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Hitting Time for Random Walks on the Joint Sierpinski Networks

Xiaobei Liu* Xiaoyan Li
School of Mathematical Sciences, Jiangsu University, Zhenjiang, 212013, P. R. China
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Abstract: We propose a Joint Sierpinski Network model ($JSN(S,n)$) in this paper where n denotes the generation index of the network, which is generated by embedding the Sierpinski Network $S_n \circ N$ into model variable S. The $JSN(S,n)$ can be composed of m regions. In addition, we study and calculate the exact expression of the hitting time on the $JSN(S,n)$ with a trap node, and find that the hitting time is determined by the number of iterations n and model variable S.

Keywords: Joint Sierpinski Network; random walks; hitting time

1 Introduction

Complex networks[1, 2] play an increasingly important role in the real world and social realistic complex systems[3–5], such as global transportation network, the world wide web network[6] and so on. Many scholars have devoted themselves to studying and understanding the dynamical and structural properties of complex network models[7, 8]. In this paper, we introduce the Joint Sierpinski Network. Inspired by the literature and the idea[9–11], the Joint Sierpinski Network model is derived on the basis of the Sierpinski Network. The calculation progress of the hitting time of its is given out in this paper.

2 Preliminaries

2.1 The Sierpinski Network $S_n \circ N$

The $S_n \circ N$ can be seen from the self-similarity that it is generated by splicing 3^x regions $S_{n-x} \circ N$, where $x \in \{1, 2, ..., n-1\}$. Furthermore, For the convenience of subsequent explanation, we adopt two node labelling methods for $S_n \circ N$, one is letter marking to show the nodes. As shown in figure 1, the three outermost corner nodes on the $S_n \circ N$ have been denoted as A, B and C. And the other way of labeling is that all nodes be labeled sequentially from the top to the bottom by the site index i. The total time to absorption node and the hitting time are denoted as $T_{total}(n)$ and $\overline{T}(n)$ on the $S_n \circ N$, the analytical expression[12] can be obtained as follows, respectively. $T_{total}(n) = \frac{25}{4} \cdot 5^n \cdot 3^n + 3 \cdot 5^n - \frac{1}{4} \cdot 3^n$, $\overline{T}(n) = \frac{25 \cdot 5^n \cdot 3^n + 12 \cdot 5^n - 3^n}{2(5 \cdot 3^n + 1)}$.

Next, we calculate the hitting time when the three outermost corner nodes A, B and C are set as absorptive nodes, then $T_i^3(n)$ represents the trapping time from node i to the three absorption nodes A, B and C, where $i \in N_n/\{A, B, C\}$. In addition, the total time to absorption and the hitting time are denoted as $T_{total}^3(n)$ and $\overline{T}^3(n)$, respectively. The relationship between $T_{total}^3(n)$ and $T_{total}(n)$ on the $S_n \circ N$ as follow:

$$\begin{aligned} T_{total}(n) &= \sum_{i \in N_n} T_i^3(n) + \left(\frac{N_n - 3}{3} + 1\right) \cdot 2T_B(n) \\ &= T_{total}^3(n) + \frac{2N_n}{3} T_B(n). \end{aligned}$$

***Corresponding author**. *E-mail address*: 2388711593@qq.com

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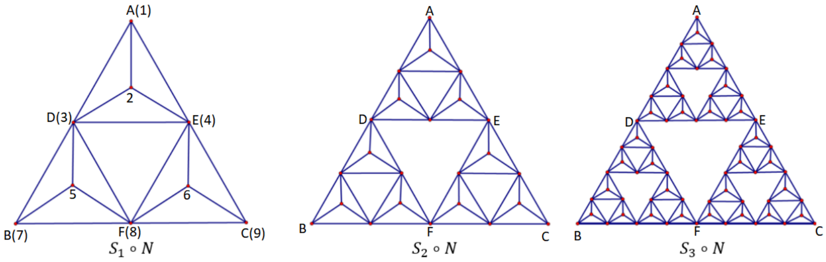

Figure 1: the first three generations of the $S_n \circ N$

Where $T_B(n) = 3 \cdot 5^n$. Therefore, it can be obtained that $T^3_{total}(n)$ and $T_{total}(n)$ satisfy

$$\begin{aligned} T^3_{total}(n) &= T_{total}(n) - \frac{2N_n}{3} T_B(n) \\ &= \frac{3^n(5 \cdot 5^n - 1)}{4}. \end{aligned} \tag{1}$$

The expression of the hitting time is obtained.

$$\begin{aligned} \overline{T}^3(n) &= \frac{1}{N_n - 3} T^3_{total}(n) \\ &= \frac{3^n(5 \cdot 5^n - 1)}{2(5 \cdot 3^n - 3)}. \end{aligned}$$

2.2 The Joint Sierpinski Network $JSN(S, n)$

The Joint Sierpinski Network, which is a network model generated by embedding the $S_n \circ N$ into model variable S, is defined as $JSN(S, n)$, where n is the generation. For model variable S, it can be assembled by splicing m triangles by overlapping nodes and has at most one common node between any two triangles. Let set of the three outermost vertices of each triangle is denoted as $\Omega^i_n \in \{A^i, B^i, C^i\}(i = 1, ..., m)$, the set of all nodes of model variable S is denoted as Ω_n. That is, the number of nodes is defined as l on the model variable S,

$$\Omega_n = \Omega^1_n \cup \Omega^2_n \cup ... \cup \Omega^m_n,$$

namely $|\Omega^i_n| = 3(i = 1, 2, ..., m)$ and $|\Omega_n| = l$.

Therefore, the $JSN(S, n)$ can be composed of m regions, each of which is denoted as $S^{(i)}_n \circ N$, where $i \in \{1, 2, ..., m\}$ is labeled according to the relative position of the region. Let the set of all nodes of the $JSN(S, n)$ is denoted as θ_n, the set of vertices of each region $S^{(i)}_n \circ N$ is denoted as $\theta^i_n(i = 1, 2, ..., m)$. Therefore, there is

$$\theta_n = \theta^1_n \cup \theta^2_n \cup ... \cup \theta^m_n.$$

In addition, we defined the set of nodes except the nodes on the model variable S as $\overline{\Omega}_n$ on the $JSN(S, n)$, namely, $\overline{\Omega}_n = \theta_n / \Omega_n$. In other words, Ω_n and $\overline{\Omega}_n$ are two complementary sets that make up all the nodes of the $JSN(S, n)$. So the set of nodes of each region except the three outermost vertices is defined as $\overline{\Omega}^i_n(i = 1, 2..., m)$. For the nodes belonging to this set, these will be labeled sequentially from the top to the bottom by site index j. Therefore, in order to accurately locate every node on the $JSN(S, n)$, we need to introduce two dimensional array (i, j), where i represents the region $S^{(i)}_n \circ N$ of the node, and j represents the location indicator of the node in this region. And all the sets of sites satisfie

$$\overline{\Omega}_n = \overline{\Omega}^1_n \cup \overline{\Omega}^2_n \cup ... \cup \overline{\Omega}^m_n.$$

Based on the above analysis, we redefined the number of nodes as $N(S, n)$ and the number of edges as $E(S, n)$ on the

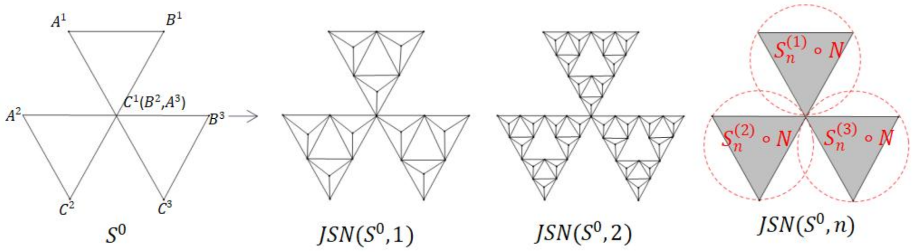

Figure 2: model variable S^0, the first two generations and nth generation

$JSN(S,n)$, where $N(S,n) = |\theta_n|$. Consequently, it is easy to know that the following relationship is correct.

$$\begin{aligned} N(S,n) &= m \cdot N_n - (3m - l) \\ &= \frac{5m}{2} \cdot 3^n - \frac{3m}{2} + l, \end{aligned} \tag{2}$$

$$E(S,n) = m \cdot 6 \cdot 3^n. \tag{3}$$

3 Exact formula of hitting time on the $JSN(S,n)$

In this section, we calculate the exact formula of the hitting time in general case on the $JSN(S,n)$, where the absorptive node can be selected to any node in the set Ω_n. Since Ω_n and $\overline{\Omega}_n$ are two complementary sets that make up all the nodes of the $JSN(S,n)$, we calculate the trapping time of each node belonging to these two sets separately, then add the obtained results to get the total absorption time.

The mean first passage time($MFPT$) from node (i,j) to node (i',j') on the $JSN(S,n)$ is denoted as $T_{(i,j),(i',j')}(S,n)$, the average time to absorption of node (i,j) is denoted as $T_{(i,j)}(S,n)$. And the total absorption time and hitting time on the $JSN(S,n)$ are denoted as $T_{total}(S,n)$ and $\overline{T}(S,n)$, respectively.

First, we calculate the sum of the trapping time of nodes in set Ω_n that is composed of all nodes on the mode variable S, so the number of nodes in this set will not change as the number of iterations increases, namely $\Omega_1 = \Omega_2 = ... = \Omega_n$. According to the structure of model variable S and the definition of the $JSN(S,n)$, it is easy to know that the iteration will only occur within these m equilateral triangles. Therefore, the $T_{a,b}(S,n)$ from site a to site b will increases proportionally as the number of iterations n increases, where $a, b \in \Omega_n$.

$$T_{a,b}(S,n) = 5^{n-1} \cdot T_{a,b}(S,1) \tag{4}$$

According to the above analysis, the analytic expression of trapping time of nodes in set Ω_n is that

$$\begin{aligned} \sum_{a \in \Omega_n} T_a(S,n) &= 5^{n-1} \cdot \sum_{a \in \Omega_1} T_a(S,1) \\ &= 5^{n-1} \cdot T(S,1) \end{aligned} \tag{5}$$

Since both Ω_n and $T_a(S,1)$ are independent of the number of iterations n and only determined by the model variable S and the location of the absorptive node, we use random variable $T(S,1)$ to represent the total trapping time of the nodes that belong to set Ω_1 on the $JSN(S,1)$, where the model variable S is generally finite and simply connected network. And $T(S,1)$ can be calculated by the matrix algorithm.

Next, we calculate the sum of the trapping time of nodes in set $\overline{\Omega}_n$. For any site (i,j) belonged to set $\overline{\Omega}_n^i (i = 1,2,...,m)$, its path to the absorptive node can be divided into the following two processes: the first one is that random walker performs a random walk from node (i,j) to any of the outermost vertices belonging to set Ω_n^i in region $S_n^{(i)} \circ N (i = 1,2,...,m)$ for the first time. The second process is that the walker finally reaches the absorptive node after leaving that vertex. Since the node (i,j) is randomly selected from set $\overline{\Omega}_n^i$ according to uniform distribution, the probability of the random walker reaching the three outermost vertices A_n^i, B_n^i and C_n^i for the first time is equal. Therefore, the sum of the trapping time of all nodes belonging to the each region in the first path is the sum of the absorption time on the $S_n \circ N$ with three absorptive nodes that we analyzed in the previous section. So, from the above analysis, the following relationship can be obtained:

$$\begin{aligned}\sum_{i=1}^{m}\sum_{(i,j)\in\overline{\Omega}_n^i} T_{(i,j)}(S,n) &= \sum_{i=1}^{m}\left[\sum_{(i,j)\in\overline{\Omega}_n^i} T^3_{(i,j)}(n) + \frac{N_n-3}{3}\sum_{a\in\Omega_n^i} T_a(S,n)\right] \\ &= m\cdot T^3_{total}(n) + \frac{N_n-3}{3}\sum_{i=1}^{m}\sum_{a\in\Omega_n^i} T_a(S,n) \\ &= m\cdot T^3_{total}(n) + \frac{N_n-3}{3}\sum_{i=1}^{m}\left(5^{n-1}\cdot\sum_{a\in\Omega_1^i} T_a(S,1)\right) \\ &= m\cdot T^3_{total}(n) + \frac{5\cdot 3^{n-1}-1}{2}\cdot 5^{n-1}\sum_{i=1}^{m}\sum_{a\in\Omega_1^i} T_a(S,1) \end{aligned} \tag{6}$$

If the site a belonging to set Ω_1^i connects more than one triangle, the $T_a(S,1)$ of this node will be accumulated multiple times, and the cumulative times are determined by the number of triangles connected by the node. Since the model variable S is assembled by splicing triangles by overlapping nodes, the number of triangles connected by each node is one third of the node degree. In addition, the total time to absorption with degree weight on the $JSN(S,1)$ is defined as $T_d(S,1) = \sum_{a\in\Omega_n} d_a T_a(S,1)$, which can also be calculated by the matrix algorithm. Thus, the above can be further written as

$$\begin{aligned}\sum_{i=1}^{m}\sum_{(i,j)\in\overline{\Omega}_n^i} T_{(i,j)}(S,n) &= m\cdot T^3_{total}(n) + \frac{5\cdot 3^{n-1}-1}{2}\cdot 5^{n-1}\sum_{a\in\Omega_1^i}\frac{d_a}{3}T_a(S,1) \\ &= m\cdot T^3_{total}(n) + \frac{5\cdot 3^{n-1}-1}{6}\cdot 5^{n-1}\cdot T_d(S,1) \end{aligned} \tag{7}$$

Consequently, we can obtain the sum of the average absorption time of all nodes on the $JSN(S,n)$, $T_{total}(S,n)$, which satisfies

$$\begin{aligned} T_{total}(S,n) &= \sum_{a\in\Omega_n} T_a(S,n) + \sum_{i=1}^{m}\sum_{(i,j)\in\overline{\Omega}_n^i} T_{(i,j)}(S,n) \\ &= 5^{n-1}\cdot T(S,1) + m\cdot T^3_{total}(n) + \frac{5\cdot 3^{n-1}-1}{6}\cdot 5^{n-1}\cdot T_d(S,1) \\ &= 5^{n-1}\cdot T(S,1) + m\cdot\frac{3^n(5\cdot 5^n-1)}{4} + \frac{5\cdot 3^{n-1}-1}{6}\cdot 5^{n-1}\cdot T_d(S,1) \end{aligned} \tag{8}$$

Then, it can be obtained that the analytic expression of the hitting time on the $JSN(S,n)$ as follows

$$\begin{aligned} \overline{T}(S,n) &= \frac{1}{N(S,n)-1}T_{total}(S,n) \\ &= \frac{(6T(S,1)-T_d(S,1))\cdot 2\cdot 5^{n-1} + (45m+2T_d(S,1))\cdot 3^{n-1}\cdot 5^n - m\cdot 3^{n+1}}{6(5m\cdot 3^n - 3m + 2l - 2)} \end{aligned} \tag{9}$$

From the above analysis, we can see that the hitting time is determined by model variable S, the number of iterations n and the location of the absorptive node.

4 hitting time of the $JSN(S^0,n)$

In this subsection, we will describe the hitting time of different absorption nodes when the model variable is S^0. First, referring to the $JSN(S^0,1)$ of figure 2, we label all the nodes from top to bottom and obtain the transition probability matrix M_0 of order 25 with no absorption as follows.

$$M_0 = \begin{pmatrix} A_{11} & A_{12} & O \\ A_{21} & A_{22} & A_{23} \\ O & A_{32} & A_{33} \end{pmatrix}_{25\times 25}$$

See appendix A for detailed elements of matrix M_0.

4.1 first category of the absorptive node

First, we consider the total time to absorption and the hitting time when the node C^1 is set as the absorptive node. The transition probability matrix of the $JSN(S^0, 1)$ with absorptive node C^1 is denoted as M_0'.

$$M_0' = \begin{pmatrix} A_{11} & A_{12} & O \\ A_{21} & A_{22} & B_{23} \\ O & B_{32} & B_{33} \end{pmatrix}_{24\times 24}$$

It is obtained by removing row and column of trap node C^1 in matrix M_0, where block matrix B_{23} and B_{32} are obtained by removing the elements of first column and row of block matrix A_{23} and A_{32}, respectively. The first row and column of A_{33} is simultaneously removed to get block matrix B_{33}. By substituting matrix M_0' into $T = (I - M')^{-1}e$, where e is $(N-1)$-dimensional columnvector and all elements are 1, I is the identity matrix of order $N-1$. The vector T_0' consisting of the hitting time of all nodes on the $JSN(S^0, 1)$, except the absorptive node C^1, can be obtained as follows:

$$T_0' = [15\ 15\ 15\ 15\ 15\ 12\ 12\ 9\ 15\ 12\ 12\ 15\ 15\ 9\ 9\ 15\ 15\ 12\ 12\ 15\ 15\ 15\ 15\ 15]^{\mathrm{T}}$$

If vertex C^1 is set as the absorptive node, the sum of the trapping time and the trapping time with degree weights of nodes belonged to set Ω_n on the $JSN(S^0, 1)$ are respectively denoted as:

$$T'(S^0, 1) = 90,$$

$$T_d'(S^0, 1) = 270.$$

Consequently, substituting these numerical result into Eq (8) and Eq (9), respectively, it can be derived that the analytical expressions of the total time to absorption $T_{total}'(S^0, n)$ and the hitting time $\overline{T}'(S^0, n)$ on the $JSN(S^0, n)$.

$$T_{total}'(S^0, n) = 3 \cdot \frac{25}{4} \cdot 5^n \cdot 3^n + 9 \cdot 5^n - \frac{3}{4} \cdot 3^n, \tag{10}$$

$$\overline{T}'(S^0, n) = \frac{25 \cdot 5^n \cdot 3^n + 12 \cdot 5^n - 3^n}{2(5 \cdot 3^n + 1)}. \tag{11}$$

It can be found that $T_{total}'(S^0, n)$ is three times of $T_{total}(n)$, while $\overline{T}'(S^0, n)$ is equal to $\overline{T}(n)$. As we described in the previous section, $JSN(S^0, n)$ is composed of three regions $S_n^{(i)} \circ N (i = 1, 2, 3)$. And node C^1 is in the center of the network $JSN(S^0, n)$. When it is set as absorptive node, the random walker cannot reach other regions before reaching the absorption node, whether it starts from any region. Therefore, the random walk time on each region can be considered separately and the absorption time on each region corresponds to the absorption time on the $S_n \circ N$ with absorptive node A.

4.2 second category of the absorptive node

Next, we consider the total time to absorption and the hitting time when the node A^1 is set as the absorptive node. The transition probability matrix with absorption node A^1 is denoted as M_0''.

$$M_0'' = \begin{pmatrix} C_{11} & C_{12} & O \\ C_{21} & A_{22} & A_{23} \\ O & A_{32} & A_{33} \end{pmatrix}_{24\times 24}$$

This is obtained by removing row and column of trap node A^1 in matrix M_0, where the first row and column of A_{11} is simultaneously removed to get block matrix C_{11}. Block matrix C_{12} and C_{21} are obtained by removing the elements of first row and column of block matrix A_{12} and A_{21}, respectively. Thus, the matrix M_0'' is substituted into $T = (I - M')^{-1}e$ to obtain the each element of vector T_0'' as follows:

$$T_0'' = [36\ 45\ 27\ 45\ 42\ 51\ 57\ 90\ 87\ 75\ 87\ 90\ 90\ 84\ 84\ 90\ 90\ 87\ 87\ 90\ 90\ 90\ 90\ 90]^{\mathrm{T}}$$

The sum of the trapping time and the total time to absorption with degree weight of nodes belonging to Ω_n are expressed as $T''(S^0, 1)$ and $T_d''(S^0, 1)$, which the absorptive node is the site A^1. Then, it can be obtained that

$$T''(S^0, 1) = 480,$$

$$T_d''(S^0, 1) = 1890.$$

The exact formula of the total time to absorption and the hitting time can be expressed as

$$T_{total}''(S^0, n) = \frac{29}{4} \cdot 5^{n+1} \cdot 3^{n+1} + 33 \cdot 5^n - \frac{3}{4} \cdot 3^n, \tag{12}$$

$$\overline{T}''(S^0, n) = \frac{29 \cdot 5^{n+1} \cdot 3^{n+1} + 132 \cdot 5^n - 3^{n+1}}{6(5 \cdot 3^n + 1)}. \tag{13}$$

When other nodes belonged to the second class are set as absorptive nodes, the analytical expressions of the total time to absorption and the hitting time on the $JSN(S^0, n)$ are all equal to the above expressions. In addition, we know that the hitting time characteristics the diffusion efficiency of network model. So it can be found that the hitting time when node A^1 is set as the absorptive node is several times larger than when node C^1 is set as the absorptive node, which also indicates that when node C^1 is set as the absorptive node, its absorption efficiency will be higher. Finally, the hitting time obtained in the above two ways of setting the absorptive node is numerically simulated, as can be seen from the figure 3.

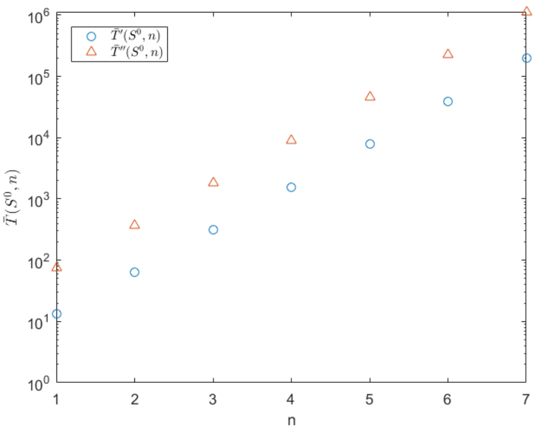

Figure 3: the numerical results of the hitting time over the $JSN(S^0, n)$ with different absorption node

5 Conclusions

In this paper, we propose a kind of Joint Sierpinski Network, which is defined as $JSN(S, n)$. The total time of the random walker and the general exact expression of the hitting time are gived, respectively. The $JSN(S, n)$ is generated by embedding the Sierpinski Network $S_n \circ N$ into model variable S, which consists of m regions. Therefor, the $JSN(S, n)$ can be composed of m regions and the hitting time of the $JSN(S, n)$ varies with the model variable S. In addition, we study the hitting time on a general model $S_n \circ N$ with trap site F. It is worth emphasizing that the $JSN(S, n)$ is an extension of the $S_n \circ N$. Figure 3 shows that different trap node leads to different numerical results of the mean time to absorption at the trap for the same model variable S.

Appendix A

From the transition probability matrix M_0, we have

$$A_{11}=\begin{pmatrix} 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix} A_{12}=\begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 \end{pmatrix} A_{21}=\begin{pmatrix} \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & 0 \\ 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_{22}=\begin{pmatrix} 0 & \frac{1}{6} & \frac{1}{6} & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{6} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & \frac{1}{6} & 0 \end{pmatrix} A_{23}=\begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \cdots & 0 \\ \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \cdots & 0 \end{pmatrix}_{5\times 15}$$

$$A_{32}=\begin{pmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 & \frac{1}{9} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{15\times 5} A_{33}=\begin{pmatrix} 0 & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{1}{6} & \frac{1}{6} & & & & \\ 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & & & & \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & & & & \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ & & & & & & & \frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix}_{15\times 15}$$

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