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The Three-factor Option Decision Model of Coal Development Investment Project Based on Jump-Diffusion Coal Price

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Abstract: It is significant for scientific and rational decision-making method of coal resources development investment (for short CRDI) to evaluate reasonably the value of coal resources investment, to make the best coal resources investment program, and to timely manage the risks and flexibility in investment process. On the basis of analyzing the option characteristics of CRDI, CRDI could be regarded as a compound real option. In fact, when there are a lot of abrupt events, jump diffusion process can better fit the changing tendency of spot price; risk-free interest rate and convenience yield were not constant. A three-factor model is set up when coal spot price follows jump-diffusion process, interest rate follows HJM model and convenience yield follows mean-reverting process. Through a real example application, we found the model can get higher values.

Keywords: coal resources development investment; real option; stochastic convenience yield; stochastic interest rates; jump-diffusion; three-factor model

1 Introduction

Coal resource is Chinese first energy. It's predicted that by 2020a coal will still be Chinese primary energy, its proportion in the composition of the production and consumption of Chinese one-off energy will not be below 60%, and not below 50% within 50 years. Therefore, the position of coal is steady and long-term in Chinese energy constitution. Through investigation and research, we predict that in 2020 coal capacity will approach to 3.5 billion tons, newly increasing coal capacity about 0.05 billion tons every year, newly increasing investment about 30∼35 billion Yuan. Thus we can see that in quite a long period of time in the future, the scale of coal resource investment and development will be very large and in order to make these investments effectively used in complicated and changeable market, a set of scientific and reasonable methods must be set up to properly evaluate, decide and manage these coal resource investment projects.

The discounted cash flows (DCF) is considered as the main approach to valuate CRDI at home and abroad. On the background of global competition, CRDI is affected by more and more uncertain factors, which makes the inherent defects and limitations of DCF stand out much more. In fact, CRDI has some characteristics as follows. First, future income and interest are uncertain, and we can only know their probability distribution. The second is "the flexibility". CRDI has some room to choose the time of genuine investment. At the evaluating time point, NPV is less than 0, we can choose to wait; once market becomes better and NPV is more than 0, we can choose investment; if the market doesn't become better for a very long period time, we can choose to give it up permanently; after investment, we also have all kinds of alternatives according to market such as enlarging mining scale, pausing mining, stopping mining permanently or closing mines and so on. These flexibilities raise the value of CRDI. Obviously, traditional DCF cannot consider these characteristics of CRDI and catch the flexibilities, alternatives and uncertainties existing in whole investment decision so that the value of CRDI was undervalued. All in all, a scientific, proper coal resource investment decision and managing method must be found. Real option is first put forward by Stewart C. Myers in 1977, and develops on the basis of financial option theories set up by Black, Scholes and Merton[1∼2]. Using option theory, Brennan and Schwrts, Paddock and Siege, Zhang Neng-fu and

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Cai Si-jing, Liao Zuohong and Li Xiaozhao, Zhang Jinsuo, researched on evaluations of natural resources investments, mineral resources assets[3−8]. However, risk-free interest rate and convenience yield were constant in these researches; the coal spot price follows Geometrical Brownian Motion. In fact, when there are a lot of abrupt events, jump diffusion process can better fit the changing tendency of spot price. Therefore, this paper tries to set up a three-factor model of evaluating CRDI under stochastic convenience yield, stochastic interest rate, and jump diffusion coal spot price.

2 The characteristics of coal resource investment

In China, government divides mine right into exploring right and mining right. Now suppose exploring stage has been finished and government has received the information about reserve and cost. Once government grants mining license, the owner of mining right comes into the development stage. In development stage, the owner of mining right can choose to delay developing, continue developing or stop developing. It's obvious that every stage above has flexibility which can realize the optimum allocation of coal resource and avoid the risks of project failure to the full extent.

In the process of coal developing, there are uncertainties on coal price, coal reserve, developing cost and developing conditions, which will diminish generally with the increase of information and passage of time. The owner of mining right should adapt himself to these uncertainties actively during the validity of mining license, so as to timely adjust produce and operation. Develop, delay developing, enlarge produce scale, reduce produce scale and abandon mining can all change the current flow and the risk distribution, all of which should be valuable. For example, choosing to develop can make the owner of mining rights get normal investment revenue and occupy strategic activeness in market competition, choosing to delay developing can bring value from waiting, choosing to abandon mining can bring the revenue from transferring risks. The important norm by which the owner of mining rights chooses to develop, delay developing, enlarge produce scale, reduce produce scale or abandon mining is the relationship between the coal spot price and coalmine full operational cost. After waiting a period of time and predicts coal price is still less than the complete cost of tonnage coal, coal mining rights owner can choose to abandon developing in order to avoid larger loss. Therefore, CRDI is a decision as to whether develop, delay developing, enlarge produce scale, reduce produce scale or abandon developing, and so on, investment management is management of such options. The combined effects of each option constituted a process of investment characteristics of compound option. Factors of CRDI value include coal spot price, the volatility of coal spot price, risk-free rate, reserve levels, convenience yield, operating costs, and coal development scale. Therefore, under the no-arbitrage pricing theory and risk-neutral principle, the evaluation model of CRDI could be established based on real option theory.

3 The model

3.1 The assumptions

In order to set up the three-factor model under the stochastic convenience yield, stochastic spot coal price and stochastic interest rate, several assumptions are made as follows:

Assumption 1 The basic construction of coalmine has been finished, namely, the coalmine has mining conditions.

Assumption 2 There is no consideration of the costs caused by opening or closing coalmine or permanently abandoning coalmine. This makes the flexibility of exercising mining rights won't be influenced, so the value of coal reserves can be regarded as the difference between coal price and mining cost.

Assumption 3 There is no transaction cost of CRDI. That is, when the market is sufficiently efficient, obtaining or losing CRDI won't produce new cost because of "market friction". On the basis of the assumption, the price relationship between coal future and coal spot can be determined with the pricing principle of no-arbitrage.

Assumption 4 The interest rate is risk-free.

Assumption 5 The cost of mining coal resources is constant. Thus, the convenience yield can directly be determined by the price changes of coal spot and coal future. In reality, because the mining activity is influenced by geological condition, mining technology, the macro-economic policies and market conditions, the mining cost may have some changes. This will be discussed in our another paper.

3.2 Evaluation model

Suppose that the per unit value of CRDI at time$t$ isV_t, the coal spot price at time$t$ isS_t, the coalmine full operational cost is C_o. When $t = T$, the value of CRDI at development date is as follows

$$V_T = \max(S_T - C_o, 0) \tag{1}$$

In fact, when there are a lot of abrupt events, jump diffusion process can better fit the changing tendency of spot price. Therefore, the coal spot price S_t follows jump-diffusion process. That is

$$\frac{dS_t}{S_t} = (r - \delta - \lambda\overline{J})dt + \sigma_S dZ_S(t) + Jdg \tag{2}$$

Where, r is risk-free interest rate, $\delta$is the net convenience yield of coal reserves,$\sigma_S$is the volatility rate of $S_t$conditional on no jumps, $dZ_S(t)$is an increment to a Wiener process, $\lambda$is the (annual) frequency of jump, $J$is the jump size of each jump, $E(J) = \overline{J}$,$g$is the Poisson counter with intensityλ:$\Pr ob\{dg = 1\} = \lambda dt$and $\Pr ob\{dg = 0\} = 1 - \lambda dt$. According to the analysis in References, we know δ follows the mean-reverting process, that is

$$d\delta = [k_\delta(\hat{\alpha} - \delta) - \lambda_\delta\sigma_\delta]dt + \sigma_\delta dZ_\delta \tag{3}$$

Where, $\hat{\alpha}$is the long-term mean convenience yield,$k_\delta$is the speed of adjustment reflecting the supply and demand information, dZ_δ is an increment to a Wiener process, $\lambda_\delta$is the market price of risk for convenience yield of coal reserves.

The spot interest rate $r$is given by the following risk-neutralized Heath, Jarrow, and Merton (HJM) no-arbitrage process,

$$dr = \left(\frac{\partial f}{\partial v}(t,v) + k_r f(t,v) + \frac{\sigma_r^2}{2k_r}\left(1 - e^{-2k_r(v-t)}\right) - k_r r\right)dt + \sigma_r dZ_r \tag{4}$$

Where σ_r is the volatility rate of risk-free interest rate. The equation (4) is a Markov process. It is preference-free and depends on only two parameters $k_r$and σ_r, $dZ_r$is an increment to a Wiener process, the instantaneous standard deviation at time t of the T maturity forward rate is $\sigma_r e^{-k_r(T-t)}$. The forward interest rate term $f(t,T)$ corresponds to the instantaneous forward rate at time $t$for date $T > t$.

Suppose that$cov\,(dZ_S, dZ_\delta) = \rho_1 dt$ represents the correlation between the process (2) and the process (3),$cov\,(dZ_\delta, dZ_r) = \rho_2 dt$ represents the correlation between the process (3) and the process (4), and $cov\,(dZ_S, dZ_r) = \rho_3 dt$ represents the correlation between the process (2) and the process (4).

About valuation of derivative security conditional on stochastic convenience yields, interest rates, and jump diffusion in spot price, Hillard and Reis had set a valuation model of commodity futures and options under stochastic convenience yields, interest rates, and jump diffusions in the spot in 1998[9]. This paper will set up a three-factor model following the method of Hillard and Reis's research.

Now, we construct a coal future market, the price of coal future F which underlying asserts is coal product follows this process,

$$\widehat{dF} = \frac{\partial F}{\partial S}\sigma_S S dZ_S + \frac{\partial F}{\partial \delta}F_\delta\sigma_\delta dZ_\delta + \frac{\partial F}{\partial r}\sigma_r dZ_r - \frac{\partial F}{\partial S}S\overline{k^*}\lambda^* dt + \frac{\partial F}{\partial S}SJ^* dg \tag{5}$$

The Feynman-Kac solution for future price is

$$F\,(t,T) = E_t\,(S_T) \tag{6}$$

The reference [9] had given the solution to equation (6), the three-factor future price is

$$F\,(S_t,\delta_t,t,T) = S_t A\,(\tau)\,D_1\,(\tau)\,D_2\,(\tau)\,D_2\,(\tau)\,e^{-H_c(\tau)\delta_t}\frac{1}{B\,(t,T)} \tag{7}$$

Where, $B\,(t,T)$is the value at time $t$of a zero-coupon bound with maturity T,

$$A\,(\tau) = \exp\left[\frac{(H_c - \tau)\left(k_\delta^2\hat{\alpha} - k_\delta\lambda_\delta\hat{\alpha} - \sigma_\delta^2/2 + \rho_1\sigma_S\sigma_\delta k_\delta\right)}{k_\delta^2} - \frac{\sigma_\delta^2 H_c^2}{4k_\delta}\right]$$

$$D_1\,(\tau) = \exp\left[\frac{\rho_2\sigma_r\sigma_\delta}{k_\delta + k_r}\left(\frac{H_r - \tau}{k_r} + \frac{H_c - \tau}{k_\delta} + H_\delta H_r\right)\right] D_2\,(\tau) = \exp\left[\frac{\rho_3\sigma_r\sigma_S}{k_r}(\tau - H_r)\right]$$

$$D_3\,(\tau) = \exp\left[-(H_r - \tau)\frac{\sigma_r^2}{k_r^2} - \frac{\sigma_r^2 H_r^2}{2k_r}\right] H_c\,(\tau) = \frac{1 - e^{-k_\delta\tau}}{k_\delta} H_r\,(\tau) = \frac{1 - e^{-k_r\tau}}{k_r}.$$

Substituting $\frac{\partial F}{\partial S}$, $\frac{\partial F}{\partial \delta}$and $\frac{\partial F}{\partial r}$in equation (5), gives

$$\widehat{dF} = F\sigma_S dZ_S - FH_\delta(\tau)\sigma_\delta dZ_\delta + FH_r\sigma_r dZ_r - F\overline{J^*}\lambda^* dt + FJ^* dg \tag{8}$$

To further simplify the process in (8), define another standard Wiener process Z_F and a parameterσsuch that

$$\sigma dZ_F = \sigma_S dZ_S - H_\delta(\tau)\sigma_\delta dZ_\delta + H_r(\tau)\sigma_r dZ_r \tag{9}$$

Then, equation (9) reduces to

$$\widehat{dF/F} = -\overline{J^*}\lambda^* dt + \sigma dZ_F + J^* dg \tag{10}$$

Where J^* is the jump size of the price of coal reserves future, and

$$\begin{aligned} &\sigma^2(\sigma_S, \sigma_\delta, \sigma_r, \rho_1, \rho_2, \rho_3, k_r, k_\delta, \tau) \\ &= \sigma_S^2 + \sigma_\delta^2 H_c(\tau)^2 + \sigma_r^2 H_r(\tau)^2 - 2\sigma_S\sigma_\delta\rho_2 H_c(\tau) \\ &+2\sigma_S\sigma_r\rho_3 H_r(\tau) - 2\sigma_\delta\sigma_r\rho_3 H_c(\tau)H_r(\tau) \end{aligned} \tag{11}$$

Following the method of Hillard and Reis's research ideas, we give a three-factor model when coal spot price follows jump-diffusion process, interest rate follows HJM model and convenience yield follows mean-reverting process:

$$V(t, T_1, T) = B(t, T? \tag{12}$$

Where, $b(n) = -\overline{k^*}\lambda^* + \frac{n\ln(1+J^*)}{\tau_1}$,$j_{1n} = \frac{\ln\frac{F(t,T)Z(t,T_1,T)}{C_O} + b(n)\tau_1 + \frac{1}{2}(\sigma^2 + n\sigma_J^2)}{\sqrt{\sigma^2 + n\sigma_J^2}}$,$j_{2n} = d_{1n} - \sqrt{\sigma^2 + n\sigma_J^2}$,

$$\begin{aligned} Z(t, T_1, T) &= \exp\left[-\tfrac{\rho_3\sigma_s\sigma_r}{k_r}(\tau_1 - H_r(\tau_1))\right] \cdot \exp\left[-\tfrac{\sigma_r^2}{k_r^2}(\tau_1 - H_r(\tau_1) - \tfrac{e^{-k_r(T-T_1)} - e^{-k_r\tau}}{k_r})\right] \\ &\cdot \exp\left[\tfrac{e^{-k_r\tau_1} - e^{2k_r\tau - k_r(T_1+T)}}{2k_r} + \tfrac{\rho_2\sigma_\delta\sigma_r}{k_\delta k_r}(\tau_1 - H_r(\tau_1) - \tfrac{e^{-k_c(T-T_1)} - e^{-k_c\tau}}{k_\delta})\right] \\ &\cdot \exp\left[\tfrac{e^{-k_\delta\tau_1} - e^{(k_\delta+k_r)\tau} - k_r(T+T_1)}{k_r + k_\delta}\right] \end{aligned}$$

On the term of $T_1 = T$equation (12) could give

$$V(t, T) = B(t, T)\sum_{n=0}^{\infty}\left[\frac{e^{-\lambda^*\tau_1}(\lambda^*\tau_1)^n}{n!}\right]\left[F(t, T)Z(t, T)e^{b(n)\tau_1}N(j_{1n}) - C_O N(j_{2n})\right] \tag{13}$$

Where, $T$is the date of CRDI, $T - t$ is the holding time of CRDI, $N(j_{1n})$and$N(j_{2n})$are the probability of standardized normal distribution with parameters $j_{1n}$and j_{2n}, which is $N(j_{in}) = \int_{-\infty}^{j_{in}} f(z)dz$, $f(z)$is the density function of standardized normal distribution,

$$b(n) = -\overline{k^*}\lambda^* + \frac{n\ln(1 + J^*)}{\tau_1}, j_{1n} = \frac{\ln\frac{F(t,T)Z(t,T)}{C_O} + b(n)\tau_1 + \frac{1}{2}(\sigma^2 + n\sigma_J^2)}{\sqrt{\sigma^2 + n\sigma_J^2}} \quad , \quad j_{2n} = d_{1n} - \sqrt{\sigma^2 + n\sigma_J^2} \quad ,$$

$$\begin{aligned} Z(t, T) &= \exp\left[-\tfrac{\rho_3\sigma_s\sigma_r}{k_r}(\tau_1 - H_r(\tau_1))\right] \cdot \exp\left[-\tfrac{\sigma_r^2}{k_r^2}(\tau_1 - H_r(\tau_1) - \tfrac{e^{-k_r(T-T_1)} - e^{-k_r\tau}}{k_r})\right] \\ &\cdot \exp\left[\tfrac{e^{-k_r\tau_1} - e^{2k_r\tau - 2k_rT}}{2k_r} + \tfrac{\rho_2\sigma_\delta\sigma_r}{k_c k_r}(\tau_1 - H_r(\tau_1) + \tfrac{1 - e^{-k_\delta\tau}}{k_\delta})\right] \\ &\cdot \exp\left[\tfrac{e^{-k_\delta\tau_1} - e^{(k_\delta+k_r)\tau} - 2k_rT}{k_r + k_\delta}\right] \end{aligned}$$

4 An example

A coalmine in Shaanxi Province has 61.95 million tons of residual coal reserves, with annual production capacity of 1.2 million tons and the recovery rate of 0.7, we can get that the residual reserves are extracted completely in the 25^{th} year. In 2010, the predictive capital investment is 35080.09 million Yuan, the coal price on the coalmine's entrance is 200 Yuan per ton (without taxes), the coalmine full operational cost per unit of output is 120 Yuan per ton, the average rate of return in mining industry is 10%. Through the market survey and analysis, the volatility rate of coal market price is 0.175. Risk-free interest rate is 5 %, we can get that $\sigma_r$is 0.1318 and $\rho^*$is 0.245 using daily data of 7-day repo rates from July 22, 1996 to December 30, 2009.

Based on the above, we have $Q_0 = 61.95$Mt, $q_t = 1.2$Mt, S_t=200 Yuan/t, I=35080.09 M Yuan, R=10%, r=5%,$C_o = 120$Yuan/t.

The typical DCF formula to valuate CRDI is as follows

$$V^* = \sum_{t=1}^{n} [S'_t - C^s_t - F^1_t - F^2_t - T'_t)/(1+r)^n] \tag{14}$$

Where$V^*$is the value of CRDI based on DCF method, S'$_t$is sale income in thet^{th} year, C$_t$is the full operational cost in thet^{th} year, $F^1_t$is the compensation fee for resources in thet^{th} year, $F^2_t$is the resources tax in thet^{th} year, T'$_t$is the enterprise income tax in thet^{th} year, $n$is the residual extraction time of coal reserves(its unit is year). In this case, $S_t$is 200 yuan/ tS'$_t$is 2400M Yuan/aC$_o$ is 144 M Yuan /aF$_{t1}$andF$_{t2}$is 6.36 M Yuan /aT'_t=28.8 M Yuan/a. Substituting these parameters the above typical DCF formula, we can get

$$\mathrm{V_{DCF}} = \sum_{t=1}^{25} \left[(24000 - 14400 - 636 - 2880) \cdot \frac{1}{(1+0.05)^t} \right]$$
$$= 954.0109 \mathrm{Myuan}$$

Assumed that the coefficient of mean-reverting$a$is 0.1 and θ is 0.04. Using the one-factor model based on options, the value of this CRDI is 1021.0218Myuan.

In fact, the project value may be influenced by emergency incident, two conditions are thought about: with jump and with no jump. Fig.1 illustrates the dependence of the guarantee value on the jumping degrees and the intensity of the emergency incident. From Fig.1, when the other parameters constant, it can be seen that the value of the CRDI is increasing with more bigger exercise rights. In order to quantifying how the CRDI value changes when emergency incident occur at the different forms, we draught the Fig.2 at the exercise rights is equal to 13. From Fig.2, when the other parameters constant, it can be seen that the CRDI increases with the rising θ or λ. Therefore, before quantifying the value of the CRDI, it is necessary to forecast the jump degree and intensity of the emergency incident, in order to prevent and control the risks at different possible point.

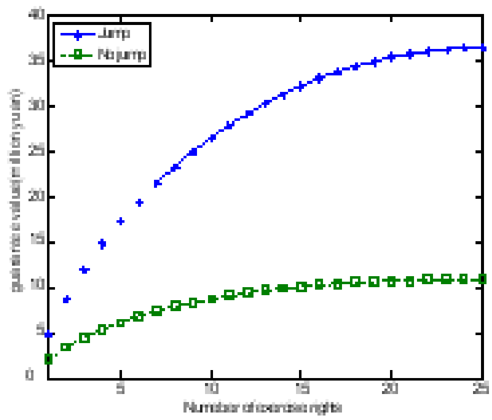

Figure 1: the value of CRDI with jump or no jump

5 Conclusions pyright

It is significant for scientific and rational decision-making method of for short CRDI to evaluate reasonably the value of coal resources investment. CRDI is a decision as to whether develop, delay developing, enlarge produce scale, reduce produce scale or abandon developing, and so on, investment management is management of such options. Under the no-arbitrage pricing theory and risk-neutral principle, the three-factor evaluation model of CRDI is established based on real option theory. Through a practical instance, it's tested that the three-factor model can better reflect the value of this CRDI.

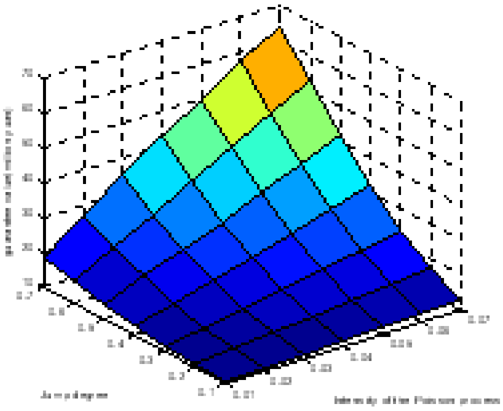

Figure 2: the value of CRDI with difference jump size

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