Numerical Solution of the Falkner-Skan Equation with Stretching Boundary by Collocation Method

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Abstract: In the present paper, a solution for the boundary value problem over a semi-infinite interval has been obtained by transforming the two-dimensional laminar boundary equations into a nonlinear ordinary equation using similarity variables. Moreover, a collocation method is proposed to solve the Falkner-Skan equation. This method is based on rational Legendre functions and convert the Falkner-Skan equation to a system of nonlinear algebraic equations. The results are tabulated and compared with some other methods.

Keywords: Falkner-Skan equation; Collocation method; Rational Legendre functions; Semi-infinite interval; Nonlinear ODE

1 Introduction

Many science and engineering problems arise in unbounded domain. Different spectral methods have been proposed for solving problems in unbounded domains. However, the most common approach is to apply polynomials that are orthogonal over unbounded domains, such as the Hermite spectral method and the Laguerre spectral method [1–8].

Guo [9–11] proposed a method that proceeds by mapping the original problem in an unbounded domain to a problem in a bounded domain, and then used the appropriate Jacobi polynomials to approximate the resulting problems.

Another approach is replacing infinite domain with \([-L, L]\) and semi-infinite interval with \([0, L]\) by choosing \(L\), sufficiently large. This method is named domain truncation [12].

Another effective direct approach for solving such problems is based on rational approximations. Christov [13] and Boyd [14, 15] developed some spectral methods on unbounded intervals by using mutually orthogonal systems of rational functions. Boyd [15] defined a new spectral basis, named rational Chebyshev functions on the semi-infinite interval, by mapping to the Chebyshev polynomials. Guo et al. [16] introduced a new set of rational Legendre functions which are mutually orthogonal in \(L^2(0, +\infty)\). They applied a spectral scheme using the rational Legendre functions for solving the Korteweg-de Vries equation on the half line. Boyd et al. [17] applied pseudospectral methods on a semi-infinite interval and compared rational Chebyshev, Laguerre and mapped Fourier sine.

Parand et al. [18–27] applied the spectral method to solve nonlinear ordinary differential equations on semi-infinite intervals. Their approach was based on rational Tau and collocation methods.

Falkner-Skan equation is categorized as a boundary layer problem. The boundary layer theory was first proposed by Prandtl in 1904. It asserts that the viscous effect would be confined to a thin shear layer adjacent to the solid boundary in the case of a motion with very little viscosity. Hence, the fluid motion is split into two parts: near the boundary the viscosity effect is important and the fluid is said to be viscous, and far away from the boundary the fluid viscous effect is unimportant and can be treated as an inviscid fluid [28].

There is a large body of literature on the solution of Falkner-Skan equation, see Hartree [29], Cebeci and Keller [30], Hildyard [31], Yang and Chien [32], Soundalgekar [33], Forbrich [34], Thomas and Harris [35], Liu and Lyu [36], Brodie and Banks [37], Assassa and Ghazy [38], Summers [39], Massoudi and Ramezani [40], Fazio [41], N. S. Asaithambi [42], Heeg et al. [43], Morgan et al. [44] and Liao [45].

There is a numerical solution to the Falkner-Skan equation in [46], [47] and [28] using Runge-Kutta integration scheme with shooting method and Lie-group shooting method, respectively to solve this problem. Asaithambi presented

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several numerical methods based on finite-element methods, shooting and finite-differences [48–50]. Yao [51] developed an analytical technique, named Homotopy analysis method, and gave an analytical approximation solution to the Falkner-Skan equation. [52] solved this equation analytically for some value of $\beta$. [53] built a solution from the asymptotic method of matched expansions at order one. [54–56] solved the MHD Falkner-Skan using Hankel-padé method, finite-difference scheme and homotopy analysis method, respectively. Kuo adopted the differential transformation method to investigate the velocity and shear stress fields [57] and the temperature field [58] associated with the Falkner-Skan boundary-layer problem. [59] solved the nonlinear ordinary differential equation using Padé Adomian decomposition method (ADM) to obtain series solution.

Here we use rational Legendre collocation method to solve Falkner-Skan equation which is a nonlinear differential equation on a semi-infinite interval. The main point of our analysis lies in the fact that there is no reconstruction of the problem on the finite domain. We show that our results agree well with the results of other methods, which demonstrate the viability of the new technique. In this sense, this method has the potential to provide a wider applicability. On the other hand, the comparison of the results obtained by this method and the others shows that the new method provides accurate solutions.

The organization of the paper is as follows:
In Section 2, we describe the problem formulation. In the next section, we describe the formulation of rational Legendre functions. In section 4, we present our solutions for this problem, summarize the application of this method for Falkner-Skan wedge flow and compare it with the existing methods in the literature. The conclusions are described in the final section.

2 Problem formulation

In this section, we introduce two-dimensional steady-state boundary layer equation with boundary conditions as follows [60, 61]:

$$\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \\
v \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= U \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2},
\end{align*}$$

\tag{1}

\begin{align*}
\{ y &= 0 \quad u=0,v=0, \\
y &= \infty \quad u=U(x).
\end{align*}

where $x, y, u, v$ are dimensionless variables using reference length $x_0$ and $\nu, U$ are kinematic viscosity and potential velocity respectively. By using newly defined variables and dimensionless variables as follows, where $\eta$ is a dimensionless variable similarity solution, $\psi$ is the stream function and $m$ is the power of length coordinate $x$, of the velocity of the potential flow $U(x) = u_1 x^m$ [60, 61]:

$$\begin{align*}
\eta &= \sqrt{\frac{m+1}{2}} \sqrt{\frac{U}{\nu x}} y; \quad \psi = \sqrt{\frac{2}{m+1}} \sqrt{\nu x U f(\eta)} \\
u &= U f'(\eta), \quad v = -\sqrt{\frac{m+1}{2}} \sqrt{\frac{\nu U}{x}} (f + \frac{m-1}{m+1}\eta f') \\
m &= \frac{\beta}{2-\beta}, \quad 2m = \beta
\end{align*}$$

\tag{2}

\tag{3}

\tag{4}

thus Eq. (1) is converted to the third-order nonlinear Falkner-Skan equation as follows:

$$f''' + f f'' + \beta(1-f'^2) = 0,$$

\tag{5}

with the boundary conditions

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1,$$

\tag{6}

The wall mass transfer $f(0)$ which is defined here as zero, can sometimes can get nonzero value, namely $f(0) = \gamma$, showing the strength of the mass transfer at the wall, i.e. $\gamma < 0$ corresponds to injection and $\gamma > 0$ corresponds to suction and $\beta$ is a parameter of the streamwise pressure gradient.

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3 Rational Legendre functions properties

The well-known Legendre polynomials are orthogonal in the interval $[-1, 1]$ with respect to the weight function $\rho(y) = 1$ and can be determined with the aid of the following recurrence formula:

$$
\begin{align*}
P_0(y) &= 1, \\
P_1(y) &= y, \\
(n + 1)P_{n+1}(y) &= (2n+1)yP_n(y) - nP_{n-1}(y) \quad n \geq 1.
\end{align*}
$$

(7)

The rational Legendre (RL) functions are defined as follows

$$
R_n(x) = P_n\left(\frac{x+L}{x+L}\right),
$$

where the constant parameter $L$ sets the length scale of the mapping. Boyd [62] offered guidelines for optimizing the map parameter $L$ for rational Chebyshev functions, which is useful for rational Legendre functions, too.

$R_n(x)$ is the $n$th eigenfunction of the singular Sturm-Liouville problem

$$
\frac{(x+L)^2}{L} (xR_n'(x))' + n(n+1)R_n(x) = 0, \quad x \in [0, \infty), \quad n = 0, 1, 2, \ldots
$$

(8)

Thus RL functions satisfy:

$$
\begin{align*}
R_0(x) &= 1, \\
R_1(x) &= \frac{x-L}{x+L}, \\
(n+1)R_{n+1}(x) &= (2n+1)\frac{2x}{x+L}R_n(x) - nR_{n-1}(x) \quad n \geq 1.
\end{align*}
$$

(9)

3.1 Function approximation

Let $w(x) = \frac{2L}{(x+L)^2}$ denotes a non-negative, integrable, real-valued function over the interval $I = [0, \infty)$. We define:

$$
L^2_w(I) = \{ v : I \rightarrow \mathbb{R} \mid v \text{ is measurable and } ||v||_w < \infty \},
$$

(10)

where

$$
||v||_w = \left( \int_0^\infty |v(x)|^2 w(x) dx \right)^{\frac{1}{2}},
$$

(11)

is the norm induced by the scalar product

$$
< u, v >_w = \int_0^\infty u(x)v(x)w(x) dx.
$$

(12)

Thus $\{ R_n(x) \}_{n \geq 0}$ denote a system which is mutually orthogonal under (12), i.e.,

$$
< R_n, R_m >_w = \frac{2}{2n+1} \delta_{nm},
$$

(13)

where $\delta_{nm}$ is the Kronecker delta function. This system is complete in $L^2_w(I)$. For any function $u \in L^2_w(I)$ the following expansion holds

$$
u(x) = \sum_{k=0}^{+\infty} a_k R_k(x),
$$

(14)

with

$$
a_k = \frac{< u, R_k >_w}{||R_k||^2_w}.
$$

(15)

The $a_k$s are the expansion coefficients associated with the family $\{ R_k(x) \}$.
3.2 Rational Legendre interpolation approximation

Canuto et al. [63] and Gottlieb et al. [64] introduced Gauss-integration. Further, Guo et al. [16] introduced rational Legendre-Gauss points. Let
\[ \mathbb{R}_N = \text{span}\{R_0, R_1, \ldots, R_N\}, \]
and \( \{y_j\}_{j=0}^N \) be \( N+1 \) roots of the polynomial \( P_{N+1}(x) \). These points are known as Legendre-Gauss points. We define
\[ x_j = L \frac{1+y_j}{1-y_j}, \quad j = 0, 1, \ldots, N, \]
which are named as rational Legendre-Gauss nodes. In fact, these points are zeros of the function \( R_{N+1}(x) \). The relations between rational Legendre orthogonal systems and rational Gauss integrations are as follows:
\[ \int_{-1}^{1} u(x)w(x)\,dx = \sum_{j=0}^{N} u(x_j)w_j, \quad \forall u \in \mathbb{R}_{2N+1}, \]
where
\[ w_j = \frac{2L}{(x_j + L)^2x_j[R_{N+1}(x_j)]^2}, \quad j = 0, \ldots, N, \]
are the corresponding weights with the \( N+1 \) rational Legendre-Gauss nodes. The interpolating function of a smooth function \( u \) on a semi-infinite interval is denoted by \( I_Nu \). It is an element of \( \mathbb{R}_N \) and is defined as
\[ I_Nu(x) = \sum_{j=0}^{N} a_k R_k(x). \]

To obtain the order of convergence of rational Legendre approximation, we first define the space
\[ H_{r,w,A}(I) = \{v : v \text{ is measurable and } \|v\|_{r,w,A} < \infty\}, \]
where the norm is induced by
\[ \|v\|_{r,w,A} = \left( \sum_{k=0}^{r} \frac{(x+1)^{\frac{r}{2}+k}}{x^{k}} \left( \frac{d}{dx} v(x) \right)^2 \right)^{\frac{1}{2}}, \]
and \( A \) is the Sturm-Liouville operator as follows:
\[ Av(x) = -w^{-1}(x) \frac{d}{dx} \left( x \frac{d}{dx} v(x) \right). \]
We have the following theorem for the convergence:

**Theorem 1** For any \( v \in H_{r,w,A}(I) \) and \( r \geq 0 \),
\[ \|I_Nv - v\|_w \leq cN^{-r}\|v\|_{r,w,A}. \]

A complete proof of the theorem and discussion on convergence is given in [16]. To apply a pseudospectral approach, we consider the residual \( Res(x) \) when the expansion is substituted into the governing equation. It requires that \( a_k \)'s be selected so that the boundary conditions are satisfied, but make the residual zero at as many (suitable chosen) spatial points as possible.
4 Solving Falkner-Skan wedge flow equation with Rational Legendre functions

In this section, we apply the collocation method to find solutions of Falkner-Skan equation. To solve this equation, we expect to approximate this equation with

\[ I_N f(x) = \sum_{i=0}^{N} a_i R_i(x), \]  

(26)

and form residual function, but should pay attention that

\[ \lim_{x \to \infty} R'_i(x) = 0, \]  

(27)

so

\[ \lim_{x \to \infty} \frac{dI_N f(x)}{dx} = 0. \]  

(28)

it means that the third boundary condition doesn’t satisfy. Hence, we should use secondary function \( f_0(x) = \frac{x^3}{(x + L)^2} \) and approximate \( f(x) \) as

\[ f(x) \approx f_0(x) + I_N f(x), \]  

(29)

so that the third boundary condition will satisfy.

Now we form the residual function:

\[
\begin{align*}
Res(x) &= \frac{d^3}{dx^3} f_0(x) + \frac{d^3}{dx^3} I_N f(x) + (f_0(x) + I_N f(x))\left(\frac{d^2}{dx^2} f_0(x) + \frac{d^2}{dx^2} I_N f(x)\right) + \\
&\quad \frac{d^2}{dx^2} I_N f(x) + \beta\left[1 - \left(\frac{d}{dx} f_0(x) + \frac{d}{dx} I_N f(x)\right)^2\right] = 0,
\end{align*}
\]

(30)

\( I_N y \) is a good approximation of function \( y \) if it is zero on the whole domain. In other words, we should select coefficients \( a_i \)’s so that the residual function approaches zero on the most of the domain. The collocation method for Falkner-Skan equation is to find \( I_N y \in \mathbb{R}_N \) such that

\[
\begin{align*}
Res(x_j) &= 0, \quad j = 0, \ldots, N - 2, \\
I_N y(x_0) &= \gamma, \\
\left. \frac{dI_N y(x)}{dx} \right|_{x=0} &= 0,
\end{align*}
\]

(31) \hspace{1cm} (32) \hspace{1cm} (33)

where the \( x_j \)’s are rational Legendre-Gauss nodes. This generates a set of \( N + 1 \) nonlinear equations that can be solved by Newton method for unknown coefficients \( a_i \)’s.

As shown in Tables 1-3 where the comparison of collocation method solution of \( f''(0) \) with the analytical method, namely \( \text{HAM} [51] \) and numerical results by 4th-order Runge-Kutta method, combined with Newton-Raphson technique with the integral distance \( \eta_{\infty} = 10 \), the integral step size \( \delta \eta = 0.001 \) and the error of \( f''(0) \) less than \( 1 \times 10^{-6} \) is arised when \( \gamma \) varies from \(-1\) to \(7\) and some fixed \( \beta \)s are evaluated to be \(0, 1\) and \(2\), respectively. It is found that the results of the present work agree well with those by other methods [42, 51]. This verifies the validity of the present work.

Besides, the influences of \( \beta \) and \( \gamma \) on the solution curves of \( f'(\eta) \) are further analyzed, where some fixed \( \beta \)s are evaluated to be \(0, 1\) and \(2\), respectively, when the parameter \( \gamma = 5 \) is chosen, and some fixed \( \gamma \)a are evaluated to be \(-1, 0, 1\) and \(5\), respectively, when the parameter \( \beta = 1 \) is unchanged as well, as shown in Figs. 1 and 2. This also shows the validity of the present work.

5 Conclusion

In this study, the collocation method with Rational Legendre functions is employed to solve the Falkner-Skan equation, which is a nonlinear boundary value problem. A comparison is also made between the results of the present method and other numerical/analytical methods. It is found that the results of the present work agree well with those obtained by other methods. The validity of this method is based on this assumption that it converges by increasing the number of collocation points.

Our aim was to apply an accurate and well-conditioned method which gives more accurate answers without reformulating the equation to bounded domains. Numerical results indicate the convergence and effectiveness of the present approach.

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Table 1: Results for collocation method using rational Legendre for $\beta = 0$ in comparison with numerical result by [42] and analytical result by [51].

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<th>Numerical result[42]</th>
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Table 2: Results for collocation method using rational Legendre for $\beta = 1$ and comparison with numerical result by [42] and analytical result by [51].

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Table 3: Results for collocation method using rational Legendre for $\beta = 2$ and comparison with numerical result by [42] and analytical result by [51].

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Figure 1: $f'(\eta)$ by collocation method using rational Legendre for different value of $\beta$ and $\gamma = 5$

Figure 2: $f'(\eta)$ by collocation method using rational Legendre for different value of $\gamma$ and $\beta = 1$

References


IINNS homepage: http://www.nonlinearscience.org.uk/


IJNS homepage: http://www.nonlinearscience.org.uk/