New Exact Solutions for (2+1)-Dimensional Kadomtsev-Petciashivili Equation

Libo Yang1,∗, Dianchen Lu2
1 faculty of mathematics and physics, Huaiyin Institute of Technology, Jiangsu, 223003, P.R.China
2 Nonlinear Scientific Research Center, Jiangsu University, Jiangsu, 212013, P.R.China
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Abstract: In this paper, using extended (\(G/G'\))-expansion method, we construct several new exact solutions for (2+1)- dimensional K-P(Kadomtsev-Petciashivili) equation. These solutions include hyperbolic function solutions, trigonometric function solutions, and rational function solutions. And when the constant of the hyperbolic function solutions are set at special values we obtain solitary wave solutions.

Keywords: extended (\(G/G'\))-expansion method; (2+1)- dimensional K-P equation; exact solutions; solitary wave solution

1 Introduction

In recent years, the investigation of exact solutions to nonlinear evolution equations plays an important role in the nonlinear physical phenomena. Searching for explicit solutions of nonlinear evolution equations by using various different methods is the main goal for many researchers, and several powerful methods have been proposed to construct exact solutions for nonlinear partial differential equations, such as homogeneous balance method, the hyperbolic function method, the F-expansion method, inverse scattering method, projective Riccati equations method, Jacobian elliptic functions expansion method and so on. Ref.[1-8]. By using these methods we obtained many valuable exact solutions for nonlinear evolution equations. But constructing new methods and searching for new useful exact solutions for nonlinear evolution equations are meaningful. Last, Wang eta. presented the (\(G/G'\))-expansion method Ref.[9] and using this methods constructed many new exact solutions for nonlinear evolution equations Ref.[10-11]. In this paper, we obtained several new exact solutions of (2+1)- dimensional K-P equation by using this extend (\(G/G'\))-expansion method.

This paper is arranged as follows. In section 2, we briefly describe the extend (\(G/G'\))-expansion. In section 3, we obtain several solutions for (2+1)- dimensional K-P equation. In section 4, some conclusions are given.

2 Method

Consider a given nonlinear evolution equation, with variables \(x, y, z, \ldots\) and \(t\)

\[
F(u, u_x, u_y, u_z, \ldots, u_t, u_{xx}, u_{yy}, u_{zz}, \ldots) = 0 ,
\]

where \(u = u(x, y, z, \ldots, t)\) is an unknown function, \(F(u, u_x, u_y, u_z, \ldots, u_t, u_{xx}, u_{yy}, u_{zz}, \ldots)\) is a polynomial with the variables \(u, u_x, u_y, \ldots\).

We make the gauge transformation

\[
u(x, y, \ldots, t) = u(\xi)\quad \xi = kx + ly + mz + \ldots + st ,
\]

where \(k, l, m, \ldots, s\) are nonzero constants to be determined later.

Substituting (2) into Eq.(1) yields a complex ordinary differential equation of \(u(\xi)\), namely

\[
F(u, u', u'', \ldots) = 0 ,
\]

∗Corresponding author. E-mail address: yanglibo80@126.com
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where \( u' = \frac{du}{dx} \), \( u'' = \frac{d^2u}{dx^2} \), \( F(u, u', u'', \ldots) \) is a polynomial with the variables \( u \) and \( u' \). We assume Eq. (3) has the following formal solutions:

\[
u(\xi) = a_0 + \sum_{i=1}^{n} a_i \left( \frac{G'}{G} \right)^i
\]

where \( G = G(\xi) \) satisfies the second order LODE

\[
G'' + \lambda G' + \mu G = 0,
\]

where \( G' = \frac{dG}{d\xi}, G'' = \frac{d^2G}{d\xi^2}, \ldots \), \( a_0, a_i (i = 1, 2, \ldots, n) \), \( \lambda, \mu \) are constants to be determined later, and \( a_n \neq 0 \). \( \xi = kx + ly + mz + \ldots + st \) is a function with the variables \( x, y, z, \ldots, t \). The positive integer \( n \) can be determined by considering the homogeneous balance between governing nonlinear terms and the highest order derivatives of \( u \) in Eq. (3).

Substituting (4) into Eq.(3) and using second order LODE (5), namely, by Eq. (5), we have \( \left( \frac{G'}{G} \right)' = -\lambda \left( \frac{G'}{G} \right) - \mu - \left( \frac{G'}{G} \right)^2 \), collecting all terms with the same order of \( \left( \frac{G'}{G} \right) \) together, the left-hand side of Eq.(3) is converted into another polynomial in \( \left( \frac{G'}{G} \right) \). Setting the coefficients of \( \left( \frac{G'}{G} \right)^i \) (\( i = 1, 2, \ldots, n \)) to zero yields a set of nonlinear algebraic equations (NAEs) in \( a_0, a_i (i = 1, 2, \ldots, n) \), \( k, l, m, \ldots, s, \lambda, \mu \), solving the NAEs, we obtain the solution of NAEs. Substituting these results into (4), we can obtain several exact solutions of Eq.(1). Where \( \frac{G'}{G} \) can be obtained by general solution of Eq. (5), the results as following:

(a) When \( \lambda^2 - 4\mu > 0 \), we have

\[
G' = \frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \frac{\sqrt{\lambda^2 - 4\mu} \xi + C_1 \sinh \frac{\lambda \xi}{2}}{C_1 \sinh \frac{\lambda \xi}{2} + C_2 \cosh \frac{\lambda \xi}{2}},
\]

(b) When \( \lambda^2 - 4\mu < 0 \), we have

\[
G' = \frac{\lambda}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \frac{-C_1 \sin \frac{\lambda \xi}{2} - C_2 \cos \frac{\lambda \xi}{2}}{C_1 \cos \frac{\lambda \xi}{2} - C_2 \sin \frac{\lambda \xi}{2}},
\]

(c) When \( \lambda^2 - 4\mu = 0 \), we have

\[
G' = \frac{\lambda}{2} \frac{C_2}{C_1 + C_2 \xi}.
\]

### 3 The new exact solutions of the (2+1)- dimensional K-P equation

Let us consider the (2+1)- dimensional K-P equation with following form:

\[
(u_t + 6uu_x + uu_{xxx})_x + \alpha uu_{yy} = 0,
\]

(9)

We introduce a gauge transformation

\[
u(x, y, t) = v(\xi) \quad \xi = x + ly - \eta t,
\]

(10)

where \( l \) and \( \eta \) are nonzero constants to be determined later. Substituting (10) into (9), we have

\[
(\alpha l^2 - \eta)u'' + 6(uu')' + u'''' = 0,
\]

(11)

Directly integral the Eq.(11) twice and set integral constant to zero, we have

\[
(\alpha l^2 - \eta)u + 3u^2 + u'' = 0,
\]

(12)

By balancing the governing nonlinear terms and the highest order derivatives in Eq.(12)((n + 2) = 2n), we obtain \( n = 2 \), thus we assume Eq.(ref12) has the following solutions

\[
u(\xi) = a_0 + a_1 \left( \frac{G'}{G} \right) + a_2 \left( \frac{G'}{G} \right)^2, \quad a_2 \neq 0,
\]

(13)
Substituting (13) into Eq.(12) and using second order LODE (5), collecting all terms with the same order of \( \frac{G'}{G} \) together, setting the coefficients of to zero yields nonlinear algebraic equations. Specific as follows

\[
(\alpha l^2 - \eta) a_0 + 3a_0^2 + (a_1 \lambda + 2a_2 \mu) \mu = 0 (\alpha l^2 - \eta) a_1 + 6a_0 a_1 + a_1 \lambda^2 + 6a_2 \lambda \mu + 2a_1 \mu = 0 \\
(\alpha l^2 - \eta) a_2 + 3(a_1^2 + 2a_0 a_2) + 4a_2 \lambda^2 + 8a_2 \mu + 3a_1 \lambda = 0 \\
6a_1 a_2 + 10a_2 \lambda + 2a_1 = 0 \\
3a_2^2 + 6a_2 = 0
\]

Solving the algebraic equations above, yields

\[
a_0 = -\frac{1}{6} ([\lambda^2 + 8 \mu] \mp [\lambda^2 - 4 \mu]), a_1 = -2 \lambda, a_2 = -2, \alpha l^2 - \eta = -\lambda^2 - 6 \mu - 6a_0 , \quad (14)
\]

Following, we discuss the value of \( \lambda^2 - 4 \mu \).

Case 1: when \( \lambda^2 - 4 \mu > 0 \), yields

\[
a_0 = -\frac{1}{6} ([\lambda^2 + 8 \mu] \mp (\lambda^2 - 4 \mu)], a_1 = -2 \lambda, a_2 = -2, l = l, \eta = \alpha l^2 + \lambda^2 + 6 \mu + 6a_0 = \alpha l^2 \pm (\lambda^2 - 4 \mu)
\]

Expression(13) can be written as

\[
u(\xi) = -\frac{1}{6} ([\lambda^2 + 8 \mu] \mp (\lambda^2 - 4 \mu]) - 2\lambda (\frac{G'}{G}) - 2\frac{G''}{G}, \quad (15)
\]

where \((G')^2\) is given by (6) and from (6) we yield

\[
(G')^2 = \frac{\lambda^2}{4} - \frac{\lambda \sqrt{\lambda^2 - 4 \mu}}{2} \frac{C_1 \cosh \frac{\lambda^2 - 4 \mu}{2} \xi + C_2 \sinh \frac{\lambda^2 - 4 \mu}{2} \xi}{C_1 \sinh \frac{\lambda^2 - 4 \mu}{2} \xi + C_2 \cosh \frac{\lambda^2 - 4 \mu}{2} \xi} \\
+ \frac{\lambda^2 - 4 \mu}{4} \left( \frac{C_1 \cosh \frac{\lambda^2 - 4 \mu}{2} \xi + C_2 \sinh \frac{\lambda^2 - 4 \mu}{2} \xi}{C_1 \sinh \frac{\lambda^2 - 4 \mu}{2} \xi + C_2 \cosh \frac{\lambda^2 - 4 \mu}{2} \xi} \right)^2
\]

where \( C_1 \) are \( C_2 \) arbitrary constants. Substituting (6) and (16) into (15), yields the general hyperbolic function solutions of Eq. (1)

\[
u_1(x, y, t) = \frac{\lambda^2 - 4 \mu}{2} - \frac{\lambda^2 - 4 \mu}{2} \left( \frac{C_1 \cosh \frac{\lambda^2 - 4 \mu}{2} \xi + C_2 \sinh \frac{\lambda^2 - 4 \mu}{2} \xi}{C_1 \sinh \frac{\lambda^2 - 4 \mu}{2} \xi + C_2 \cosh \frac{\lambda^2 - 4 \mu}{2} \xi} \right)^2 , \quad (17)
\]

where \( \eta = x + ly - (\alpha l^2 + \lambda^2 - 4 \mu)t \).

\[
u_2(x, y, t) = \frac{\lambda^2 - 4 \mu}{6} - \frac{\lambda^2 - 4 \mu}{2} \left( \frac{C_1 \cosh \frac{\lambda^2 - 4 \mu}{2} \xi + C_2 \sinh \frac{\lambda^2 - 4 \mu}{2} \xi}{C_1 \sinh \frac{\lambda^2 - 4 \mu}{2} \xi + C_2 \cosh \frac{\lambda^2 - 4 \mu}{2} \xi} \right)^2 , \quad (18)
\]

where \( \eta = x + ly - (\alpha l^2 - \lambda^2 + 4 \mu)t \).

Specifically, when \( C_1 = 0, C_2 \neq 0, \mu = 0 \), general solutions (17),(18) become the other two solitary wave solutions of Eq. (1)

\[
u_3(x, y, t) = \frac{\lambda^2}{2} \sech^2 \frac{|\lambda| \xi}{2} , \quad (19)
\]

where \( \eta = x + ly - (\alpha l^2 + \lambda^2)t \).

\[
u_4(x, y, t) = \frac{\lambda^2}{6} (-2 + 3 \sech^2 \frac{|\lambda| \xi}{2} , \quad (20)
\]
where \( \eta = x + ly - (\alpha l^2 - \lambda^2) t \).

Case 2: when \( \lambda^2 - 4\mu < 0 \), yields

\[
a_0 = -\frac{1}{6}[(\lambda^2 + 8\mu) \pm (\lambda^2 - 4\mu)], \quad a_1 = -2\lambda, \quad a_2 = -2, \quad l = l, \quad \eta = \alpha l^2 + \lambda^2 + 6\mu + 6a_0 = \alpha l^2 - (\lambda^2 - 4\mu)
\]

Substituting above into (13) we have

\[
u(\xi) = -\frac{1}{6}[(\lambda^2 + 8\mu) \pm (\lambda^2 - 4\mu)] - 2\lambda (\frac{G'}{G}) - 2(\frac{G'}{G})^2,
\]

(21)

where \( (\frac{G'}{G})^2 \) is given by (7) and from (7) we yield

\[
(\frac{G'}{G})^2 = \frac{\lambda^2}{4} - \frac{\lambda\sqrt{4\mu - \lambda^2} - C_1 \sin \frac{\sqrt{4\mu - \lambda^2} \xi}{2} + C_2 \cos \frac{\sqrt{4\mu - \lambda^2} \xi}{2}}{C_1 \cos \frac{\sqrt{4\mu - \lambda^2} \xi}{2} + C_2 \sin \frac{\sqrt{4\mu - \lambda^2} \xi}{2}}
\]

\[
+ \frac{4\mu - \lambda^2}{4} \left( \frac{C_1 \sin \frac{\sqrt{4\mu - \lambda^2} \xi}{2} + C_2 \cos \frac{\sqrt{4\mu - \lambda^2} \xi}{2}}{C_1 \cos \frac{\sqrt{4\mu - \lambda^2} \xi}{2} + C_2 \sin \frac{\sqrt{4\mu - \lambda^2} \xi}{2}} \right)^2,
\]

(22)

Substituting (7) and (22) into (21), yields the general trigonometric function solutions of Eq. (1)

\[
u_5(x, y, t) = \frac{\lambda^2 - 4\mu}{6} - \frac{4\mu - \lambda^2}{2} \left( \frac{C_1 \sin \frac{\sqrt{4\mu - \lambda^2} \xi}{2} + C_2 \cos \frac{\sqrt{4\mu - \lambda^2} \xi}{2}}{C_1 \cos \frac{\sqrt{4\mu - \lambda^2} \xi}{2} + C_2 \sin \frac{\sqrt{4\mu - \lambda^2} \xi}{2}} \right)^2,
\]

(23)

where \( \eta = x + ly - (\alpha l^2 - \lambda^2 + 4\mu)t \).

Specifically, when \( C_1 = 0, C_2 \neq 0, \mu = 0 \), general solutions (23)(24) become the other two trigonometric function solutions of Eq. (1)

\[
u_7(x, y, t) = \frac{\lambda^2}{6} (-2 + 3 \sec^2 \frac{\lambda \xi}{2}),
\]

(25)

where \( \eta = x + ly - (\alpha l^2 - \lambda^2)t \).

\[
u_6(x, y, t) = \frac{\lambda^2}{2} \sec^2 \frac{\lambda \xi}{2},
\]

(26)

where \( \eta = x + ly - (\alpha l^2 + \lambda^2)t \).

Case 3: when \( \lambda^2 - 4\mu = 0 \), yields

\[
a_0 = -\frac{1}{6}[(\lambda^2 + 8\mu)], \quad a_1 = -2\lambda, \quad a_2 = -2, \quad l = l, \quad \eta = \alpha l^2 + \lambda^2 + 6\mu + 6a_0 = \alpha l^2
\]

Substituting above \( a_0, a_1, a_2 \) and \( \eta \) into (13) we have

\[
u(\xi) = -\frac{1}{6}[(\lambda^2 + 8\mu) - 2\lambda (\frac{G'}{G}) - 2(\frac{G'}{G})^2,
\]

(27)

where \( (\frac{G'}{G})^2 \) is given by (8) and from (8) we yield

\[
(\frac{G'}{G})^2 = \frac{\lambda^2}{4} - \frac{\lambda C_2}{C_1 + C_2 \xi} + \left( \frac{C_2}{C_1 + C_2 \xi} \right)^2
\]

(28)

Substituting (8) and (28) into (27), yields the general rational function solutions

\[
u_9(x, y, t) = \frac{\lambda^2 - 4\mu}{3} - 2(\frac{C_2}{C_1 + C_2 \xi})^2,
\]

(29)

where \( \eta = x + ly - \alpha l^2 t \).
4 Conclusion

By using this extended \((\frac{G'}{G})\)-expansion method we obtained seven groups new exact solutions of (2+1)-dimensional K-P equation. This method is concise and easy understand. This method we can be used to other nonlinear evolution equations.

References


IJNS homepage: http://www.nonlinearscience.org.uk/