n\omega + \sigma \omega) - r(n\omega - \omega)} - \frac{2\gamma_2^2}{\eta_2 + \beta_1^{-1}L_2^2}.
\]

Thus,

\[
\| e(t) \| \leq \sqrt{\| V(0)\|_r e^{-r\tau} + \frac{\psi}{1-e^{-r}} + \frac{2\gamma_2^2}{r} + \sqrt{\| V(0)\|_r e^{-r\tau} + \frac{\psi}{1-e^{-r}} + \frac{2\gamma_2^2}{r}}}.
\]

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Namely, the projective lag synchronization error system (5) converges exponentially to a small region $D$ containing the origin, where

$$D = \{ e \in R^n \| e \| \leq \sqrt{\frac{\psi}{1-\sigma^2} + \frac{2\gamma^2}{r^2}} \},$$

Thus, the projective lag synchronization between the master system (1) and the slave system (2) is achieved with an error level $\sqrt{\frac{\psi}{1-\sigma^2} + \frac{2\gamma^2}{r^2}}$. The proof is completed. ■

In the following, we now present a numerically tractable projective lag synchronization conditions. Let

$$\beta_1 = \frac{L_f}{\sqrt{\lambda_{max}(B_2^2 B_2^T)}}, \quad \beta_2 = \frac{L_g}{\sqrt{\lambda_{max}(C_2^2 C_2^T)}}, \quad \beta_3 = \beta_8 = 1,$$

and

$$\eta_1^* = 2k - \lambda_M(A_2 + A_2^T) - 2L_f \sqrt{\lambda_M(B_2^2 B_2^T)} - L_g \sqrt{\lambda_M(C_2^2 C_2^T)} - 1,$$

$$\eta_2^* = \lambda_M(A_2 + A_2^T) + 2L_f \sqrt{\lambda_M(B_2^2 B_2^T)} + L_g \sqrt{\lambda_M(C_2^2 C_2^T)} + 1,$$

where $\eta_1^* \geq \eta_1, \eta_2^* \leq \eta_2$.

We can obtain the following corollary from Theorem 1.

**Corollary 3** Suppose that the parameter mismatch satisfies $F_3 \leq 0$. If there exist positive scalars $k$, projective factor $\lambda$ and $\sigma$ satisfying $0 < \sigma < 1$ such that

$$p = r(\sigma \omega - \tau) - (\eta_2^* + L_g \sqrt{\lambda_{max}(C_2^2 C_2^T)}) (\omega - \sigma \omega) > 0,$$

where $r$ is the unique solution of the equation:

$$-r = -\eta_1^* + L_g \sqrt{\lambda_{max}(C_2^2 C_2^T)} e^{\tau r}.$$  \hspace{1cm} (13)

Then system (1) and (2) are projective lag-synchronized with an error level

$$\sqrt{\lambda_{max}(C_2^2 C_2^T)} (\omega - \sigma \omega),$$

where $\zeta = (\xi + \mu) c \eta_2^* + L_g \sqrt{\lambda_{max}(C_2^2 C_2^T)} (\omega - \sigma \omega) - \mu$, $\xi = \frac{\gamma^2 \eta_1^*}{\eta_2^* + L_g \sqrt{\lambda_{max}(C_2^2 C_2^T)}}$ and $\mu = \frac{\gamma^2 \eta_1^*}{\eta_2^* + L_g \sqrt{\lambda_{max}(C_2^2 C_2^T)}}$.

**Remark 4** If the $\tau$ and $\omega$ are given, one can determine the feasible region $M$ of control parameters $(k, \sigma)$ from Corollary 1.

$$r^* = (\eta_2^* + L_g \sqrt{\lambda_{max}(C_2^2 C_2^T)}) (\omega - \sigma \omega) / (\sigma \omega - \tau).$$

where $r^* < r$.

Then the control strength $k$ can be estimated as follows:

$$k > k^* = \frac{1}{2} \lambda_M(A_2 + A_2^T) + L_f \sqrt{\lambda_M(B_2^2 B_2^T)} + \frac{1}{2} L_g \sqrt{\lambda_M(C_2^2 C_2^T)} + \frac{1}{2} \left( r^* + L_g \sqrt{\lambda_M(C_2^2 C_2^T)} e^{\tau^* r} \right).$$

From (15), we can determine the feasible region $M$ of control parameters $(k, \sigma)$,

$$M = \{(k, \sigma) | k > k^* = \frac{1}{2} \lambda_M(A_2 + A_2^T) + L_f \sqrt{\lambda_M(B_2^2 B_2^T)} + \frac{1}{2} L_g \sqrt{\lambda_M(C_2^2 C_2^T)} + \frac{1}{2} \left( r^* + L_g \sqrt{\lambda_M(C_2^2 C_2^T)} e^{\tau^* r} \right), 0 < \sigma < 1 \}.$$

**Remark 5** Notice that when projector factor $\lambda = 1$ and $\lambda = -1$, the projective lag synchronization scheme used in this paper can be extended to investigate the lag synchronization of delayed chaotic systems with parameter mismatch and lag anti-synchronization of delayed chaotic systems with parameter mismatch respectively.

**Remark 6** If we choose $\alpha = 0$, this model in this paper becomes the projective synchronization of delayed chaotic systems with parameter mismatch.

**Remark 7** Projective lag synchronization becomes quasi-synchronization where $\alpha = 0$ and $\lambda = 1$. With the presence of projective factor and the delay, the synchronization problem can be more complex, and it is quite different from the known paper [19].
4 Numerical example

In this section, we present an example to show the validity of the projective lag synchronization scheme made in the previous section.

Example: Consider the Lu oscillator [20], which is given by following delayed differential equations:

\[
\dot{x}(t) = -Cx(t) + Af(x(t)) + Bg(x(t-1)),
\]

where

\[
C_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 2 & -0.1 \\ -5 & 3.2 \end{pmatrix}, \quad B_1 = \begin{pmatrix} -1.5 & -0.1 \\ -0.18 & -2.5 \end{pmatrix},
\]

and \(f(x(t)) = g(x(t)) = \tanh(x(t))\).

This model was studied by Lu in [20]. Figure 1 shows the chaotic attractor of the Lu oscillator. In this example, we observe that \(\gamma_1 = 4\), \(\lambda = 0\) and \(\tau = 1\). The corresponding slave system is given by

\[
\dot{y}(t) = -Cy(t) + Af(y(t)) + Bg(y(t-1)),
\]

where

\[
C_2 = \begin{pmatrix} 0.99 & 0 \\ 0 & 1.1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 2.1 & -0.1 \\ -5.1 & 3.1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} -1.5 & -0.11 \\ -0.16 & -2.4 \end{pmatrix},
\]

and \(f(y(t)) = g(y(t)) = \tanh(y(t))\).

Note that parameter mismatches satisfy that \(\beta_3^{-1} \| A \| \lambda^2 + \beta_4^{-1} (\| B_2 \| + \| B_1 \|) L_f^2 \lambda^2 + \beta_5^{-1} (\| C_2 \| + \| C_1 \|) L_g^2 \lambda^2 \leq \gamma_2 = 0.023\). Also, we can plot the relationship curve between \(k^*\) and \(\sigma\) after giving \(\omega = 9\), as shown in Figure 2. For numerical simulation, we set \(\omega = 9\), \(\sigma = 0.9\), \(\lambda = 0.9\), \(\alpha = 0.02\), \(k = 40\), and plot the projective lag synchronization error curve, as shown in Figure 3. Consequently, we estimate the error bound is \(D = \{ e \in R^n | \| e \| \leq 0.247 \}\).
Figure 2: Given $\omega = 9$, the relationship between $k^*$ and $\sigma$.

Figure 3: Synchronization error curve with $\omega = 9, \lambda = 0.9, \sigma = 0.9$ and $k = 40$.

5 Conclusions

In this paper, we have studied the effect of parameter mismatch on the projective lag synchronization for coupled chaotic systems by means of periodically intermittent control. A sufficient condition for projective lag synchronization has been established based on Lyapunov stability theory and intermittent control techniques. Numerical simulations have shown the validity of theoretical result.

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