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A Novel Hybrid Method of Forecasting Crude Oil Prices Based on Multi-Layer Symbolic Pattern Network

Xin Wang, Mei Sun*
Energy Development and Environmental Protection Strategy Research Center, Jiangsu University, Zhenjiang, Jiangsu, 212013, China
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Abstract: Faced with the high-dimensional characteristics of time series, symbolization is used to reduce the dimensionality and effectively eliminate redundancy and irrelevant features. Several existing network prediction methods, which based on the topology of complex networks, only consider the target sequence itself, which ignored the influence of related factors. In this paper, we improve the single-layer network and construct a Multi-layer Symbolic Pattern Network prediction model by considering the interaction between different variables with time lag. The basic idea is to link the directed weighted networks based on causation entropy, then extract the multi-layer network topology to jointly predict the target sequence. The 2018/1-2018/12 fluctuation trend and of crude oil futures prices are predicted, with auxiliary variables S&P 500 Index. The results show that the multi-layer network improves the directional prediction accuracy of oil price over the single-layer network. By optimizing the selection process of the neural network training set, the improved LSTM-based time series prediction model proposed in this paper has higher prediction accuracy and smaller errors than the single LSTM neural network and ARIMA model.

Keywords: Multi-layer Symbolic Pattern Network; Crude Oil Price; Long Short-Term Memory Neural Network; Time Series Prediction

1 Introduction

Digging out the hidden effective information of time series has a great effect and significance. As data mining technology, time series forecasting is an important aspect of time series analysis. Common prediction methods are based on traditional econometric models, ANNs and SVM. Symbolic time series analysis (STSA) [1] is used to represent time series and explore its essential characteristics. The symbolization method has the advantages of high storage cost performance, fast calculation speed, ignoring abnormal points and noise reduction. In addition, the use of complex network science to characterize the dynamic characteristics of nonlinear time series has gradually become a new multidisciplinary method, and it is a hot spot in the study of nonlinear dynamics and complex systems, such as the Visibility Graph, recurrence plots, coarse-grained networks and ordinal networks. Complex network theory shows strong adaptability and anti-noise ability in characterizing the interaction between various real system components. The different dimensions of high-dimensional time series do not exist in isolation but are connected and mutually restricted. The causation entropy (CSE) [2] based on information theory has gradually developed into a popular method to explore the causal structure of complex systems or high-dimensional time series.

There is no literature that considers both the interactions between high-dimensional time series and applying higher-order Markov processes to construct network prediction model. Therefore, this paper uses multiple related time series to construct a multi-layer symbolic pattern network, and by extracting the information of the multi-layer network topology, combined with the Long Short-Term Memory (LSTM) [3] neural network, a novel time series forecasting hybrid model is proposed. This article takes the WTI international crude oil futures price sequence as an example, constructs a multi-layer symbolic model network based on oil price fluctuations and related factors, and then predicts the fluctuation trend and

***Corresponding author**. *E-mail address*: sunm@ujs.edu.cn

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fluctuation rate of oil price fluctuations, providing a new solution for international crude oil price forecasts. The main contributions of this paper are:

(1) We use the principle of maximum entropy to calculate the optimal symbolization threshold and determine the approximate generation partition of the phase space division.

(2) Use causal entropy to describe the time-lag interaction between different sequences, and establish a multi-layer directed weighted symbolic mode network with the international crude oil price fluctuation symbolic mode network as the main network.

(3) Apply the rollover framework to configure the time series fluctuation trend prediction model based on the multi-layer symbolic pattern network to fully obtain new information or shocks, discard old data over time.

(4) Combine the multi-layer symbolic model network prediction model with LSTM and filter out the similar fluctuation fragments in historical data according to the prediction result of the oil price fluctuation trend.

The rest of the article is organized as follows: Section 2 establishes a multi-layer symbolic pattern network, a multi-layer network-based time series fluctuation trend prediction method and an improved LSTM-based time series mixed prediction model. Section 3 takes WTI crude oil futures price as an example to predict its volatility trend and volatility of oil prices and verifies the effectiveness of the method proposed in this article. At the end of the paper, we present our conclusions.

2 Model Construction

2.1 Multi-layer Symbolic Pattern Network

The symbolic pattern network is proposed based on the data fluctuation network [4]. For a time series $\{x_t|t = 1, \cdots, T\}$, the corresponding symbolic sequence $\{s_t|t = 1, \cdots, T\}$ is obtained by the topological partition of the phase space. For the diversity of nodes in the network and the complexity of the edges, we get $S = \lfloor (T - L)/l + 1 \rfloor$ symbolic patterns by Takens theorem and coarse graining algorithm (see Figure 5), In this paper, for the continuity of the symbolic pattern, the time delay is selected as 1. Record the set of symbolic patterns as $\{X_t^i|X_t^i =' s_{t-L+1} \cdots s_t', t \geq L, i \in [1, M], M \leq S\}$. There are *M* kinds of symbolic patterns, and *i* stands for the *i*-th one. Repeated symbolic patterns are regarded as one node in the network. The number of times the link appears is defined as weight of the edge.

There are two symbolic pattern sequences *X* and *Y*, ignoring the instantaneous influence, there may be an influence with time lag between the two sequences, gradually add the historical moment state of *X* to the set of influencing factors on *Y*, calculate the information flow of the set of influencing factors on the current moment of *Y*, when CSE between them reaches the maximum, then the corresponding *k* is the maximum impact delay between the two sequences. To determine the Markov order of the symbolic pattern sequence, solve the mutual information maximum model [5]:

$$k = \arg\max_k C_{X_{t-1},\ldots,X_{t-k} \to Y_t | X_{t-\setminus\{t-1,\ldots,t-k\}}}, \tag{1}$$

$$C_{X_{t-1},\ldots,X_{t-k} \to Y_t | X_{t-\setminus\{t-1,\ldots,t-k\}}} = I(Y_t; X_{t-1}, \ldots, X_{t-k} | Y_{t^-}, X_{t-\setminus\{t-1,\ldots,t-k\}}), \tag{2}$$

Having obtained the time lag of the effect of *X* on *Y*, the following is to determine which historical moments within the $\{t - 1, \ldots, t - k\}$ moment of *X* directly affect the current state of *Y*. The historical moments of *Y* and other historical moments of *X* need to be added to the causal entropy formula as known conditions. Let $Q_t^Y = \{t-1, \ldots, t-k_Y+1, t-k_Y\}$ and $Q_t^{X\to Y} = \{t-1, \ldots, t-k_{X\to Y}+1, t-k_{X\to Y}\}$ denote the initial set of historical moments in which *Y* itself and *X* have an influence on the state of *Y* at the current moment, respectively. The casual structure between the two sequences is as follows:

$$P_t^{X\to Y} = \{t' | C_{X_{t'} \to Y_t | X_{Q_t^{X\to Y} \setminus t'}} > 0, \forall t' \in Q_t^{X\to Y}\}, \tag{3}$$

$$C_{X_{t'} \to Y_t | X_{Q_t^{X\to Y} \setminus t'}} = I(Y_t; X_{t'} | Y_{t^-}, X_{t-\setminus t'}) = I(Y_t; X_{t'} | Y_{P_t^Y}, X_{Q_t^{X\to Y} \setminus t'}), \tag{4}$$

where P_t^Y defines the causal structure of *Y*, and $P_t^{X\to Y}$ defines the causal structure of *X* on *Y*, i.e., representing the t moment state of *Y* subject to *X* of the direct impact of each moment in the $P_t^{X\to Y}$.

The pattern at each moment is used as a node. A connected edge is generated when a certain historical moment has a direct impact on the current state of the moment. Connected edges have direction and weight, and the weight of connected edges is the number of transitions between the two modes to construct a single-layer symbolic pattern network. According to Section(), map the sequences *Y* and *X* to the networks G_Y and G_X, respectively, and link the nodes in G_X corresponding to each time in the set $P_t^{X\to Y}$ to the nodes at time t in G_Y, to achieve interlayer connectivity (with direction and weights) to obtain a multi-layer directed weighted network.

2.2 Trend prediction and Evaluation Indicators

Based on the idea of pattern reproduction [6], it is reasonable to infer that the symbolic pattern corresponding to the node at time *t* to be predicted in the future must be an element in $Cd^Y(t)$. The following is a detailed analysis of the two possible situations of the target node (from the perspective of *Y*): (1) If $Y_{P_t^Y}$ has no out-neighbor node in G_Y, then the future node that appears at *t* can be estimated according to the trend extrapolation method. That is, if $Y_{t-1} =' s_1 s_2 s_3 s_4 s_5'$, then $Y_t =' s_2 s_3 s_4 s_5 s_5'$. (2) If $Y_{P_t^Y}$ has out-neighbor nodes in G_Y, then check in the multi-layer network to find all the out-neighbor nodes from nodes at P_t^Y and $P_t^{X\to Y}$. The set of out-neighbor nodes are respectively:

$$N^{out(Y)}(Y_{P_t^Y}) = \{Y^{i_1}, Y^{i_2}, ..., Y^{i_m}\}, i_1, i_2, ..., i_m \in [1, M_Y], \tag{5}$$

$$N^{out(X\to Y)}(X_{P_t^{X\to Y}}) = \{Y^{j_1}, Y^{j_2}, ..., Y^{j_n}\}, j_1, j_2, ..., j_n \in [1, M_Y], \tag{6}$$

The next step is to combine the above two sets of out-neighbor nodes, the new set of out-neighbor nodes obtained is recorded as $Cd^Y(t) = N^{out(Y)}(Y_{P_t^Y}) \oplus N^{out(X\to Y)}(X_{P_t^{X\to Y}})$, where $\oplus$ defines an operation including merging the same patterns, deleting redundant patterns, and supplementing missing patterns. We choose the principle of the greatest node strength, that is, select the element with the greatest strength from $Cd^Y(t)$ as the node at *t*.

The prediction method proposed in this section is used to predict the symbolic pattern at time *t*. Assuming that the predicted symbolic pattern is $\widehat{Y}_t =' a_1 a_2 a_3 a_4 a_5'$, it means that the predicted symbol of the symbol sequence at time *t* is $\widehat{s}_t = a_5$.

When predicting the volatility trend of the time series, to measure the prediction accuracy of the proposed model, so we use the following indicators to measure the prediction accuracy:

(1) Directional prediction accuracy.

$$D_{stat} = \frac{1}{N}\sum\nolimits_{t=1}^{N} a_t \times 100\%, \tag{7}$$

where s_t and $\hat{s}_t$ are the true and the predicted symbol, respectively. If the prediction is correct, the value of a_t is 1, otherwise it is 0.

(2) Prediction accuracy value-added. Suppose there are two prediction models with the same parameter *k*. The directional prediction accuracy curves of the two models with the parameter *k* on the same set of data are f_1 and f_2. The definition of the value-added in accuracy is given below:

$$VA_{21} = \frac{1}{k_2 - k_1}\int_{k_1}^{k_2} (f_2 - f_1)\mathrm{d}k. \tag{8}$$

This indicator describes the average improvement in prediction accuracy between the two models.

2.3 Time Series Volatility Forecast

2.3.1 MSPN-LSTM

LSTM is widely used in the field of time series classification and time series prediction. This section uses MATLAB's Deep Learning Toolbox to configure the LSTM network and predict the volatility of crude oil prices. The prediction requires historical fluctuation data, the target node at time t, and the predicted symbolic pattern. Assuming that the target node at time *t* is known to be $s_{t-L}s_{t-L+1}\cdots s_{t-1}$, the symbolic pattern at time *t* is $s_{t-L+1}s_{t-L+2}\cdots s_t$ according to the model prediction in Section 2. On this basis, the volatility at time *t* is predicted. Specific steps are as follows:

Step 1: Traverse all historical symbolic patterns and compare them with the predicted symbolic patterns. When the two are consistent, record the volatility vector corresponding to the historical symbolic pattern and save it in tempHistory.

Step 2: Traverse all the elements in tempHistory. The first *L*-1 number of each element is the input, and the last number is the output.

Step 3: Take the first *L*-1 number of the volatility vector corresponding to the symbolic pattern at time *t* obtained by prediction as input, and substitute it into the neural network trained in step 2 to obtain the predicted volatility at time *t*.

The basic idea of the hybrid neural network prediction model is to filter the neural network training set and look for segments with similar fluctuations in history as the training set of the model.

To measure the forecasting performance of different forecasting models and compare the predicted value with the true value, two loss functions are selected in this paper: Root Mean Square Error (RMSE) and Mean Absolute Error (MAE).

3 Empirical Analysis

3.1 Data

The importance of crude oil is self-evident. Due to the asymmetric information exchange between the oil price and the stock market, the influence relationship between the two has always been a hotspot of academic research [7]. This paper selects WTI crude oil futures prices (https://www.eia.gov) and S&P 500 Index (SPX) (https://finance.yahoo.com), and takes oil price volatility as the forecast target, and SPX as the forecast auxiliary variable.

3.2 Symbolization

The data is divided into two parts, with monthly oil price up to 2017/12 as the model training set and the 2018/1- 2018/12 data as a testing set. Then the log-return sequence is calculated by $r_t = \ln y_t - \ln y_{t-1}$, where .

In this paper, we set $r_0 \geq 0$ as the threshold, then log-return series is converted to a symbolic sequence:

$$s_t = \begin{cases} r, r_t > r_0 \\ e, -r_0 \leq r_t \leq r_0. \\ d, r_t < -r_0 \end{cases} \tag{9}$$

This section mainly explores the fluctuating trend of the oil price series. Therefore, the symbol space is set to $\{r, e, d\}$. For the volatility series of the selected two sequences, the maximum entropy principle is used to optimize the symbolization threshold of each sequence. The optimal symbolization thresholds of oil price and S&P 500 obtained are 0.029877 and 0.018917, respectively. The probability of $\{r, e, d\}$ of oil price are 35%, 34%, and 31%.

3.3 Causal Structure

For the international crude oil price fluctuation trend sequence, use the G-P algorithm [8] to determine the embedding dimension m = 3. For the continuity of the fluctuation trend, the time delay parameter is $\tau = 1$. The size of the sliding window in the sliding window technology corresponds to the embedding dimension, that is, L = 3. To obtain as many symbolic patterns as possible, the sliding step length l = 1. To simplify the model and the convenience of calculation, assume that each symbolic pattern sequence itself satisfies the first-order Markov process.

First calculate the Markov order, then calculate CSE of the historical state and the current state to eliminate redundant information and obtain the causal structure. As shown in Figure 1, it is found that the mutual information between the oil price and SPX increases to a maximum value when $k = 5$, indicating that the state of SPX in $Q_t^{SPX \to Oil} = \{t-1, t-2, t-3, t-4, t-5\}$ already contains all the historical information (SPX to Oil) about the state at t of the oil price. Thus, the Markov order of SPX on the oil price is 5. After determining the Markov order, by calculating the causation entropy at $\forall t' \in Q_t^{SPX \to Oil}$ and the current state of the oil price, if CSE is greater than 0, it indicates that the state of supply at t' has a direct effect on the current state of the oil price, otherwise it indicates that there is no direct effect, so the causal structure between SPX and the oil price symbolic pattern sequence is $P_t^{SPX \to Oil} = \{t-1, t-5\}$.

According to the causal structure between variables, the nodes in the auxiliary network link the current moment in the crude oil price symbolic pattern network. Then a directed inter-layer link is established between the network layers.

3.4 Oil price fluctuation trend forecast

3.4.1 Analysis of network prediction model results

According to the time series volatility trend prediction model in Section 2, for the fluctuation trend prediction of crude oil futures prices, and the program runs through MATLAB R2019b.

In this paper, we configure forecasting models by the rollover framework [9], which is generally applied to portfolio theory. Set the oil price as the main forecast sequence and SPX as the auxiliary forecast sequence to construct a two-layer directed weighting network. Figure 2 shows the directional prediction accuracy curve of the model under the candidate node prediction criteria. The blue dotted line is the single-layer network prediction accuracy curve, the red solid line is the multi-layer network prediction accuracy curve, and the abscissa is the rollover framework scale of the prediction model.

From Figure 2, it is found that from the perspective of the two-layer symbolic network prediction model, the model "Oil + SPX" has a better prediction effect, especially when the rolling scale is small, the prediction accuracy of the

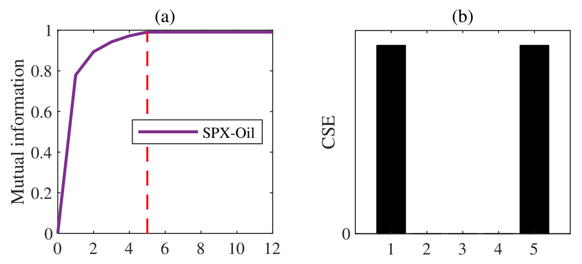

Figure 1: Markov order and causal structure based on mutual information and causation entropy form SPX to oil price

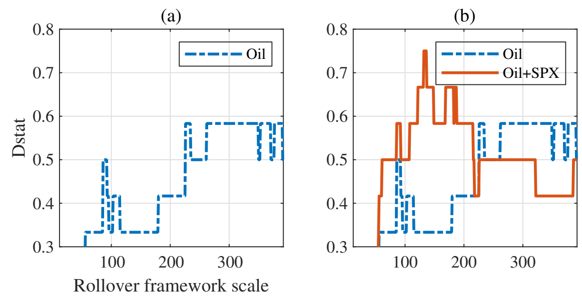

Figure 2: The prediction accuracy curve

multi-layer network is significantly better than that of the single-layer network, but when the rolling scale is small, the prediction accuracy of the multi-layer network is significantly better than that of the single-layer network. When the scale of the rollover framework is large, there is a negative benefit. The highest prediction accuracy is 58.33% and 75% respectively, the directional prediction accuracy of the multi-layer network prediction model is 28.57% higher than that of the single-layer network prediction model, and the increase in the prediction accuracy is 0.0492.

3.5 Improved LSTM-based time series forecasting

3.5.1 Experiment description and parameter settings

To verify the effectiveness of the proposed model, with the volatility at a specified time as the forecast target, this part sets up the following two sets of experiments: (1) The basic LSTM neural network time series prediction model compares the use of the first two data to predict the third data LSTM time series prediction model; (2) Based on symbol prediction, the fluctuating symbol status at the moment to be predicted is divided into three situations: “known”, “unknown”, “predicted”.

3.5.2 Case1: Comparison of prediction results of a single LSTM neural network

In case 1, we take the volatility of WTI crude oil futures prices from 2018/1 to 2018/12 as the forecast target. Figure 3 is a schematic diagram of LSTM-1 and LSTM-2 models.

Figure 4 shows the volatility prediction value of the oil price by LSTM-1 and LSTM-2 and compares with the true volatility of the oil price series. It can be seen that the results of LSTM-1 have large deviations and poor results. This is mainly because the oil price fluctuates frequently, and the basic LSTM neural network cannot learn the oil price volatility at the last moment and the current volatility well. Therefore, it is difficult to predict the volatility of oil prices at the current moment only from the previous volatility. The prediction result of the LSTM-2 model is closer to the actual fluctuation, but the predicted value often fluctuates around 0, and the predicted value and the actual value have opposite fluctuation trends, such as 2018/2, 2018/3, 2018/6, 2018/8 and 2018/9. Compared with LSTM-1, LSTM-2 takes the oil price volatility at the previous moment as an input variable in the prediction model and obtains the predicted value of oil price fluctuations by two-dimensional input. The error of the LSTM-1 and LSTM-2 models and the running time of the program are shown in Table 1. It can be seen from the table that the RMSE and MAE of LSTM-2 are both lower than LSTM-1. However, due to the additional consideration of one-dimensional variables, the running time has also been greatly increased from 46s to 163s.

3.5.3 Case2: Comparison of prediction results of MSPN-LSTM hybrid prediction model

In case 2, the LSTM neural network time series prediction model based on the multi-layer directed weighted network proposed in this section is applied. According to the state of the fluctuation symbol at the current moment, the model can be divided into three categories, corresponding to the current symbol is known, the current symbol is unknown, and the current symbol uses the prediction. Figure 5 shows examples of the construction of the training set and test set of the above three MSPN-LSTM models.

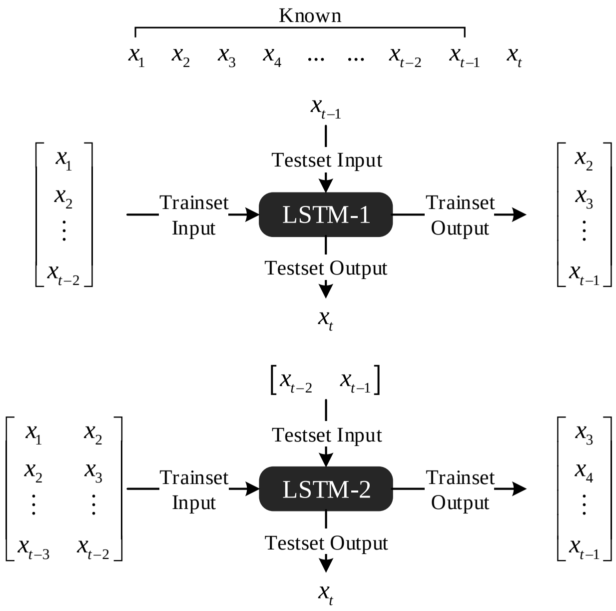

Figure 3: Comparison of LSTM-1 and LSTM-2

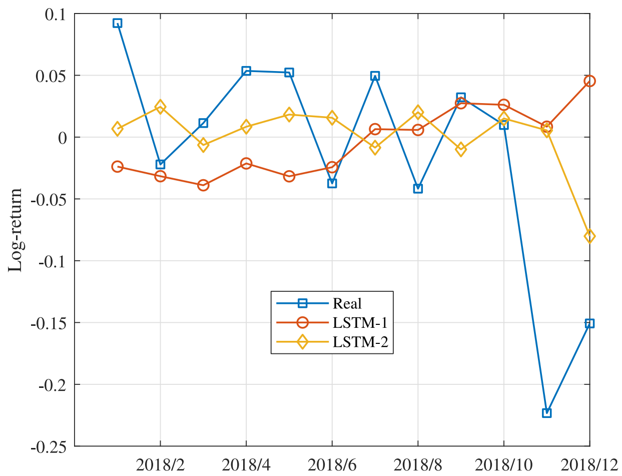

Figure 4: Comparison of the predicted value with the real

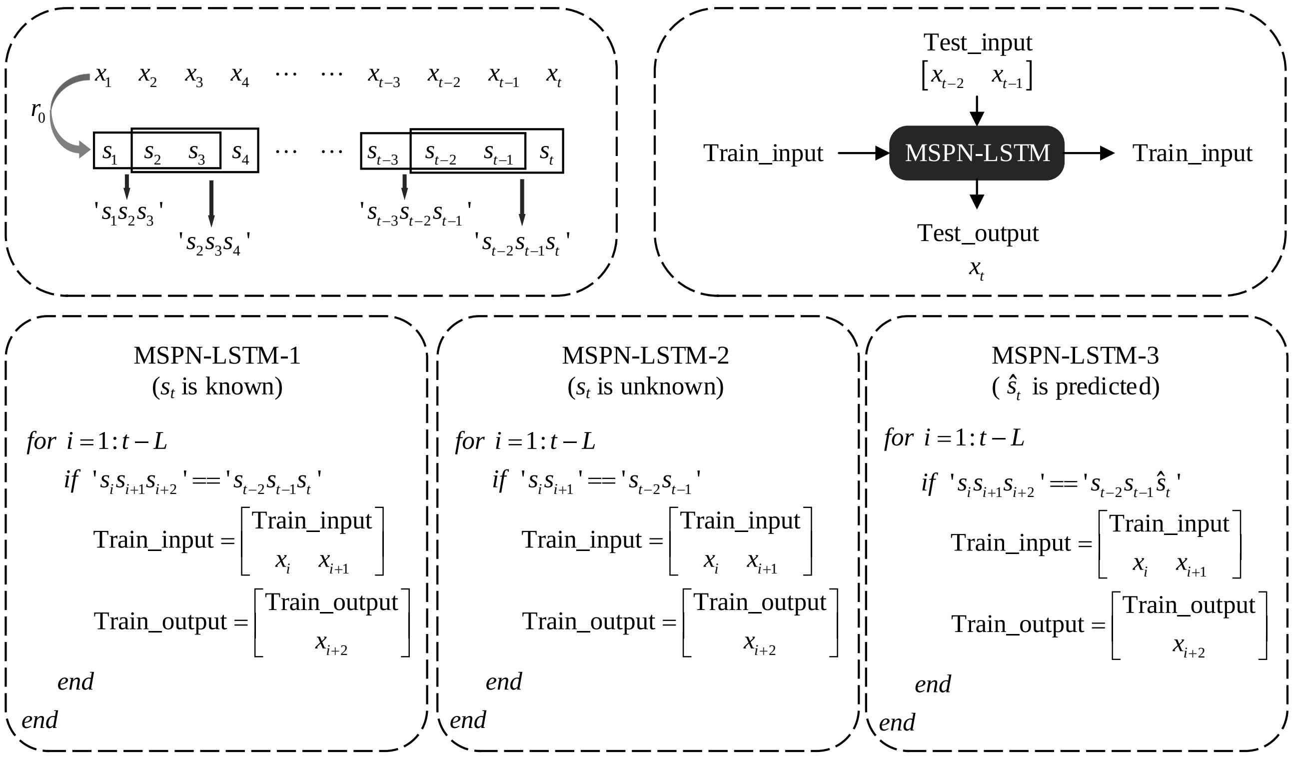

Figure 5: Comparison of input and output of three MSPN-LSTM models

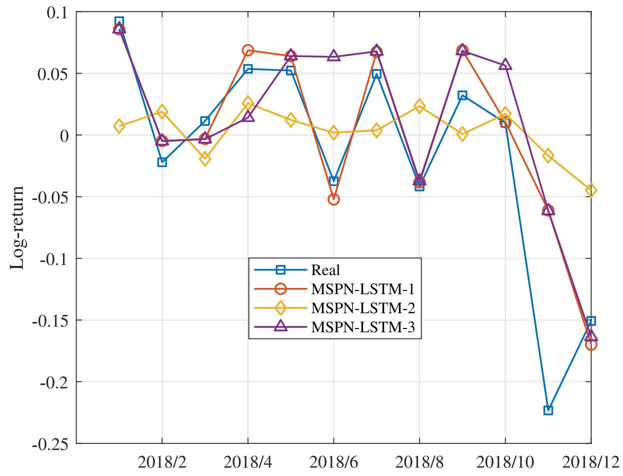

Figure 6: Comparison of the predicted value with the real

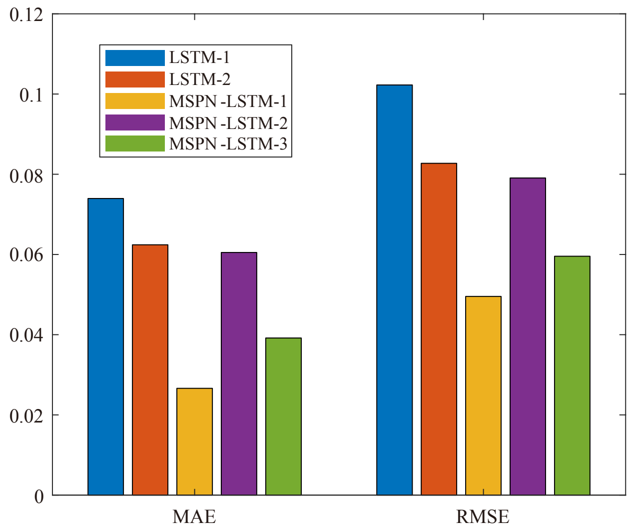

Figure 7: Error of LSTMs and MSPN-LSTMs

Table 1: The prediction error and program running time of LSTMs and MSPN-LSTMs

	RMSE	**MAE**	**Time (unit: s)**
LSTM-1	0.102261253654519	0.0739550199166667	46.297733
LSTM-2	0.0827308633699711	0.0624134329166667	163.2178
MSPN-LSTM-1	0.0495418634649424	0.0266253454166667	116.1305
MSPN-LSTM-2	0.0790692420633821	0.06049588	106.0897
MSPN-LSTM-3	0.0595576016272246	0.0391753218333333	111.8402

Figure 6 records the volatility prediction values at corresponding moments output by different hybrid models. It can be seen from the selection method of the training set that when the accuracy of the fluctuation trend prediction model in Chapter 4 reaches 100%, MSPN-LSTM-3 is transformed into MSPN-LSTM-1. Analyze the errors of the three prediction models and calculate their RMSE and MAE respectively. It can be seen from Table 1 that the calculation time of the three models is equivalent. From the RMSE point of view, MSPN-LSTM-1 has the smallest error 0.0495, MSPN-LSTM-3 reached 0.0596. Compared with MSPN-LSTM-2, the former has obvious advantages. From the perspective of MAE, MSPN-LSTM-1 and MSPN-LSTM-3 have the similar error, and MSPN-LSTM-2 has the largest error.

3.5.4 Error comparative analysis

The RMSE and MAE of the five prediction models of the established comparative experiment are compared and analyzed, and the results are shown in Figure 7.

As can be seen from Figure 7, among the five models proposed in this chapter, MSPN-LSTM-1 has the smallest prediction error, followed by MSPN-LSTM-3, and LSTM-1 has the worst prediction effect. MSPN-LSTM-1 assumes that all fluctuation trends are predicted correctly, so the prediction effect is the best. The MSPN-LSTM-3 is based on the actual oil price fluctuation trend prediction, so it can better reflect the actual prediction performance of the model. For MSPN-LSTM-3, the LSTM neural network prediction model combined with the multi-layer symbolic pattern network reduces the RMSE and MAE by 28% and 37%, respectively, compared with the single LSTM prediction model LSTM-2.

In summary, combining the multi-layer network symbolic prediction model with the LSTM neural network time series prediction model can significantly improve the prediction accuracy of the model, but the prerequisite for using MSPN-LSTM is to accurately predict the fluctuation trend at the corresponding moment. Because if the volatility trend forecast is incorrect or unpredictable, not only will it not bring obvious forecast gains, but it will also increase the complexity of the operation, and the gain will not be worth the loss.

4 Conclusion

This article focuses on the time series forecasting problem. Considering the high-dimensional characteristics of time series data and the large amount of noise, a single traditional econometric model and modern neural network methods cannot accurately predict time series. Theory has gradually become a popular method in time series analysis. Therefore, a novel time series data hybrid forecasting model that can not only characterize the dynamic characteristics between different dimensions of the time series, but also has high forecasting accuracy has been established. The main conclusions of this article are as follows:

(1) Taking the forecast of WTI crude oil futures price volatility as an example, a multi-layer symbolic model network is constructed from multiple time series, and the single-layer network and the multi-layer network model are compared and analyzed from the perspectives of the prediction direction accuracy and the increase in the prediction accuracy. Verified the superiority of the multi-layer symbolic pattern network.

(2) Based on the prediction of oil price fluctuation trend, the specific volatility rate of oil price is predicted, combined with the currently popular LSTM neural network, the selection process of the training set and test set in the neural network prediction model is optimized, and the historical data is Volatility fragments with consistent volatility trends are input to LSTM model training, thereby shielding the impact of unmatched volatility fragments on the prediction accuracy. The results show that the LSTM prediction model combined with the multi-layer symbolic mode network and the single LSTM neural network prediction model have higher performance Forecast accuracy and smaller forecast errors.

In the future, improvements will be made in the following aspects: (1) Increase the consideration of mutation points. (2) Consider the variable Markov order. (3) Choose more auxiliary variables that have a significant impact on the forecast sequence.

Acknowledgments

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