Soret and Dufour Effects on Mixed Convection in a Non-Darcy Micropolar Fluid

D.Srinivasacharya *, Ch.RamReddy
Department of Mathematics
National Institute of Technology, Warangal-506004, A.P., India.
(Received 31 August 2010 , accepted 28 October 2010 )

Abstract: The Soret and Dufour effects on the steady, laminar mixed convection heat and mass transfer along a semi-infinite vertical plate embedded in a micropolar fluid saturated non-Darcy porous medium with heat and mass flux conditions are investigated. The non-linear governing equations and their associated boundary conditions are initially cast into dimensionless forms using similarity transformations. The resulting system of equations is then solved numerically using the Keller-box method. The numerical results are compared and found to be in good agreement with previously published results as special cases of the present investigation. The non-dimensional velocity, microrotation, temperature and concentration profiles are displayed graphically for different values of coupling number, Forchheimer number, Soret and Dufour numbers. In addition, the skin-friction coefficient, Nusselt number and Sherwood number are shown in a tabular form.

Keywords: Mixed convection; Non-Darcy porous medium; Micropolar fluid; Soret and Dufour effects

1 Introduction

The analysis of mixed convection boundary layer flow along a vertical plate embedded in a fluid saturated porous media has received considerable theoretical and practical interest. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. Several authors have studied the problem of mixed convection in different surface geometries. The analysis of convective transport in a porous medium with the inclusion of non-Darcian effects has also been a matter of study in recent years. The inertia effect is expected to be important at a higher flow rate and it can be accounted for through the addition of a velocity squared term in the momentum equation, which is known as the Forchheimer’s extension of the Darcy’s law. A detailed review of convective heat transfer in Darcy and non-Darcy porous medium can be found in the book by Nield and Bejan [1].

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier and Ficks laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Dufour effect was recently found to be of order of considerable magnitude such that it cannot be neglected [Eckert and Drake[2]]. Dursunkaya and Worek[3] studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams[4] presented the same effects on mixed convective and mass transfer steady laminar boundary layer flow over a vertical flat plate with temperature dependent viscosity. Postelnicu[5] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Both free and forced convection boundary layer flows with Soret and Dufour have been addressed by Abreu [6]. Alam and Rahman [7] investigated the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction.

* Corresponding author. E-mail address: dsc@nitw.ac.in, dsrinivasacharya@yahoo.com

Copyright © World Academic Press, World Academic Union
IUNS.2011.04.15/470
Recently, the effect of Soret and Dufour parameters on free convection heat and mass transfer from a vertical surface in a doubly stratified Darcian porous medium has been reported by Lakshmi Narayana and Murthy[8].

The study of non-Newtonian fluid flows has gained much attention from researchers because of its applications in biology, physiology, technology and industry. In addition, the effects of heat and mass transfer in non-Newtonian fluid also have great importance in engineering applications such as thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Several investigators have extended many of the available convection heat and mass transfer problems to include the non-Newtonian effects. Many of the non-Newtonian fluid models describe the nonlinear relationship between stress and the rate of strain. But the micropolar fluid model introduced by Eringen[9] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, they can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphological sediments, haematological suspensions, liquid crystals, lubricants etc.

The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and porous media is presented by Lukaszewicz[10]. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering and industrial manufacturing processes. The problem of mixed convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators. Ahmad[11] studied the boundary layer flow of a micropolar fluid over a semi-infinite plate. Laminar mixed convection boundary layer flow of a micropolar fluid from an isothermal vertical flat plate has been considered by Jena and Mathur[12]. Markin and Mahmood[13] obtained similarity solutions for the mixed convection flow over a vertical plate for the case of constant heat flux condition at the wall. Gorla [14] presented asymptotic boundary layer solutions in order to study the combined convection from a vertical semi-infinite plate to a micropolar fluid with uniform heat flux. The heat transfer process in a two-dimensional steady hydromagnetic natural convective flow of a micropolar fluid over an inclined permeable plate subjected to a constant heat flux condition have been analyzed numerically by Rahman [15]. Although the Soret and Dufour effects of the medium on the heat and mass transfer in a micropolar fluid is important, very little work has been reported in the literature. Beg [16] analyzed the two dimensional coupled heat and mass transfer of an incompressible micropolar fluid past a moving vertical surface embedded in a Darcy- Forchheimer porous medium in the presence of significant Soret and Dufour effects. A mathematical model for the steady thermal convection heat and mass transfer in a micropolar fluid saturated Darcian porous medium in the presence of significant Dufour and Soret effects and viscous heating is presented by Rawat and Bhargava[17].

The aim of the present paper is to investigate the Soret and Dufour effects on the mixed convection from a semi-infinite vertical plate embedded in a stable, micropolar fluid saturated non-Darcy porous medium with uniform and constant heat and mass flux conditions. The Keller-box method given in Cebeci and Bradshaw[18] is employed to solve the nonlinear system of this particular problem. The effects of micropolar parameter, non-Darcy parameter, Soret and Dufour numbers are examined and are displayed through graphs. The results are compared with relevant results in the existing literature and are found to be in good agreement.

2 Mathematical Formulation

Consider a steady, laminar, incompressible, two-dimensional mixed convective heat and mass transfer along a semi infinite vertical plate embedded in a free stream of micropolar fluid saturated non-Darcy porous medium. The free stream velocity which is parallel to the vertical plate is $u_\infty$, temperature is $T_\infty$ and concentration is $C_\infty$. Assume that the fluid and the porous medium have constant physical properties. The fluid flow is moderate and the permeability of the medium is low so that the Forchheimer flow model is applicable and the boundary effect is neglected. The fluid and the porous medium are in local thermodynamical equilibrium. Choose the coordinate system such that $x$-axis is along the vertical plate and $y$-axis normal to the plate. The physical model and coordinate system are shown in Fig.(1). The plate is maintained at uniform and constant heat and mass fluxes $q_u$ and $q_m$ respectively. In addition, the Soret and Dufour effects are considered.

Assuming that the Boussinesq and boundary-layer approximations hold and using the Darcy-Forchheimer model and Dupuit-Forchheimer relationship [1], the governing equations for the micropolar fluid are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$ (1)
\[
\frac{\rho}{\varepsilon^2} \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \left( \mu + \kappa \right) \frac{\partial^2 u}{\partial y^2} + \kappa \frac{\partial \omega}{\partial y} + \rho g^* \left( \beta_T (T - T_\infty) + \beta_C (C - C_\infty) \right) + \frac{\mu}{K_p} (u_\infty - u) + \frac{\rho_b}{K_p} (u_\infty^2 - u^2) \quad (2)
\]

\[
\frac{\rho j}{\varepsilon} \left( \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} \right) = \gamma \frac{\partial^2 \omega}{\partial y^2} - \kappa \left( 2\omega + \frac{1}{\varepsilon} \frac{\partial u}{\partial y} \right) \quad (3)
\]

\[
\frac{u}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{DK_T}{C_sC_p} \frac{\partial^2 C}{\partial y^2} \quad (4)
\]

\[
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{D}{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (5)
\]

where \( u \) and \( v \) are the Darcy velocity components in \( x \) and \( y \) directions respectively, \( \omega \) is the component of microrotation whose direction of rotation lies in the \( xy \)-plane, \( T \) is the temperature, \( C \) is the concentration, \( g^* \) is the acceleration due to gravity, \( \rho \) is the density, \( \mu \) is the dynamic coefficient of viscosity, \( b \) is the empirical constant, \( \varepsilon \) is the porosity, \( \beta_T \) is the coefficient of thermal expansion, \( \beta_C \) is the coefficient of solutal expansions, \( \kappa \) is the vortex viscosity, \( j \) is the micro-inertia density, \( \gamma \) is the spin-gradient viscosity, \( \alpha \) is the effective thermal diffusivity, \( D \) is the effective solutal diffusivity of the medium, \( C_p \) is the specific heat at constant pressure, \( C_s \) is the concentration susceptibility, \( T_m \) is the mean fluid temperature and \( K_T \) is the thermal diffusion ratio. The subscript \( \infty \) indicates the condition at the outer edge of the boundary layer. The last two terms on the right hand side of Eq.(2) stand for the first-order (Darcy) resistance and second-order porous inertia resistance, respectively.

The boundary conditions are

\[
\begin{align*}
  u &= 0, \quad v = 0, \quad \omega = 0, \quad q_w = -k \frac{\partial T}{\partial y}, \quad q_m = -D \frac{\partial C}{\partial y}, \quad a t \quad y = 0 \quad (6a) \\
  u &= u_\infty, \quad \omega = 0, \quad T = T_\infty, \quad C = C_\infty \quad a s \quad y \to \infty \quad (6b)
\end{align*}
\]

where \( k \) is the thermal conductivity of the fluid. The boundary condition \( \omega = 0 \) in Eq.(6a), represents the case of concentrated particle flows in which the microelements close to the wall are not able to rotate.
In view of the continuity Eq. (1), we introduce the stream function \( \psi \) by

\[
\eta = \frac{y}{x} Re^{1/2}, \quad \psi = \nu Re^{1/2} f(\eta), \quad \omega = \frac{\nu}{x^2} Re^{3/2} g(\eta),
\]

Substituting Eq. (7) in Eqs. (2)-(5) and then using the following similarity transformations

\[
\theta(\eta) = \frac{T - T_\infty}{q_w x} Re^{1/2}, \quad \phi(\eta) = \frac{C - C_\infty}{q_m x} Re^{1/2}
\]

we get the following nonlinear system of differential equations.

\[
\left( \frac{1}{(1 - N)} \right) f'''' + \frac{1}{2\epsilon} f''' + \left( \frac{N}{1 - N} \right) g'' + g, \theta + g, \phi + \frac{1}{Da Re} (1 - f') + \frac{F_s}{Da} (1 - f'^2) = 0
\]

\[
\lambda g'' + \frac{1}{2\epsilon} f' g' + \frac{1}{2\epsilon} f' g - \left( \frac{N}{1 - N} \right) J(2g + \frac{1}{\epsilon} f'') = 0
\]

\[
\frac{1}{Pr} \theta'' + \frac{1}{2} f' \theta' - \frac{1}{2} f' \theta + Da \phi'' = 0
\]

\[
\frac{1}{Sc} \phi'' + \frac{1}{2} f' \phi' - \frac{1}{2} f' \phi + S, \theta'' = 0
\]

where the primes indicate partial differentiation with respect to \( \eta \) alone, \( \nu \) is the kinematic viscosity, \( Gr = \frac{g^* \beta_T q_w x^4}{k \nu^2} \) is the thermal Grashof number, \( Ge = \frac{g^* C q_m x^4}{D \nu^2} \) is the solutal Grashof number, \( Re = \frac{u_\infty x}{\nu} \) is the local Reynolds number, \( g_s = \frac{G_r}{Re^{5/2}} \) is the temperature buoyancy parameter, \( g_c = \frac{G_c}{Re^{5/2}} \) is the mass buoyancy parameter, \( Pr = \frac{\nu}{\alpha} \) is the Prandtl number, \( Sc = \frac{\nu}{\mu} \) is the Schmidt number, \( Da = \frac{K_p}{x^2} \) is the local Darcy number, \( F_s = \frac{b}{x} \) is the local Forchheimer number, \( J = \frac{D}{\rho \nu} \) is the micro-inertia density and \( \lambda = \frac{\gamma}{\rho^2 \nu} \) is the spin-gradient viscosity, \( N = \frac{K}{\mu + \kappa} \)

\( 0 \leq N < 1 \) is the Coupling number \([19] \), \( D_f = \frac{DK_T q_m k}{C_s C_p v q_m D} \) is the Dufour number and \( S_r = \frac{DK_T q_w D}{T_m \nu q_m k} \) is the Soret number.

The boundary conditions (6) in terms of \( f, g, \theta, \phi \) become

\[
\eta = 0: \quad f(0) = 0, \quad f'(0) = 0, \quad g(0) = 0, \quad \theta'(0) = -1, \quad \phi'(0) = -1
\]

\[
\eta \to \infty: \quad f'(\infty) = 1, \quad g(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0.
\]

If \( Da \to \infty, \epsilon = 1, D_f = 0 \) and \( S_r = 0 \), the problem reduces to mixed convection heat and mass transfer on a semi infinite vertical plate with uniform heat and mass fluxes in a micropolar fluid without Soret and Dufour effects. We notice that for \( N = 0 \), where the flow, temperature and concentration fields are unaffected by the microstructure of the fluid and the microrotation component is a passive quantity. Hence, in the limit, as \( N \to 0 \), the governing Eqs.(1)-(5) reduce to the corresponding equations for a mixed convection heat and mass transfer in a viscous fluids.

The wall shear stress, heat and mass transfers from the plate respectively are given by

\[
\tau_w = \left[ (\mu + \kappa) \frac{\partial u}{\partial y} + \kappa \omega \right]_{y=0}
\]

\[
q_w = -k \left[ \frac{\partial T}{\partial y} \right]_{y=0}
\]

\[
q_m = -D \left[ \frac{\partial C}{\partial y} \right]_{y=0}
\]
The non-dimensional skin friction coefficient $C_f = \frac{2\tau_w}{\rho U_*^2}$, the local Nusselt number $Nu_x = \frac{q_{w,x}}{k(T_w - T_\infty)}$ and local Sherwood number $Sh_x = \frac{q_w x}{D(C_w - C_\infty)}$, where $U_*$ is the characteristic velocity, are given by

$$C_f Re^{1/2} = \left( \frac{2}{1 - N} \right) f''(0), \quad (15a)$$

$$\frac{Nu_x}{Re^{1/2}} = \frac{1}{\theta(0)}, \quad \frac{Sh_x}{Re^{1/2}} = \frac{1}{\phi(0)}. \quad (15b)$$

3 Results and Discussions

The flow Eqs. (9) and (10) which are coupled, together with the energy and concentration Eqs. (11) and (12), constitute non-linear nonhomogeneous differential equations for which closed-form solutions cannot be obtained. Hence the governing Eqs. (9) to (12) have been solved numerically using the Keller-box implicit method [Cebeci and Bradshaw [18]]. The method has the following four main steps:

i. Reduce the system of Eqs. (9) to (12) to a first order system;

ii. Write the difference equations using central differences;

iii. Linearize the resulting algebraic equations by Newton’s method and write them in matrix-vector form;

iv. Use the block-tridiagonal-elimination technique to solve the linear system.

This method has been proven to be adequate and give accurate results for boundary layer equations. In the present study, the boundary conditions for $\eta$ at $\infty$ are replaced by a sufficiently large value of $\eta$ where the velocity approaches one and microrotation, temperature and concentration approach zero. The value of $\eta_{\infty}$ is taken as 10 and a grid size of $\eta$ as 0.01. Also, the solutions for the dimensionless velocity, angular momentum, temperature and concentration function as shown graphically in Figs. (2-9). In order to study the effects of micropolar parameter $N$, Forchheimer number $F_s$, Soret number $S_r$ and Dufour number $D_f$ explicitly, computations were carried out for the cases of $g_s = 1.0$ and $g_r = 0.1$, $Pr = 0.71$ (air), $Sc = 0.22$ (hydrogen at 25°C and 1 atmosphere pressure), $Re = 200$, $\epsilon = 0.3$ and $Da = 1.0$. The values of Soret number $S_r$ and Dufour number $D_f$ are chosen such a way that their product is constant according to their definition provided that the mean temperature $T_m$ is kept constant. The values of micropolar parameters $J = 0.1$ and $\lambda = 1.0$ are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen[9].

In order to verify the accuracy of the present method, the results are compared with those cases reported by Cebeci and Bradshaw [18], Yin [20], Chamkha [21] and Lin and Lin [22], as shown in Tables (1) and (2) and the comparisons are found to be in a very good agreement. Therefore, the developed code can be used with great confidence to study the problem considered in this paper.

In Figs. (2-5), the effects of the coupling number $N$ on the dimensionless velocity, microrotation, temperature and concentration are presented for fixed values of Forchheimer number $F_s$, Soret and Dufour numbers. As $N$ increases, it can be observed from Fig.(2) that the maximum velocity decreases in amplitude and the location of the maximum velocity moves farther away from the wall. Since $N \to 0$ corresponds to viscous fluid, the velocity in case of micropolar fluid is less compared to that of viscous fluid case. From Fig.(3), we observe that the microrotation completely negative within the boundary layer. As $N \to 0$, the microrotation tends to zero because in the limit $N$ tends to zero the micro polarity is lost and the fluid is to behave as non-polar fluid. It is clear from Fig.(4) that the temperature boundary layer increases with the increase of coupling number $N$. It can be seen from Fig.(5) that the concentration boundary layer of the fluid increases with the increase of coupling number $N$. The temperature and concentration in case of micropolar fluids is more than that of the Newtonian fluid case.

The dimensionless velocity component for different values of Forchheimer number $F_s$ with $N = 0.3$, $S_r = 2.0$ and $D_f = 0.03$ is depicted in Fig.(6). It shows the effects of Forchheimer (inertial porous) number on the velocity. In the absence of Forchheimer number (i.e., when $F_s = 0$), the present investigation reduces to a mixed convection heat and mass transfer in a micropolar fluid saturated porous medium with Soret and Dufour effects. It is observed from Fig.(6) that velocity of the fluid decreases with increase in the value of the non-Darcy parameter $F_s$. The increase in non-Darcy parameter implies that the porous medium is offering more resistance to the fluid flow. This results in reduction of the velocity profile. From Fig.(7), it can be observed that the microrotation changes in sign from negative to positive within the boundary layer. The dimensionless temperature for different values of Forchheimer number $F_s$ for $N = 0.3$, $S_r = 2.0$
and \( D_f = 0.03 \), is displayed in Fig. (8). An increase in Forchheimer number \( F_s \), increase temperature values, since as the fluid is decelerated, energy is dissipated as heat and serves to increase temperatures. As such the temperature is minimized for the lowest value of \( F_s \) and maximized for the highest value of \( F_s \) as shown in Fig.(8). Fig.(9) demonstrates the dimensionless concentration for different values of Forchheimer number with \( N = 0.3, S_r = 2.0 \) and \( D_f = 0.03 \). It is clear that the concentration of the fluid increases with the increase of Forchheimer number. The increase in non-Darcy parameter reduces the intensity of the flow and increases the thermal and concentration boundary layer thickness.

Fig.(10) displays the non-dimensional velocity for different values of Soret number \( S_r \) and Dufour number \( D_f \) with fixed values of coupling number \( N \) and Forchheimer number \( F_s \). It is observed that the velocity of the fluid increases with the increase of Dufour number (or decrease of Soret number). From Fig.(11), it can be noted that the microrotation changes in sign from negative to positive within the boundary layer. The dimensionless temperature for different values of Soret number \( S_r \) and Dufour number \( D_f \) for \( N = 0.3 \) and \( F_s = 0.5 \), is shown in Fig.(12). It is clear that the temperature of the fluid increases with the increase of Dufour number (or decrease of Soret number). Fig. (??) demonstrates the dimensionless concentration for different values of Soret number \( S_r \) and Dufour number \( D_f \) for \( N = 0.3 \) and \( F_s = 0.5 \). It is seen that the concentration of the fluid decreases with increase of Dufour number (or decrease of Soret number).

The variations of \( f''(0), \frac{1}{\theta(0)} \) and \( \frac{1}{\phi(0)} \) which are proportional to the local skin-friction coefficient, heat and mass transfer rates are shown in Table(3) for different values of the coupling number with fixed \( F_s = 0.5, S_r = 2.0 \) and \( D_f = 0.03 \). It shows that the skin friction factor is lower for micropolar fluid than the Newtonian fluids \( N = 0 \). Since micropolar fluids offer a great resistance (resulting from vortex viscosity) to the fluid motion and causes larger skin friction factor compared to Newtonian fluid. The heat and mass transfer rates decrease with the increasing values of coupling number. Further, it can be noticed that the heat and mass transfer coefficients are more in case of viscous fluids. That is, as \( N \) increases, the thermal and solutal boundary layer thickness become large, thus give rise to a small values of local heat and mass transfer rates. Since the skin friction coefficient as well as heat and mass transfer are lower in the micropolar fluid compared to the Newtonian fluid, which may be beneficial in flow, temperature and concentration control of polymer processing. Therefore, the presence of microscopic effects arising from the local structure and micromotion of the fluid elements reduce the heat and mass transfer coefficients. The opposite nature can be found in the case of Forchheimer number. Hence the non-Darcy parameter has an important role in controlling the flow field. Finally, the effects of Dufour and Soret number on the local skin-friction coefficient and the rate of heat and mass transfer are shown in this table. The behavior of these parameters is self-evident from the Table(3) and hence are not discussed for brevity.

| Table 1: Values of \( \frac{1}{2} \) for fixed values of \( N = 0, Da \rightarrow \infty, \epsilon = 1, \lambda \rightarrow 0, \mathcal{J} = 0, g_s = 0 \) and \( g_c = 0 \) |
|-----------------|-----------------|-----------------|-----------------|
| 0.33206 | 0.332057 | 0.332206 | 0.33206 |

4 Conclusions

In this paper, a boundary layer analysis for mixed convection heat and mass transfer in a non-Darcy micropolar fluid over a vertical plate with uniform heat and mass flux conditions in the presence of Soret and Dufour effects is considered. Using the similarity variables, the governing equations are transformed into a set of nonsimilar parabolic equations where numerical solution has been presented for a different values of parameters. The higher values of the coupling number \( N \) (i.e., the effect of microstructure becomes significant) result in lower velocity distribution and but higher wall temperature, wall concentration distributions in the boundary layer compared the Newtonian fluid case. The numerical results indicate that the skin friction coefficient as well as rate of heat and mass transfer in the micropolar fluid are lower compared to that of the Newtonian fluid. The higher values of the Forchheimer number \( F_s \) indicate lower velocity distribution and but higher wall temperature, wall concentration distributions. Also, the skin friction coefficient as well as rates of heat and mass transfer increase as Forchheimer number increases. The present analysis has also shown that the flow field is appreciably influenced by the Dufour and Soret effects.

\[ \text{IJNS homepage: } \text{http://www.nonlinearscience.org.uk/} \]
Table 2: Values of $Nu_x/Re^{1/2}$ for various values of Prandtl number with $N = 0$, $Da \to \infty$, $\epsilon = 1$, $Sr = 0$, $D_f = 0$, $g_s = 0$, $g_c = 0$ and $Sc \to 0$

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>Lin and Lin[20]</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.0775587</td>
<td>0.0775588</td>
</tr>
<tr>
<td>0.1</td>
<td>0.200655</td>
<td>0.200657</td>
</tr>
<tr>
<td>1.0</td>
<td>0.458971</td>
<td>0.458973</td>
</tr>
<tr>
<td>10</td>
<td>0.979788</td>
<td>0.979710</td>
</tr>
<tr>
<td>100</td>
<td>2.15196</td>
<td>2.152223</td>
</tr>
</tbody>
</table>

Table 3: Effects of skin friction, heat and mass transfer coefficients for varying values of coupling, Forchheimer, Soret and Dufour numbers.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$F_s$</th>
<th>$S_r$</th>
<th>$D_f$</th>
<th>$f''(0)$</th>
<th>$\frac{1}{\theta(0)}$</th>
<th>$\frac{1}{\phi(0)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>1.38202</td>
<td>0.58347</td>
<td>0.29316</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>1.29585</td>
<td>0.57444</td>
<td>0.29002</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>1.20376</td>
<td>0.56418</td>
<td>0.28637</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>1.10422</td>
<td>0.55235</td>
<td>0.28208</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>0.99500</td>
<td>0.53844</td>
<td>0.27692</td>
</tr>
<tr>
<td>0.6</td>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>0.87271</td>
<td>0.52168</td>
<td>0.27055</td>
</tr>
<tr>
<td>0.7</td>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>0.73202</td>
<td>0.50068</td>
<td>0.26238</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>0.56428</td>
<td>0.47267</td>
<td>0.25119</td>
</tr>
<tr>
<td>0.9</td>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>0.35518</td>
<td>0.42879</td>
<td>0.23282</td>
</tr>
<tr>
<td>0.3</td>
<td>0.1</td>
<td>2.0</td>
<td>0.03</td>
<td>1.01770</td>
<td>0.53645</td>
<td>0.27356</td>
</tr>
<tr>
<td>0.3</td>
<td>0.4</td>
<td>2.0</td>
<td>0.03</td>
<td>1.16046</td>
<td>0.55814</td>
<td>0.28358</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>2.0</td>
<td>0.03</td>
<td>1.28496</td>
<td>0.57489</td>
<td>0.29131</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0</td>
<td>2.0</td>
<td>0.03</td>
<td>1.39534</td>
<td>0.58828</td>
<td>0.29745</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>2.0</td>
<td>0.03</td>
<td>1.20376</td>
<td>0.56418</td>
<td>0.28637</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>1.6</td>
<td>0.0375</td>
<td>1.20106</td>
<td>0.56300</td>
<td>0.29770</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>1.2</td>
<td>0.05</td>
<td>1.19877</td>
<td>0.56136</td>
<td>0.30999</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>1.0</td>
<td>0.06</td>
<td>1.19794</td>
<td>0.56018</td>
<td>0.31655</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>0.8</td>
<td>0.075</td>
<td>1.19752</td>
<td>0.55855</td>
<td>0.32343</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>0.5</td>
<td>0.12</td>
<td>1.19877</td>
<td>0.55402</td>
<td>0.33445</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>0.2</td>
<td>0.3</td>
<td>1.21122</td>
<td>0.53761</td>
<td>0.34705</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>0.1</td>
<td>0.6</td>
<td>1.23506</td>
<td>0.51276</td>
<td>0.35287</td>
</tr>
</tbody>
</table>

Figure 2: Velocity profiles for various values of $N$.

Figure 3: Microrotation profiles for various values of $N$. 

*IJNS email for contribution:* editor@nonlinearscience.org.uk
Figure 4: Temperature profiles for various values of $N$.

Figure 5: Concentration profiles for various values of $N$.

Figure 6: Velocity profiles for various values of $F_s$.

Figure 7: Microrotation profiles for various values of $F_s$.

Figure 8: Temperature profiles for various values of $F_s$.

Figure 9: Concentration profiles for various values of $F_s$.

IJNS homepage: http://www.nonlinearscience.org.uk/
Figure 10: Velocity profiles for various values of $S_r$ and $D_f$.

Figure 11: Microrotation profiles for various values of $S_r$ and $D_f$.

Figure 12: Temperature profiles for various values of $S_r$ and $D_f$.

Figure 13: Concentration profiles for various values of $S_r$ and $D_f$. 

IJNS email for contribution: editor@nonlinearscience.org.uk
References


IJNS homepage: http://www.nonlinearscience.org.uk/