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Exact Solutions for Fractional Nonlinear Evolution Equations by the F-expansion Method

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Abstract: Fractional nonlinear evolution equations have been widely applied for describing various types of physical mechanism in mathematical physics and engineerings. In this paper, we study new and more exact traveling wave solutions of two nonlinear fractional differential equations with the fractional complex transform and the F-expansion method. The fractional complex transform is proposed to convert a nonlinear fractional differential equation into its ordinary differential equation by the F-expansion method along with the Jumarie's modified Riemann-Liouville derivatives. The traveling wave solutions of these two equations are obtained in terms of the hyperbolic, trigonometric, exponential and rational functions. It has been shown that the proposed method is very effectual and easily to find the exact traveling wave solutions to the fractional nonlinear evolution equations.

Keywords: Space-time fractional nonlinear evolution equations; Traveling wave solutions; Modified Riemann-Liouville derivatives; Generalized F-expansion method

1 Introduction

In recent years, fractional calculus has played a very important role in various application areas, such as modeling anomalous diffusion, heat transfer, seismic wave analysis, signal processing, control theory, image processing, and many other fractional dynamical systems [1-6]. Fractional differential equations (FDEs) are the generalized form of classical differential equations of integer order. In recent decades, The FDEs have gained much attention as they are widely used to describe various complex phenomena in the application fields, such as the fluid flow, signal processing, system identification, biology and other areas. The fractional partial differential equations have been investigated by many researchers [7].

Among the investigations for fractional differential equations, to find the exact solutions and approximate solutions of fractional differential equations is an important work. Many powerful and efficient methods have been proposed for solving the fractional differential equations so far, such as the fractional first integral method [8, 9], the fractional sub-equation method [10, 11], the (G'/G)-expansion method [12-14], the improved (G'/G)-expansion method [15], the functional variable method [16], the fractional modified trial equation method [17-19], the extended spectral method [18], the variational iteration method. Based on these methods, solutions with various forms for two given fractional differential equations have been investigated.

Recently, many authors have applied the F-expansion method [5] in various fields, however, the method has been given few solutions to the nonlinear PDEs by the F-expansion method. Chen Jiang [25] has applied the F-expansion method to solve strain wave equation appeared in microstructure solids by considering

$$F'^2(\xi) = q_0 + q_2F^2(\xi) + q_4F^4(\xi)$$

as an auxiliary differential equation. In order to obtain the standard form of this method, we have used the nonlinear differential equation

$$F'^2(\xi) = q_0 + q_1F(\xi) + q_2F^2(\xi) + q_3F^3(\xi) + q_4F^4(\xi)$$

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as an auxiliary equation for finding more comprehensive solutions to nonlinear PDEs [17-20].

In this work, we will modify the $F-$expansion method to solve nonlinear fractional partial differential equations with modified Riemann-Liouville derivative by Jumarie. In this paper, we will furthermore test the validity of $F-$expansion method by applying it to solve other fractional partial differential equations. The fractional derivative is defined in the sense of modified Riemann-liouville derivative [20].

The rest of the paper has been prepared as follows. In Section 2, the improved F-expansion method is discussed in details. Section 3 presents the application of this method to construct the exact traveling wave solutions of the nonlinear FPDEs. Conclusions and the advantages of the method are given in Section 4.

2 The improved F-expansion method

This section presents the improved F-expansion method for finding the exact traveling solutions to the nonlinear FPDEs.

One can consider the nonlinear FPDEs as the following form:

$$f(\mu + D_t^{\alpha}\mu + D_x^{\alpha}\mu + D_x^{\beta}\mu + D_t^{\alpha}D_t^{\beta}\mu + D_x^{\alpha}D_x^{\beta}\mu + ...) = 0, \alpha > 0, \beta < 1, \tag{1}$$

where x is the time variable and t is the space variable. f is a function of $\mu(x,t)$ and its partial fractional derivatives, in which higher order derivatives and nonlinear terms are involved.

Combine the independent variable x and t into $\xi = \frac{kx^{\beta}}{\Gamma(1+\beta)} \pm \frac{ct^{\alpha}}{\Gamma(1+\alpha)}$. Let us consider the traveling wave variable transformation as

$$\mu(x,t) = w(\xi), \xi = \frac{kx^{\beta}}{\Gamma(1+\beta)} \pm \frac{ct^{\alpha}}{\Gamma(1+\alpha)}, \tag{2}$$

where k and c are constants. The Jumarie's modified Riemann-Liouville [21] derivatives of order α can be defined by the following expression:

$$D_z^{\alpha}f(z) = \begin{cases} \frac{1}{\Gamma(1-\alpha)}\frac{d}{dz}\int_0^z (z-\xi)^{-\alpha}(f(\xi)-f(0))d\xi; 0 < \alpha < 1 \\ (f^{(n)}(z))^{\alpha-n}; n \le \alpha \le n+1, n \ge 1. \end{cases} \tag{3}$$

Using Eqs. (2) and (4), Eq. (1) can be converted as a nonlinear ordinary differential equation (ODE) for $u = u(\xi)$:

$$F(u, u', u'', u''', ...) = 0 \tag{4}$$

where F is a function of $u(\xi)$ and its derivatives. One can be considered the traveling wave solutions of Eq. (5) as

$$w(\xi) = \sum_{i=0}^{N} a_i F^i(\xi),$$

where the a_i are constants and satisfy $a_i \neq 0$, while $F(\xi)$ satisfies the following first order nonlinear ordinary differential equation:

$$F'^2(\xi) = q_0 + q_1F(\xi) + q_2F^2(\xi) + q_3F^3(\xi) + q_4F^4(\xi), \tag{5}$$

where q_0, q_1, q_2 are constants. Balancing the higher order derivatives with the nonlinear terms of the highest order that appeared in Eq. (5), one can get the value of the positive integer N. In this paper, we can get the formula (6) by extending Eq.(5) as follows:

$$\begin{cases} F'F = \frac{1}{2}q_1F' + q_2FF' + \frac{3}{2}q_3F^2F' + 2q_4F^3F' \\ F'' = \frac{1}{2}q_1 + q_2F + \frac{3}{2}q_3F^2 + 2q_4F^3 \\ F''' = q_2F' + 3q_3FF' + 6q_4F^2F'. \end{cases} \tag{6}$$

When $q_1 = q_3 = 0$, the solutions of Eq. (6) are in accordance with the solutions solved by the F-expansion method. We can get the solutions of Eq.(5)

$$F(\xi) = \sqrt{-\frac{q_2m^2}{q_4(2m^2-1)}}cn\left(\sqrt{\frac{q_2}{2m^2-1}}\xi\right), q_0 = -\frac{q_2^2m^2(m^2-1)}{q_4(2m^2-1)}, q_2 > 0, \tag{7}$$

$$F(\xi)=\sqrt{-\frac{q_2m^2}{q_4(m^2+1)}}sn\left(\sqrt{-\frac{q_2}{2m^2+1}}\xi\right), q_0=\frac{q_2^2m^2}{q_4(m^2+1)^2}, q_2<0, \tag{8}$$

$$F(\xi)=\sqrt{-\frac{q_2}{q_4(2-m^2)}}dn\left(\sqrt{\frac{q_2}{2-m^2}}\xi\right), q_0=-\frac{q_2^2m^2(m^2-1)}{q_4(2m^2-1)^2}, q_2>0, \tag{9}$$

where $m(0<m<1)$ is the mode of Jacobi elliptic functions (7), (8), (9). And the Jacobi elliptic functions have the following relation:

$$sn^2(\xi)+cn^2(\xi)=1, dn^2(\xi)+m^2sn^2(\xi)=1,$$

$$\frac{d}{d\xi}sn(\xi)=cn(\xi)dn(\xi), \frac{d}{d\xi}cn(\xi)=-sn(\xi)dn(\xi), \frac{d}{d\xi}sn(\xi)=-m^2cn(\xi)sn(\xi).$$

When $q_0=q_1=q_4=0$, we can get the solutions of Eq.(6)

$$F(\xi)=-\frac{q_2}{q_3}sech^2\left(\frac{\sqrt{q_2}}{2}\xi\right), q_2>0, \tag{10}$$

$$F(\xi)=-\frac{q_2}{q_3}(sech^2\left(\frac{\sqrt{-q_2}}{2}\xi\right), q_2<0, \tag{11}$$

and

$$F(\xi)=\frac{1}{q_3\xi^2}, q_2=0. \tag{12}$$

3 Applications of the method

This section presents two examples to illustrate the applicability of the proposed method to solve the space-time FDEs.

3.1 The Space and Time Fractional BBM Equation

Let us consider the space and time fractional BBM equation as follows [22]:

$$D_t^\gamma u+D_x^\sigma u_x+uD_x^\sigma u_x-D_x^{2\sigma}D_t^\gamma u=0. \tag{13}$$

Introducing the following transformation in Eq. (13):

$$\mu(x,t)=w(\xi), \xi=\frac{x^\sigma}{\Gamma(1+\sigma)}-\frac{ct^\gamma}{\Gamma(1+\gamma)}, \tag{14}$$

where c is a constant, and Eq. (13) can be rewritten to the following nonlinear ODE:

$$(1-c)w'+ww'+cw'''=0. \tag{15}$$

Integrating Eq. (15) once and setting the integration constant to zero yields

$$(1-c)w+\frac{1}{2}w^2+cw''=0. \tag{16}$$

Using the balancing principle between w'' and w^2 in Eq. (16) gives $N=2$. Therefore the solution of Eq. (16) can be written as

$$w(\xi)=a_0+a_1F(\xi)+a_2F^2(\xi), \tag{17}$$

where a_0, a_1, a_2 are constants to be determined later and $F(\xi)$ satisfies the auxiliary nonlinear ODE (5).

Substituting Eq. (17) into Eq. (16) and using Eq. (5), the left-hand side of Eq. (16) becomes a polynomial in $F^i(\xi)$. Setting the coefficients of this polynomial to zero yields a system of algebraic equations.

When $q_1 = q_3 = 0$, we can get the equations

$$\begin{cases} a_0 - ca_0 + \frac{1}{2}a_0^2 + 2ca_2q_0 = 0 \\ a_1a_2 + 2ca_1q_4 = 0 \\ a_2 - ca_2 + \frac{1}{2}a_1^2 + 4ca_2q_2 = 0 \\ \frac{1}{2}a_2^2 + 6ca_2q_4 = 0 \\ a_1 - ca_1 + a_1a_0 = 0. \end{cases} \tag{18}$$

Solving the resulting algebraic equations (18), we have

$$c = \frac{1}{1-4q_2}, a_0 = \frac{8q_2 - 32q_2^2 + \sqrt{(8q_2 - 32q_2^2)^2 - 192q_0(-1 + 8q_2 - 16q_2^2)q_4}}{2 - 16q_2 + 16q_2^2}, a_1 = 0, a_2 = \frac{12q_4}{-1+4q_2}. \tag{19}$$

Substituting Eq. (19) into Eq. (17), one can get the following solution of Eq. (13)

$$w(\xi) = \frac{8q_2 - 32q_2^2 + \sqrt{(8q_2 - 32q_2^2)^2 - 192q_0(-1 + 8q_2 - 16q_2^2)q_4}}{2 - 16q_2 + 16q_2^2} + \frac{12q_4}{-1+4q_2}F^2(\xi). \tag{20}$$

Combing the solutions of Eqs. (5), (15), (16), (17) and (20), one can obtain the following explicit solutions to the time fractional BBM equation (13):

$$\begin{aligned} w_1(\xi) =& \frac{8q_2 - 32q_2^2 + \sqrt{(8q_2 - 32q_2^2)^2 - 192q_0(-1 + 8q_2 - 16q_2^2)q_4}}{2 - 16q_2 + 16q_2^2} \\ &+ \frac{12q_4}{-1+4q_2}\left(\sqrt{-\frac{q_2m^2}{q_4(2m^2-1)}}cn\left(\sqrt{\frac{q_2}{2m^2-1}}\xi\right)\right)^2, \end{aligned} \tag{21}$$

where $q_0 = -\frac{q_2^2m^2(m^2-1)}{q_4(2m^2-1)}, q_2 > 0$,

$$\begin{aligned} w_2(\xi) =& \frac{8q_2 - 32q_2^2 + \sqrt{(8q_2 - 32q_2^2)^2 - 192q_0(-1 + 8q_2 - 16q_2^2)q_4}}{2 - 16q_2 + 16q_2^2} \\ &+ \frac{12q_4}{-1+4q_2}\left(\sqrt{-\frac{q_2m^2}{q_4(m^2+1)}}sn\left(\sqrt{-\frac{q_2}{2m^2+1}}\xi\right)\right)^2, \end{aligned} \tag{22}$$

where $q_0 = \frac{q_2^2m^2}{q_4(m^2+1)^2}, q_2 < 0$, and

$$\begin{aligned} w_3(\xi) =& \frac{8q_2 - 32q_2^2 + \sqrt{(8q_2 - 32q_2^2)^2 - 192q_0(-1 + 8q_2 - 16q_2^2)q_4}}{2 - 16q_2 + 16q_2^2} \\ &+ \frac{12q_4}{-1+4q_2}\left(\sqrt{-\frac{q_2}{q_4(2-m^2)}}dn\left(\sqrt{\frac{q_2}{2-m^2}}\xi\right)\right)^2, \end{aligned} \tag{23}$$

where $q_0 = -\frac{q_2^2m^2(m^2-1)}{q_4(2m^2-1)^2}, q_2 > 0$.

When $q_0 = q_1 = q_4 = 0$, we can get the equations

$$\begin{cases} a_0 - ca_0 + \frac{1}{2}a_0^2 = 0 \\ a_1a_2 + 5ca_2q_3 = 0 \\ a_2 - ca_2 + \frac{1}{2}a_1^2 + \frac{3}{2}ca_1q_3 + 4ca_2q_2 = 0 \\ \frac{1}{2}a_2^2 = 0 \\ a_1 - ca_1 + a_1a_0 = 0. \end{cases} \tag{24}$$

Solving the resulting algebraic Eqs. (24), we have

$$a_1 = -3q_3, a_2 = 0, a_0 = 0. \tag{25}$$

Combing the solutions of Eq. (5), (14), (16), (17) and (25), one can obtain the following explicit solutions to the time fractional BBM equation:

$$w_4(\xi) = 3q_2 sech^2\left(\frac{\sqrt{q_2}}{2}\xi\right), q_2 > 0, \tag{26}$$

$$w_5(\xi) = 3q_2 sech^2\left(\frac{\sqrt{-q_2}}{2}\xi\right), q_2 < 0, \tag{27}$$

and

$$w_6(\xi) = -\frac{3}{\xi^2}, q_2 = 0. \tag{28}$$

3.2 The Space-Time Fractional Quadratic Klein-Gordon Equation

We consider the space and time fractional Quadratic Klein-Gordon Equation as follows [23]:

$$D_t^{2\alpha}u - D_x^{2\alpha}u + \gamma u - \beta u^2 = 0, \tag{29}$$

where x is the spatial variable and t represents time variable.The nonlinear fractional Klein-Cordon equation describes many types of nonlinearities. On the other hand, the Klein-Cordon equation plays a significant role in several real world applications, for instance, nonlinear optics, quantum field theory, and the solid state physics [24].

We introduce the following transformation

$$\mu(x,t) = w(\xi), \xi = \frac{kx^\alpha}{\Gamma(1+\alpha)} - \frac{ct^\alpha}{\Gamma(1+\alpha)}, \tag{30}$$

one can rewrite Eq. (29) to the following nonlinear ODE as follows:

$$\gamma w - \beta w^2 + (c^2 - k^2)w'' = 0. \tag{31}$$

According to the balancing principle, the solution of (31) can be written as

$$w(\xi) = a_0 + a_1 F(\xi) + a_2 F^2(\xi), \tag{32}$$

where a_0, a_1, a_2 are constants to be determined later and $F(\xi)$ satisfies the auxiliary nonlinear ODE (5).

Substituting Eq. (30) and Eq. (32) into Eq. (31) and using (5), the left-hand side of Eq. (31) becomes a polynomial in$F^i(\xi)$. Setting the coefficients of this polynomial to zero yields a system of algebraic equations.

When $q_1 = q_3 = 0$, we can get the equations

$$\begin{cases} \gamma a_0 - \beta a_0^2 + 2c^2 a_2 q_0 - 2k^2 a_2 q_0 = 0 \\ -2\beta a_1 a_2 = 0 \\ \gamma a_2 - \beta a_1^2 - 2\beta a_2 q_2 - 4k^2 a_2 q_2 + 4c^2 a_2 q_4 = 0 \\ -\beta a_2^2 + 6c^2 a_2 q_4 - 6k^2 a_2 q_4 = 0 \\ \gamma a_1 - 2\beta a_1 a_0 = 0. \end{cases} \tag{33}$$

Solving the resulting algebraic Eq. (33), so we can get the solutions of Eq. (33)

Case1

$$c = -\frac{1}{2}\sqrt{4k^2 - \frac{\beta\gamma}{\sqrt{\beta^2(q_2^2 - 3q_0q_4)}}}, a_0 = \frac{\lambda - \frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}, a_1 = 0, a_2 = -\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2 - 3q_0q_4)}}, \tag{34}$$

Case2

$$c = \frac{1}{2}\sqrt{4k^2 - \frac{\beta\gamma}{\sqrt{\beta^2(q_2^2 - 3q_0q_4)}}}, a_0 = \frac{\lambda - \frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}, a_1 = 0, a_2 = -\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2 - 3q_0q_4)}}, \tag{35}$$

Case3

$$c = -\frac{1}{2}\sqrt{4k^2 - \frac{\beta\gamma}{\sqrt{\beta^2(q_2^2 - 3q_0q_4)}}}, a_0 = \frac{\lambda + \frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}, a_1 = 0, a_2 = \frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2 - 3q_0q_4)}}, \tag{36}$$

Case4

$$c=\frac{1}{2}\sqrt{4k^2-\frac{\beta\gamma}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}},a_0=\frac{\lambda+\frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta},a_1=0,a_2=\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}. \tag{37}$$

Combing the solutions of Eqs. (5), (34), (31), (32), (34), (35), (36) and (37), one can obtain the following explicit solutions to Eq. (29):

For Case1:

$$w(\xi)=\frac{\lambda-\frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}-\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}F^2(\xi), \tag{38}$$

For Case2:

$$w(\xi)=\frac{\lambda-\frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}-\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}F^2(\xi), \tag{39}$$

For Case3:

$$w(\xi)=\frac{\lambda+\frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}+\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}F^2(\xi), \tag{40}$$

For Case4:

$$w(\xi)=\frac{\lambda+\frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}+\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}F^2(\xi). \tag{41}$$

Combing the solutions of Eqs. (5), (34), (31), (32) and (38), one can obtain the following explicit solutions to Eq. (29):

$$w_1(\xi)=\frac{\lambda-\frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}-\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}\left(\sqrt{-\frac{q_2m^2}{q_4(2m^2-1)}}cn\left(\sqrt{\frac{q_2}{2m^2-1}}\xi\right)\right)^2, \tag{42}$$

where $q_0=-\frac{q_2^2m^2(m^2-1)}{q_4(2m^2-1)}, q_2>0$,

$$w_2(\xi)=\frac{\lambda-\frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}-\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}\left(\sqrt{-\frac{q_2m^2}{q_4(m^2+1)}}sn\left(\sqrt{-\frac{q_2}{2m^2+1}}\xi\right)\right)^2, \tag{43}$$

where $q_0=\frac{q_2^2m^2}{q_4(m^2+1)^2}, q_2<0$, and

$$w_3(\xi)=\frac{\lambda-\frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}-\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}\left(\sqrt{-\frac{q_2}{q_4(2-m^2)}}dn\left(\sqrt{\frac{q_2}{2-m^2}}\xi\right)\right)^2, \tag{44}$$

where $q_0=-\frac{q_2^2m^2(m^2-1)}{q_4(2m^2-1)^2}, q_2>0$.

The solutions of Eq. (39) are as the same as the solutions of Eq. (38).

Combing the solutions of Eq. (5), (34), (31), (32) and (40), one can obtain the following explicit solutions to Eq. (29):

$$w_4(\xi)=\frac{\lambda+\frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}+\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}\left(\sqrt{-\frac{q_2m^2}{q_4(2m^2-1)}}cn\left(\sqrt{\frac{q_2}{2m^2-1}}\xi\right)\right)^2, \tag{45}$$

where $q_0=-\frac{q_2^2m^2(m^2-1)}{q_4(2m^2-1)}, q_2>0$,

$$w_5(\xi)=\frac{\lambda+\frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta}+\frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}\left(\sqrt{-\frac{q_2m^2}{q_4(m^2+1)}}sn\left(\sqrt{-\frac{q_2}{2m^2+1}}\xi\right)\right)^2, \tag{46}$$

where $q_0 = \frac{q_2^2 m^2}{q_4(m^2+1)^2}, q_2 < 0$, and

$$w_6(\xi) = \frac{\lambda + \frac{\beta\gamma q_2}{\sqrt{\beta^2(q_2^2-3q_0q_4)}}}{2\beta} + \frac{3\gamma q_4}{2\sqrt{\beta^2(q_2^2-3q_0q_4)}}\left(\sqrt{-\frac{q_2}{q_4(2-m^2)}}dn\left(\sqrt{\frac{q_2}{2-m^2}}\xi\right)\right)^2, \quad (47)$$

where $q_0 = -\frac{q_2^2 m^2(m^2-1)}{q_4(2m^2-1)^2}, q_2 > 0$.

The solutions of Eq. (41) are as the same as the solutions of the Eq. (40).

4 Discussion and conclusion

In this paper, we successfully used the F-expansion method for solving fractional partial differential equations with Jumarie's modified Riemann-Liouville derivative and applied the modified F-expansion method to find exact solutions of the space and time fractional BBM equation and the space-time fractional Quadratic Klein-Gordon Equation. The solutions to the considered equations may be useful for the further analysis, such as stability analysis and mathematical Physics.

The main advantage of the F-method is that the method offers more general and huge amount of new exact traveling wave solutions with some free parameters. All the solutions obtained by the F-expansion method are given by the applied method, and in addition one can be obtained some new explicit and exact solutions. The exact solutions have its extensive potentiality to interpret the inner structures of the natural phenomena arises in mathematical physics, chemistry and biology or any natural varied instances.

Finally, it is worth mentioning that the implementation of the modified F-expansion method is very reliable and efficient, and it can also be applied to solve other fractional differential equations. In future, the plan is to study the numerical simulations for these equations along with other methods. Those results will be reported in future publications.

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