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Estimation of Parameters in the Generalized Logistic Distribution Based on Ranked Set Sampling

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Abstract: Logistic function, was used as a growth curve for the first time, by Vrhust, in the eighteenth century. Using of the logistic distribution curve for economic purposes was popular from the late nineteenth century. In later years the applications of this model as a model of growth in human populations and environmental issues were also achieved. The survival data analysis, data modeling of agricultural production and income distributions can be noted another interesting application of the Logistic function. Because of the importance of this distribution, we need optimal estimators for the parameters of this distribution. In this paper, we estimate the parameters of the logistic distribution based on Simple Random Sampling, and Ranked Set Sampling, and we will compare these two methods.

Keywords: Generalized Logistic Distribution; Order Statistics; Simple Random Sampling; Ranked Set Sampling; Maximum Likelihood Estimation

1 Introduction

Ranked Set Sampling (RSS) method was used to estimate the population mean by McIntyre for the first time in 1952 [10]. Takahasi and Wakimoto [14] studied unbiased estimators for the population mean based on RSS in 1968. The theories and applications of RSS has expressed by authors such as Dell (1972) [7]. In 1980, Stokes [13] estimated the population variance using of the data from RSS method. In later years, RSS was used to estimate the involved parameters in the statistical distributions. In 1995, Chuiv and Sinha [11] studied the estimator of the location parameter of the Cauchy distribution, and Lam and Sinha estimated the parameters of two-parameter Exponential distribution with using of RSS. Lam et al (1995) [9] studied the estimators for location and scale parameters of the Logistic distribution, Bhoj and Ahsanullah (1996) [5] estimated the parameters of the Generalized Geometric distribution based on RSS. Adetia (2000) [1] achieved estimating the parameters of the Half Logistic distribution, and Walid (2004) [16] studied the estimators of parameters of the two-parameter Logistic distribution with deta of RSS. The method of RSS was used for estimating the parameters of Bivariate Pareto distribution by Chacko and Thomas (2007) [6], and for the parameters of Bivariate Exponential distribution by AL-Saleh and Diab (2009) [3], and also for the parameter of Wibull Modified distribution by AL-Hadhrami (2010) [2].

In this paper, we will introduce the Generalized Logistic distribution. And we will study the maximum likelihood estimate (MLE) of the parameters of Logistic distribution based on a Simple Random Sample (SRS). Then we will describ the RSS, and we will study the estimation of the parameters based on the RSS, and finally we will compare these two sampling methods.

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1.1 Definition

If X is a random variable with the following cumulative distribution function (cdf):

$$F(x) = \frac{1}{(1+e^{-\frac{x-a}{b}})^r}\ ; \quad -\infty < x < \infty,\ a \in \mathbf{R},\ b > 0,\ r > 0. \tag{1}$$

Then X has the Generalized Logistic distribution with location parameter a, scale parameter b, and shap parameter r $(GL(a,b,r))$. Its density function (df) is as follows:

$$f(x) = \frac{re^{-\frac{x-a}{b}}}{b(1+e^{-\frac{x-a}{b}})^{r+1}}; \quad -\infty < x < \infty\ , a \in \mathbf{R},\ b > 0,\ r > 0. \tag{2}$$

We need optimal estimators for the parameters of this distribution. In the beginning we estimate the parameters of the distribution with the maximum likelihood method based on a SRS of size n.

2 Maximum Likelihood Estimation

Random sample $X_1, X_2, ..., X_n$ with $GL(a,b,r)$ has the likelihood function, and the log-likelihood function as follows:

$$L(\underline{x}; a, b, r) = \prod_{i=1}^{n} \frac{re^{-\frac{x_i-a}{b}}}{b(1+e^{-\frac{x_i-a}{b}})^{r+1}}\ . \tag{3}$$

$$L^* = \ln(L(\underline{x}; a, b, r)) = n\ln r - n\ln b - \sum_{i=1}^{n}\frac{x_i-a}{b} - (r+1)\sum_{i=1}^{n}\ln(1+e^{-\frac{x_i-a}{b}}). \tag{4}$$

We derive from the log-likelihood function ratio to the parameters a, b, and r. then we put them equal to zero:

$$\frac{\partial L^*}{\partial a} = \frac{n}{b} - \frac{r+1}{b}\sum_{i=1}^{n}\frac{e^{-\frac{x_i-a}{b}}}{1+e^{-\frac{x_i-a}{b}}} = 0. \tag{5}$$

$$\frac{\partial L^*}{\partial b} = \frac{-n}{b} + \sum_{i=1}^{n}\frac{x_i-a}{b^2} - \frac{r+1}{b^2}\sum_{i=1}^{n}\frac{(x_i-a)e^{-\frac{x_i-a}{b}}}{1+e^{-\frac{x_i-a}{b}}} = 0. \tag{6}$$

$$\frac{\partial L^*}{\partial r} = \frac{n}{r} - \sum_{i=1}^{n}(1+e^{-\frac{x_i-a}{b}}) = 0. \tag{7}$$

From (2.5) we have:

$$\hat{r} = \frac{n}{\sum_{i=1}^{n}(1+e^{-\frac{x_i-a}{b}})}. \tag{8}$$

The likelihood function for location parameter, is maximum where the location parameter is negative infinity. So a non-finite estimator is achieved for the location parameter. For this reason in many cases we consider the location parameter zero, and then estimate the scale and shape parameters. If we put $\hat{r}$ in the first likelihood function, we will have:

$$L(a,b) = \frac{na}{b} - \sum_{i=1}^{n}(1+e^{-\frac{x_i-a}{b}}) - n\ln\sum_{i=1}^{n}(1+e^{-\frac{x_i-a}{b}}) + H(b,x). \tag{9}$$

$H(b,x)$ is a function which does not depend on a and b. $H(b,x)$ will be maximum where a is negative infinity. Thus the MLE for the scale and shape parameters is achieved as follows when $a = 0$:

$$\hat{r} = \frac{n}{\sum_{i=1}^{n}(1+e^{-\frac{x_i}{b}})}. \tag{10}$$

$$\frac{\partial L^*}{\partial b} = \frac{-n}{b} + \sum_{i=1}^{n}\frac{x_i}{b^2} - \frac{r+1}{b^2}\sum_{i=1}^{n}\frac{x_ie^{-\frac{x_i}{b}}}{1+e^{-\frac{x_i}{b}}} = 0. \tag{11}$$

from the above equation, estimate of r is achieved as a function of b:

$$h(b) = \frac{1}{n}[\sum_{i=1}^{n} x_i - (\frac{n}{\sum_{i=1}^{n} \ln(1+e^{-\frac{x_i}{b}})} + 1)\sum_{i=1}^{n} \frac{x_i e^{-\frac{x_i}{b}}}{1+e^{-\frac{x_i}{b}}}]. \tag{12}$$

Therefore, MLE of b $(\widehat{b}_{MLE})$ is achieved from maximizing the above equation ratio to b. $H(b)$ a function of single-mode, and $(\widehat{b}_{MLE})$ which maximizes the above equation, is achieved from the Fixed Point Method $H(b) = b$.

3 Ranked set sampling

In environmental and laboratory studies, small number of units, are selected, and are measured, and are assessed chemical variables that have a negative impact on the environment. Since the measurement of the chemical units are expensive, using SRS is associated with a heavy expenditure. So, is useful providing a reasonable sampling strategy with fewer units of measurement. In the condition that measurement of the units is difficult or costly, but the units can be ranked simply with the lowest cost, RSS offers efficient methods for estimate population parameters than SRS.

For selecting a sample of size k in RSS method , we select k numbers of the samples of size k. Each of the samples of size k is ranked the variable of interest, and is selected the smallest unit level from first sample of size k, second ranked unit from the second sample of size k and unit has a larger rank from the kth sample of size k. If a larger sample size is required, In this case, these methods can be repeated s times until be measured a sample of size $n = sk$ .This n units are data from RSS method.

Let $X_{i(ic)}$, $i = 1, 2, ..., k$, $c = 1, 2, ..., s$ be sample sets of ranked from Generalized Logistic distribution of size $(n = sk)$, where k is the size of the collection, and s is the number of iterations (without loss of generality, in order to compare with SRS, we get zero for the location parameter). For simplicity $Y_{ic} = X_{i(ic)}$ is considered, and c is fixed. Y_{ic}'s are independent random variables with density function of ith order statistic:

$$g(y_{ic}) = \frac{k!}{(i-1)!(k-i)!} f(y_{ic}) \left[F(y_{ic})\right]^{i-1} \left[1 - F(y_{ic})\right]^{k-i}. \tag{13}$$

Likelihood function of sample $Y_{1c}, Y_{2c}, ..., Y_{kc}$, and the log-likelihood function are derived as follows:

$$L(\underline{y}|b, r) = \prod_{c=1}^{s}\prod_{i=1}^{k} \frac{k!}{(i-1)!(k-i)!} \times \frac{re^{-\frac{y_{ic}}{b}}[(1+e^{-\frac{y_{ic}}{b}})^r - 1]^{k-i}}{b(1+e^{-\frac{y_{ic}}{b}})^{kr+1}}. \tag{14}$$

$$\begin{aligned} L^* = \ln L(\underline{y}|b, r) &= C^* + ks\ln r - ks\ln b - \sum_{c=1}^{s}\sum_{i=1}^{k} \frac{y_{ic}}{b} \\ &+ \sum_{c=1}^{s}\sum_{i=1}^{k} (k-i)\ln\left[(1+e^{-\frac{y_{ic}}{b}})^r - 1\right] \\ &- (kr+1)\sum_{c=1}^{s}\sum_{i=1}^{k} \ln(1+e^{-\frac{y_{ic}}{b}}). \end{aligned} \tag{15}$$

where C^* is fixed.

We derivative of the log-likelihood function ratio to b and r, and we put them equal to zero:

$$\frac{\partial L^*}{\partial b} = -\frac{ks}{b} + \sum_{c=1}^{s}\sum_{i=1}^{k}(\frac{y_{ic}}{b^2}) + \sum_{c=1}^{s}\sum_{i=1}^{k}(k-i)\frac{ry_{ic}(1+e^{-\frac{y_{ic}}{b}})^{r-1}e^{-\frac{y_{ic}}{b}}}{b^2[(1+e^{-\frac{y_{ic}}{b}})^r - 1]}$$
$$-(kr+1)\sum_{c=1}^{s}\sum_{i=1}^{k}\frac{y_{ic}e^{-\frac{y_{ic}}{b}}}{b^2(1+e^{-\frac{y_{ic}}{b}})} = 0. \quad (16)$$

$$\frac{\partial L^*}{\partial r} = \frac{ks}{r} + \sum_{c=1}^{s}\sum_{i=1}^{k}(k-i)\frac{(1+e^{-\frac{y_{ic}}{b}})^r \ln(1+e^{-\frac{y_{ic}}{b}})}{[(1+e^{-\frac{y_{ic}}{b}})^r - 1]}$$
$$-k\sum_{c=1}^{s}\sum_{i=1}^{k}\ln(1+e^{-\frac{y_{ic}}{b}}) = 0. \quad (17)$$

Estimators b and r are obtained of nonlinear equations (3.4) and (3.5). These equations are solved by using numerical methods.

4 Simulation

Simulation results on MLE of the parameters of the Generalized Logistic distribution based on a random sample for different sample sizes done by Software $Maple$ with Fixed Point Method.
In MLE of parameters b and r, we have calculated estimated value, absolute bias and the MSE for $n = 12, 20, 30$, $b = 0.5, 1, 1.5, 3$ and $r = 0.5, 1, 1.5, 3$ in Table 1. According to Table 1, the MSE and bias of the MLEs of the parameters b and r in the Logistic distribution are reduced with increasing sample size.

Simulation results for estimation of the parameters of the Logistic distribution by using RSS for the different size and frequency of the rankings is performed by Software $Maple$ with Newton Raphson method. Parameter estimates by RSS are obtained the equations (3.4) and (3.5). Estimated values, absolute bias, and the MSE in RSS for $n = 12, 20, 30$ (the size of sample 5,4,3 and repetition rate 6,5,4 that $(n = ks)$), $b = 0.5, 1, 1.5, 3$ and $r = 0.5, 1, 1.5, 3$ are presented in Table 2.

According to the results in Table 2, the bias, and MSE of estimators are reduced with increasing sample size. The MSE and bias of estimator in RSS is smaller than SRS by comparing Table 1 with Table 2.

5 Conclusions

The bias and the MSE of the estimators of the parameter of the Generalized Logistic distribution based on RSS are smaller than SRS by comparing simulation results. Thus the RSS gives us more efficient estimators than SRS.

Table 1: Bias and MSE of MLEs of parameters of Logistic distribution

n	b	r	$\hat{b}$	$\hat{r}$	$Bias(b)$	$Bias(r)$	$MSE(b)$	$MSE(r)$
12	0.5	0.5	0.4509	0.5113	0.1548	0.1968	0.0374	0.0646
	0.5	1	0.4650	1.0865	0.1195	0.3369	0.0276	0.2206
	0.5	1.5	0.4715	1.6902	0.1077	0.4956	0.0182	0.5586
	0.5	3	0.5224	3.4482	0.0752	1.0360	0.1301	4.2343
	1	0.5	0.9013	0.5112	0.3102	0.1970	0.1521	0.0647
	1	1	0.9290	1.0881	0.2390	0.3384	0.0897	0.2282
	1	1.5	0.9318	1.7228	0.0719	0.5252	0.0719	0.8773
	1	3	0.8789	4.1068	0.2279	1.6987	0.0810	14.2183
	1.5	0.5	1.3512	0.5110	0.3831	0.1972	0.2314	0.0649
	1.5	1	1.3930	1.0889	0.3590	0.3393	0.2024	0.2339
	1.5	1.5	1.3900	1.7363	0.3303	0.2776	0.1691	0.9564
	1.5	3	1.2524	5.3635	0.3951	2.8097	0.2451	25.2280
	3	0.5	2.6975	0.5104	0.9374	0.1978	1.4003	0.0654
	3	1	2.7858	1.0889	0.7183	0.3394	0.8108	0.2344
	3	1.5	2.7756	1.7419	0.6650	0.5443	0.6899	0.9943
	3	3	2.4764	5.9389	0.8167	3.3843	1.0547	370.02
20	0.5	0.5	0.4813	0.5046	0.1103	0.1289	0.0189	0.0270
	0.5	1	0.4877	1.0347	0.08414	0.2134	0.0110	0.0757
	0.5	1.5	0.4892	1.5769	0.0754	0.3066	0.0088	0.1626
	0.5	3	0.5412	3.0930	0.0954	0.5578	0.0186	0.6567
	1	0.5	0.9626	0.5046	0.2207	0.1289	0.0756	0.0270
	1	1	0.9754	1.0347	0.1682	0.2134	0.0442	0.0757
	1	1.5	0.9758	1.5783	0.1512	0.3080	0.0356	0.1660
	1	3	0.9172	3.4287	0.1708	0.8260	0.0511	1.9571
	1.5	0.5	1.4439	0.5046	0.3311	0.1289	0.1704	0.0270
	1.5	1	1.4631	1.0347	0.2524	0.2134	0.0995	0.0757
	1.5	1.5	1.4630	1.5785	0.2275	0.3082	0.0805	0.1665
	1.5	3	1.3268	3.6022	0.2947	0.9940	0.1572	3.8336
	3	0.5	2.8876	0.5046	0.6625	0.1290	0.6828	0.0270
	3	1	2.9262	1.0347	0.5048	0.2134	0.3983	0.0757
	3	1.5	2.9252	1.5785	0.4553	0.3083	0.3225	0.1666
	3	3	2.6329	3.6454	0.6069	1.0364	0.6620	4.4901
30	0.5	0.5	0.4898	0.5060	0.0815	0.1005	0.0103	0.0164
	0.5	1	0.4925	1.0241	0.0637	0.1660	0.0062	0.0458
	0.5	1.5	0.4928	1.5498	0.0575	0.2355	0.0050	0.0953
	0.5	3	0.5300	3.0484	0.0711	0.4211	0.0113	0.3315
	1	0.5	0.9797	0.5060	0.1630	0.1005	0.0412	0.0164
	1	1	0.9290	1.0881	0.2390	0.3384	0.0897	0.2282
	1	1.5	0.9854	1.5500	0.1151	0.2357	0.0203	0.0955
	1	3	0.9393	3.2643	0.1300	0.5873	0.0305	0.7582
	1.5	0.5	1.4696	0.5060	0.2446	0.1005	0.0927	0.0164
	1.5	1	1.4777	1.0241	0.1925	0.1660	0.0595	0.0458
	1.5	1.5	1.4780	1.5500	0.1727	0.2357	0.0457	0.0956
	1.5	3	1.3767	3.3234	0.2217	0.6431	0.0918	0.9743
	3	0.5	2.9392	0.5060	0.4892	0.1005	0.3711	0.0164
	3	1	2.9554	1.0241	0.3824	0.1660	0.2250	0.0458
	3	1.5	2.9559	1.5500	0.3455	0.2357	0.1831	0.0956
	3	3	2.7328	3.3473	0.4603	0.6660	0.3939	1.0887

Table 2: Bias and MSE in estimating the parameters with RSS

n	b	r	$\hat{b}$	$\hat{r}$	$Bias(b)$	$Bias(r)$	$MSE(b)$	$MSE(r)$
12	0.5	0.5	0.4929	0.5265	0.1277	0.1371	0.0257	0.0281
	0.5	1	0.4872	1.0807	0.1018	0.1934	0.0224	0.0599
	0.5	1.5	0.4951	1.6344	0.0827	0.2776	0.0102	0.1274
	0.5	3	0.4912	3.4133	0.0317	0.6987	0.0083	0.8427
	1	0.5	0.9859	0.5265	0.2554	0.1371	0.1028	0.0281
	1	1	0.9902	1.6344	0.1655	0.2776	0.0410	0.1274
	1	1.5	0.9318	1.7228	0.0719	0.5252	0.0719	0.8773
	1	3	0.9824	3.4133	0.1505	1.5603	0.0335	0.8427
	1.5	0.5	1.4789	0.5265	0.3303	0.1371	0.0810	0.0281
	1.5	1	1.4911	1.0753	0.2762	0.1974	0.1179	0.0606
	1.5	1.5	1.4854	1.6344	0.2483	0.1538	0.0924	0.1274
	1.5	3	1.4736	3.4133	0.2257	0.6987	0.0755	0.8427
	3	0.5	2.9579	0.5265	0.7663	0.1371	0.9257	0.0281
	3	1	2.9822	1.0753	0.5524	0.1947	0.4717	0.0606
	3	1.5	2.9708	1.6344	0.4966	0.2776	0.3697	0.1274
	3	3	2.9473	3.4133	0.4515	0.6987	0.3022	0.8427
20	0.5	0.5	0.4751	0.4836	0.0879	0.0885	0.0126	0.0122
	0.5	1	0.4837	0.9973	0.0641	0.1301	0.0066	0.0269
	0.5	1.5	0.4859	1.5190	0.0566	0.1761	0.0051	0.0512
	0.5	3	0.4870	3.1359	0.0503	0.4244	0.0041	0.2708
	1	0.5	0.9503	0.4836	0.1759	0.0885	0.0504	0.0122
	1	1	0.9674	0.9973	0.1283	0.1301	0.0264	0.0269
	1	1.5	0.9718	1.5190	0.1133	0.1761	0.0206	0.0512
	1	3	0.9741	3.1359	0.1006	0.4244	0.0164	0.2708
	1.5	0.5	1.4255	0.4836	0.2639	0.0885	0.1134	0.0122
	1.5	1	1.4512	0.9973	0.1925	0.1301	0.0595	0.0269
	1.5	1.5	1.4577	1.5190	0.1700	0.1761	0.0464	0.0512
	1.5	3	1.4612	3.1359	0.1510	0.4244	0.0370	0.2708
	3	0.5	2.8510	0.4836	0.5278	0.0885	0.4538	0.0122
	3	1	2.9024	0.9973	0.3851	0.1301	0.2382	0.0269
	3	1.5	2.9155	1.5190	0.3400	0.1761	0.1856	0.0512
	3	3	2.9224	3.1359	0.3023	0.4244	0.1482	0.2708
30	0.5	0.5	0.4895	0.4950	0.0727	0.0720	0.0076	0.0071
	0.5	1	0.4898	1.0028	0.0551	0.1000	0.0044	0.0161
	0.5	1.5	0.4895	1.5207	0.0489	0.1419	0.0035	0.0333
	0.5	3	0.4888	3.1218	0.0436	0.3496	0.0028	0.2026
	1	0.5	0.9791	0.4950	0.1454	0.0720	0.0305	0.0071
	1	1	0.9797	1.0028	0.1103	0.1000	0.0178	0.0161
	1	1.5	0.9791	1.5207	0.0979	0.1419	0.0143	0.0333
	1	3	0.9779	3.1218	0.0872	0.3496	0.0115	0.2026
	1.5	0.5	1.4687	0.4950	0.2181	0.0720	0.0686	0.0071
	1.5	1	1.4512	0.9973	0.1912	0.1301	0.0562	0.0269
	1.5	1.5	1.4687	1.5207	0.1469	0.1419	0.0322	0.0333
	1.5	3	1.4665	3.1218	0.1308	0.3496	0.0260	0.2026
	3	0.5	2.9375	0.4950	0.4362	0.0720	0.2746	0.0071
	3	1	2.9392	1.0028	0.3309	0.1000	0.1605	0.0161
	3	1.5	2.9375	1.5207	0.2939	0.1419	0.1291	0.0333
	3	3	2.9330	3.1218	0.2616	0.3496	0.1043	0.2026

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