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Anti-synchronization of Two Different Hyperchaotic Systems via Active and Adaptive Control

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Abstract: This paper presents anti-synchronization of two different hyper-chaotic systems using active and adaptive control method. The proposed technique is applied to achieve chaos anti-synchronization for the hyperchaotic Lü and hyperchaotic Chen dynamical systems with unknown parameters. Numerica simulation results are presented to demonstrate the effectiveness of the method.

Key words: anti-synchronization; active; adaptive

1 Introduction

Since the idea of synchronizing chaotic systems was introduced by Pecora and Carroll[1] in 1990, chaos synchronization has received increasing attention due to its theoretical challenge and its great potential applications in secure communication,chemical reaction and biological systems[2].

Recently several different types of synchronizations have been proposed in the literature, for example, generalized synchronization[3–6], phase synchronization[7, 8], lag synchronization[9–11], anti-synchronization [12–14] and so on .The generalized synchronization implies the establishment of functional relations between drive and response systems. Phase synchronization is characterized in that the phase difference between two chaotic systems are locked within 2π, while their amplitudes remain chaotic and uncorrelated. Lag synchronization is described as the coincidence of two chaotic trajectories with a constant time lag. The state vectors of synchronized systems have the same absolute values but opposite signs, i.e. the sum of the output signals of two systems can converge to zero, called the anti-synchronization(AS) method.

A chaotic system is extremely sensitive to tiny variations of parameters. But in practical situation, some systems parameters cannot be exactly known in priori, the effect of these uncertainties will destroy the synchronization and even break it. Therefore, it is important and necessary to study the synchronization in such systems with unknown parameters.

In this paper, we apply active and adaptive control theory to anti-synchronize two different hyperchaotic systems. We demonstrate the technique capability on the anti-synchronization of hyperchaotic Lü and hyperchaotic Chen systems.

2 Problem and systems description

Nonlinear chaotic system

$$\begin{cases} \dot{x} = f(t,x) \\ \dot{y} = g(t,y) + u(t,x,y) \end{cases} \tag{1}$$

Where $x, y \epsilon R^n$, f and g are $R \times R^n \rightarrow R^n$ and differentiable. The first equation of the sysem (1) is master system and the second is slave system.$u(t,x,y)$ is control function. Let $e(t) = x(t) + y(t)$,our goal is to

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design the controller ,so the systems can satisfy:

$$\lim_{t\to\infty} \|e(t)\| = 0,$$

then the master system and slave system can achieve anti-synchronization.

Lü system as a typical transaction system, found by Lü and Chen, which connects the Lorenz and Chen attractors and represents the transition from one to the other .Recently, Chen Ai-min others proposed hyperchaotic Lü system[15].The hyperchaotic Lü system is described by:

$$\begin{cases} \dot{x}_1 = -a(x_1 - x_2) + x_4 \\ \dot{x}_2 = cx_2 - x_1x_3 \\ \dot{x}_3 = x_1x_2 - bx_3 \\ \dot{x}_4 = x_1x_3 + dx_4 \end{cases} \tag{2}$$

The system is based on Lü system,the first equation with a controller and the second has three equation non-controller.Then fourth equation contains an additional controller.Which a, b, c is admission to the system parameters.Gain control parameter d is to be determined.By analysis of the dynamics of the system, including the bifurcation diagram, Lyapunov exponent spectrum Poincare mapping, The system and circuit simulation experiments confirm hyperchaotic .

When $a = 36, b = 3, c = 20$ and d take other different values,System performance of the different dynamics:when$-1.03 \le d \le -0.46$, system is a periodic orbit;when$-0.46 < d \le -0.35$,system is chaotic attractor;and when $-0.35 < d \le 1.30$,there are two index greater than zero system is super chaotic attractor.

In this paper, we consider the system is hyperchaotic.Selection parameters $a = 36, b = 3, c = 20$ and $-0.35 < d \le 1.30$.When $a = 36, b = 3, c = 20$ and $d = 1$, hyperchaotic attractor as shown in Fig.1. Recently,hyperchaotic Chen system studied by Yan[16] .The system for the study of chaos synchronization

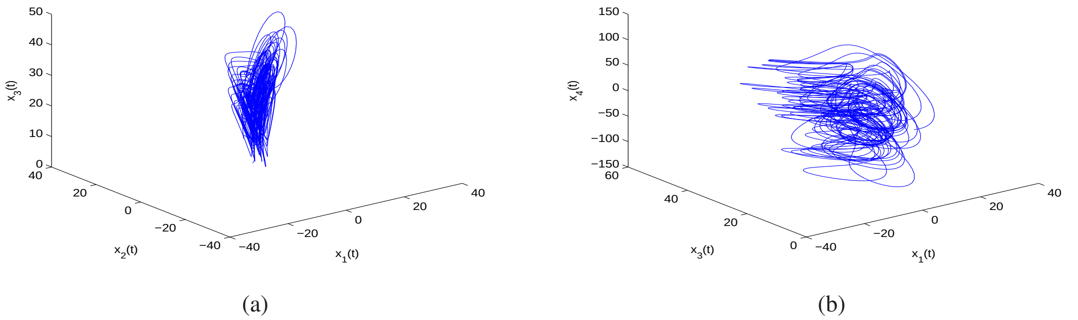


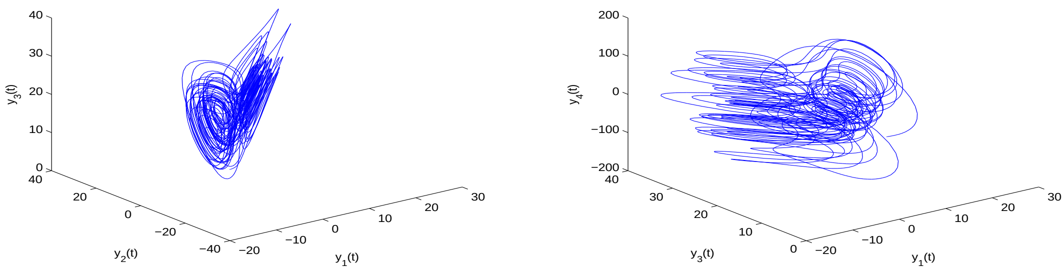
Figure 1: Hyperchaotic attractors for the Lü system. (a) Hyperchaotic attractor in (x_1, x_2, x_3) space; (b) hyperchaotic attractor in (x_1, x_3, x_4) space.

control and confidentiality of communications with a new field will be applied widely in the future, its mathematical model is:

$$\begin{cases} \dot{y}_1 = a_1(y_2 - y_1) + y_4 \\ \dot{y}_2 = d_1y_1 - y_1y_3 + c_1y_2 \\ \dot{y}_3 = y_1y_2 - b_1y_3 \\ \dot{y}_4 = y_2y_3 + r_1y_4 \end{cases} \tag{3}$$

where y_1, y_2, y_3, y_4 are state variables, and a_1, b_1, c_1, d_1, r_1 are real constants.

When $a_1 = 35, b_1 = 3, c_1 = 12, d_1 = 7$ and $0 \le r \le 0.085$, system (3) is chaotic; when $a_1 = 35, b_1 = 3, c_1 = 12, d_1 = 7$ and $0.085 \le r \le 0.798$, system (3) is hyperchaotic; when $a_1 = 35, b_1 = 3, c_1 = 12, d_1 = 7$ and $0.798 \le r \le 0.90$, system (3) is periodic . Also, it has been shown that when $a_1 = 35, b_1 = 3, c_1 = 12, d_1 = 7$ and $0.085 \le r \le 0.798$, the hyperchaotic Chen system (3) has only one equilibrium $O(0, 0, 0, 0)$, and is a forced dissipative system, which implies that the solutions of the system are bounded as $t \to \infty$.The hyperchaotic attractor as shown in Fig.2.



(a) (b)

Figure 2: Hyperchaotic attractors for the Chen system. (a) Hyperchaotic attractor in (y_1, y_2, y_3) space; (b) hyperchaotic attractor in (y_1, y_3, y_4) space.

3 Anti-synchronization via active control

To observe the anti-synchronization behavior in hyperchaotic Lü and hyperchaotic Chen systems we assume that Lü system drives the Chen system. Therefore, we define the master as the hyperchaotc Lü system (2) and slave systems as follows.

$$\begin{cases} \dot{y}_1 = a_1(y_2 - y_1) + y_4 + u_1(t) \\ \dot{y}_2 = d_1 y_1 - y_1 y_3 + c_1 y_2 + u_2(t) \\ \dot{y}_3 = y_1 y_2 - b_1 y_3 + u_3(t) \\ \dot{y}_4 = y_2 y_3 + r_1 y_4 + u_4(t) \end{cases} \tag{4}$$

We have introduced four control functions $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$.Our goal is to determine the mentioned functions. In order to estimate the control functions, we add (2) to (4). We define the AS error system as the summation of the hyperchaotic Lü system (2) and the controlled hyperchaotic Chen system (4). Let us define the states of the AS errors for the slave system (4) that is to be controlled and the controlling system (2) as:

$$\begin{cases} e_1 = x_1 + y_1 \\ e_2 = x_2 + y_2 \\ e_3 = e_3 + y_3 \\ e_4 = x_4 + y_4 \end{cases} \tag{5}$$

By adding (2) to (4) and using the notation (5) we can get;

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + (a - a_1)(y_1 - y_2) + e_4 + u_1(t) \\ \dot{e}_2 = ce_2 - (c - c_1)y_2 - x_1 x_3 - y_1 y_3 + d_1 y_1 + u_2(t) \\ \dot{e}_3 = -be_3 + (b - b_1)y_3 + x_1 x_2 + y_1 y_2 + u_3(t) \\ \dot{e}_4 = de_4 - (d - r_1)y_4 + x_1 x_3 + y_2 y_3 + u_4(t) \end{cases} \tag{6}$$

Then, by defining the active control inputs $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$ as follows:

$$\begin{cases} u_1(t) = -(a - a_1)(y_1 - y_2) - e_4 + v_1(t) \\ u_2(t) = (c - c_1)y_2 + x_1 x_3 + y_1 y_3 - d_1 y_1 + v_2(t) \\ u_3(t) = -(b - b_1)y_3 - x_1 x_2 - y_1 y_2 + v_3(t) \\ u_4(t) = (d - r_1)y_4 - x_1 x_3 - y_2 y_3 + v_4(t) \end{cases} \tag{7}$$

this leads to:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + v_1(t) \\ \dot{e}_2 = ce_2 + v_2(t) \\ \dot{e}_3 = -be_3 + v_3(t) \\ \dot{e}_4 = d_4 e_4 + v_4(t) \end{cases} \tag{8}$$

The error system (8) to be controlled is a linear system with control input $v_1(t), v_2(t)$, $v_3(t)$ and $v_4(t)$ as the function of the error states e_1, e_2, e_3 and e_4. As stated , as long as $lim_{t\to\infty}\|e(t)\| = 0$, AS between the driver and response systems is realized, that is, the hyperchaotic Lü system and hyperchaotic Chen system are anti-synchronized under a active control. According to the original method of active control,v_1, v_2, v_3 and v_4 can be rewritten as

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}$$

where A is a 3×3 constant matrix. In order to make the closed loop system (8) stable, the proper choice of the elements of the matrix A is such that the system (8) must have all eigenvalues with negative real parts. Let

$$A = \begin{pmatrix} -\lambda_1 - a & -a & 0 & 0 \\ 0 & -\lambda_2 - c & 0 & 0 \\ 0 & 0 & -\lambda_3 + b & 0 \\ 0 & 0 & 0 & -\lambda_4 - d \end{pmatrix} \tag{9}$$

with $\lambda_1 > 0, \lambda_2 > 0, \lambda_3 > 0, \lambda_4 > 0$. So the eigenvalues of the closed loop system (8) are: $-\lambda_1$, $-\lambda_2$,$-\lambda_3$, $-\lambda_4$. Thus, anti-synchronization between hyperchaotic Lü system and hyperchaotic Chen system is achieved.

4 Simulation results

Numerical simulations are carried out using MATLAB. The fourth order Runge-Kutta integration method is used to solve two systems of differential equations (2) and (4). In addition, a time step size 0.001 is employed. we assume that the control gain $(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (1, 1, 1, 1)$. We will select the parameters of hyperchaotic Lü system $a = 36, b = 3, c = 20, d = 1$ and the parameters of hyperchaotic Chen system as $a_1 = 35, b_1 = 3, c_1 = 12, d_1 = 7$ and $r_1 = 0.5$.Therefore, both Lü and Chen systems exhibit hyperchaotic behavior. The initial values of the master and slave systems are $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.3, 3.5, 4.2, 1.2)$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (1, -1, 1, 0)$. So the initial values of the error system is $(e_1(0), e_2(0), e_3(0), e_4(0)) = (1.3, -2.5, 5.2, 1.2)$.The anti-synchronization of systems (2) and (4) via active control law (7) are shown in Fig.3.

5 Anti-synchronization by adaptive control

This section ,we consider synchronization behavior between hyperchaotic Chen system and hyperchaotic Lü system via adaptive control. We assume that hyperchaotic Lü system with four unknown parameters is the drive system and hyperchaotic Chen system is the response system. The drive and response systems are defined below, respectively.

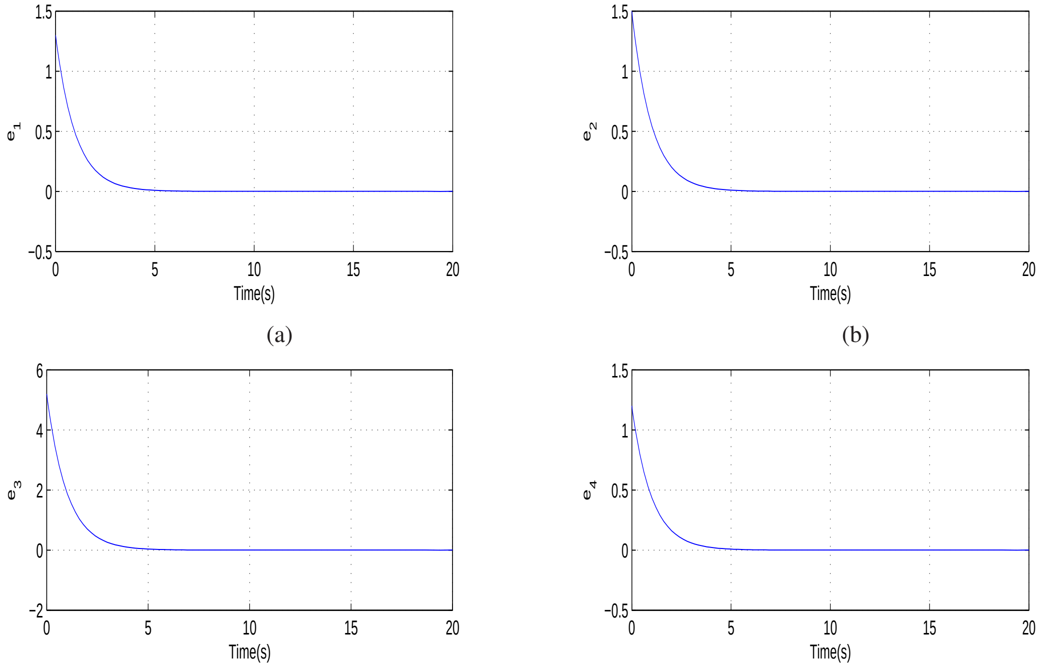
$$\begin{cases} \dot{x}_1 = -a(x_1 - x_2) + x_4 \\ \dot{x}_2 = cx_2 - x_1x_3 \\ \dot{x}_3 = x_1x_2 - bx_3 \\ \dot{x}_4 = x_1x_3 + dx_4 \end{cases} \tag{10}$$

and

$$\begin{cases} \dot{y}_1 = a_1(y_2 - y_1) + y_4 + u_1(t) \\ \dot{y}_2 = d_1y_1 - y_1y_3 + c_1y_2 + u_2(t) \\ \dot{y}_3 = y_1y_2 - b_1y_3 + u_3(t) \\ \dot{y}_4 = y_2y_3 + r_1y_4 + u_4(t) \end{cases} \tag{11}$$

where u_1, u_2, u_3, u_4 are four control functions to be designed; in order to determine the control functions to realize anti-synchronization between systems (10) and (11), we add (10) to (11) and get

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + (a - a_1)(y_1 - y_2) + e_4 + u_1(t) \\ \dot{e}_2 = ce_2 - (c - c_1)y_2 - x_1x_3 - y_1y_3 + d_1y_1 + u_2(t) \\ \dot{e}_3 = -be_3 + (b - b_1)y_3 + x_1x_2 + y_1y_2 + u_3(t) \\ \dot{e}_4 = de_4 - (d - r_1)y_4 + x_1x_3 + y_2y_3 + u_4(t) \end{cases} \tag{12}$$



(a) (b) (c) (d)

Figure 3: Dynamics of anti-synchronization errors (e_1, e_2, e_3, e_4) between hyperchaotic Lü system and hyperchaotic Chen system with time t via active control.

where $e_1 = x_1 + y_1, e_2 = x_2 + y_2, e_3 = x_3 + y_3, e_4 = x_4 + y_4$; our goal is to find proper control functions $u_i(i = 1, 2, 3, 4)$ and a parameter update rule such that system (10) globally anti-synchronizes system (11) asymptotically, i.e., $lim_{t\to\infty}\|e(t)\| = 0$, where $e = [e_1, e_2, e_3, e_4]^T$.

For two systems (10) and (11) without controls $(u_i = 0, i = 1, 2, 3, 4)$, if the initial condition $(x_1(0), x_2(0), x_3(0), x_4(0)) \neq (y_1(0), y_2(0), y_3(0), y_4(0))$, then the trajectories of two systems will quickly separate from each other and become irrelevant. However, when controls are applied, the two systems will approach anti-synchronization for any initial conditions through appropriate control functions. With this mind, we propose the following Theorem 1.

Theorem 1 *Let us now define the active control functions $u_1(t), u_2(t), u_3(t)$ and $u_4(t)$ as*

$$\begin{cases} u_1(t) = (-k_1 + \hat{a})e_1 - e_4 + \hat{a}(y_2 - y_1) + \hat{a_1}(y_1 - y_2) - \hat{a}e_2 \\ u_2(t) = (-k_2 - \hat{c})e_2 + (x_1x_3 + y_1y_3) - \hat{d_1}y_1 + \hat{c}y_2 - \hat{c_1}y_2 \\ u_3(t) = (-k_3 + \hat{b})e_3 - \hat{b}y_3 + \hat{b_1}y_3 - (x_1x_2 + y_1y_2) \\ u_4(t) = (-k_4 - \hat{a})e_4 + \hat{d}y_4 - \hat{r_1}y_4 - (x_1x_2 + y_2y_3) \end{cases} \tag{13}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{a_1}, \hat{b_1}, \hat{c_1}, \hat{d_1}, \hat{r_1}$are estimates of $a, b, c, d, a_1, b_1, c_1, d_1, r_1$ respectively and the parameters adaptive laws of $a, b, c, d, a_1, b_1, c_1, d_1, r_1$ as below

$$\begin{cases} \dot{\hat{a}} = (y_2 - y_1 + e_1 - e_2)e_1 \\ \dot{\hat{b}} = (y_3 - e_3)e_3 \\ \dot{\hat{c}} = (e_2 - y_2)e_2 \\ \dot{\hat{d}} = (e_4 - y_4)e_4 \\ \dot{\hat{a_1}} = (y_2 - y_1)e_1 \\ \dot{\hat{b_1}} = -y_3e_3 \\ \dot{\hat{c_1}} = -y_2e_2 \\ \dot{\hat{d_1}} = y_1e_2 \\ \dot{\hat{r_1}} = y_4e_4 \end{cases} \tag{14}$$

then the uncertain hyperchaotic Lü and hyperchaotic Chen system is anti-synchronized.

Proof. . According to drive system (10) and the controlled response system (11), we get the error dynamical system (12) can be described by

$$\begin{cases} \dot{e_1} = \tilde{a}(e_2 - e_1) + \tilde{a}(y_2 - y_1) + \tilde{a_1}(y_1 - y_2) - k_1 e_2 \\ \dot{e_2} = -\tilde{c}e_2 + \tilde{c}y_2 - \tilde{c_1}y_2 - \tilde{d_1}y_1 - k_2 e_2 \\ \dot{e_3} = \tilde{b} - \tilde{b}y_3 + \tilde{b_1}y_3 - k_3 e_3 \\ \dot{e_4} = -\tilde{d}e_4 + \tilde{d}y_4 - \tilde{r_1}y_4 - k_4 e_4 \end{cases} \tag{15}$$

where $\tilde{a} = \hat{a} - a, \tilde{b} = \hat{b} - b, \tilde{b} = \hat{c} - c, \tilde{d} = \hat{d} - d, \tilde{a_1} = \hat{a_1} - a_1, \tilde{b_1} = \hat{b_1} - b_1, \tilde{c_1} = \hat{c_1} - c_1, \tilde{d_1} = \hat{d_1} - d_1, \tilde{r_1} = \hat{r_1} - r_1$, Consider a Lyapunov function as

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{d}^2 + \tilde{a_1}^2 + \tilde{b_1}^2 + \tilde{c_1}^2 + \tilde{d_1}^2 + \tilde{r_1}^2) \tag{16}$$

Differentiating (16) using (15) gives

$$\begin{aligned} \dot{V} &= e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3} + e_4\dot{e_4} + \tilde{a}\dot{\hat{a}} + \tilde{b}\dot{\hat{b}} + \tilde{c}\dot{\hat{c}} + \tilde{d}\dot{\hat{d}} + \tilde{a_1}\dot{\hat{a_1}} + \tilde{b_1}\dot{\hat{b_1}} + \tilde{c_1}\dot{\hat{c_1}} + \tilde{d_1}\dot{\hat{d_1}} + \tilde{r_1}\dot{\hat{r_1}} \\ &= -(\tilde{a} + k_1)e_1^2 + \tilde{a}e_1e_2 + \tilde{a}(y_2 - y_1)e_1 + \tilde{a_1}(y_1 - y_2)e_1 - (\tilde{c} + k_2)e_2^2 + \tilde{c}y_2e_2 - \tilde{c_1}y_2e_2 \\ &\quad -\tilde{d_1}y_1e_2 + (\tilde{b} - k_3)e)_3^2 - \tilde{b}y_3e_3 + \tilde{b_1}y_3e_3 - (k_4 + \tilde{d})e_4^2 + \tilde{d}y_4e_4 - \tilde{r_1}y_4e_4 + \tilde{a_1}\dot{\hat{a_1}} \\ &\quad +\tilde{b_1}\dot{\hat{b_1}} + \tilde{c_1}\dot{\hat{c_1}} + \tilde{d_1}\dot{\hat{d_1}} + \tilde{r_1}\dot{\hat{r_1}} \end{aligned}$$

Since

$$\dot{\hat{a}} = (y_2 - y_1 + e_1 - e_2)e_1\ ,\ \dot{\hat{b}} = (y_3 - e_3)e_3\ ,\ \dot{\hat{c}} = (e_2 - y_2)e_2\ ,\ \dot{\hat{d}} = (e_4 - y_4)e_4,$$
$$\dot{\hat{a_1}} = (y_2 - y_1)e_1\ ,\ \dot{\hat{b_1}} = -y_3e_3\ ,\ \dot{\hat{c_1}} = y_2e_2\ ,\ \dot{\hat{d_1}} = y_1e_2\ ,\ \dot{\hat{r_1}} = y_4e_4$$

Then we can get

$$\dot{V} = -(k_1e_1^2 + k_2e_2^2 + k_3e_3^2 + k_4e_4^2) = -e^TPe$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}, P = \begin{pmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{pmatrix}.$$

Since $\dot{V}$ is negative semidefinite, we cannot immediately obtain that the origin of error system (12) is asymptotically stable. In fact, as $\dot{V} \le 0$,then $e_1, e_2, e_3, e_4, \tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{a_1}, \tilde{b_1}, \tilde{c_1}, \tilde{d_1}, \tilde{r_1} \in \psi_\infty$.From error system(12),we have$\dot{e_1}, \dot{e_2}, \dot{e_3}, \dot{e_4} \in \psi_\infty$.Since $\dot{V} = e^TPe$ and P is a positive-definite matrix, then we have

$$\int_0^t \lambda_{\min}(P)\,\|e\|^2 dt \le \int_0^t e^TPedt \le \int_0^t -\dot{V}dt = V(0) - V(T) \le V(0),$$

where $\lambda_{min}(P)$ is the minimum eigenvalue of positive-definite matrix P.Thus $e_1, e_2, e_3, e_4 \in \psi_2$.According to the Barbalats lemma, we have $e_1(t), e_2(t), e_3(t), e_4(t) \to 0$ as $t \to \infty$.Therefore, the drive system (10) is synchronous asymptotically with the response system (11) with the controllers (15) and the parameter estimation update law (14).This completes the proof. ■

6 Simulation results

Numerical simulations are carried out using MATLAB. The fourth order Runge-Kutta integration method is used to solve two systems of differential equations (10) and (11). In addition, a time step size 0.001 is employed. we assume that the control gain $(k_1, k_2, k_3, k_4) = (1, 1, 1, 1)$. We will select the parameters of hyperchaotic Lü system $a = 36, b = 3, c = 20, d = 1$ and the parameters of hyper chaotic Chen system as $a_1 = 35, b_1 = 3, c_1 = 12, d_1 = 7$ and $r_1 = 0.5$.Therefore, both hyperchaotic lü and hyperchaotic Chen systems exhibit hyperchaotic behavior. The initial values of the master and slave systems are $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.3, 3.5, 4.2, 1.2)$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (5, 7, 9, 11)$. So the initial values of the error system is $(e_1(0), e_2(0), e_3(0), e_4(0)) = (5.3, 10.5, 13.2, 12.2)$.The anti-synchronization of systems (10) and (11) via adaptive control law (15) and parameter update rule (14) are shown in Fig.4 and Fig.5.

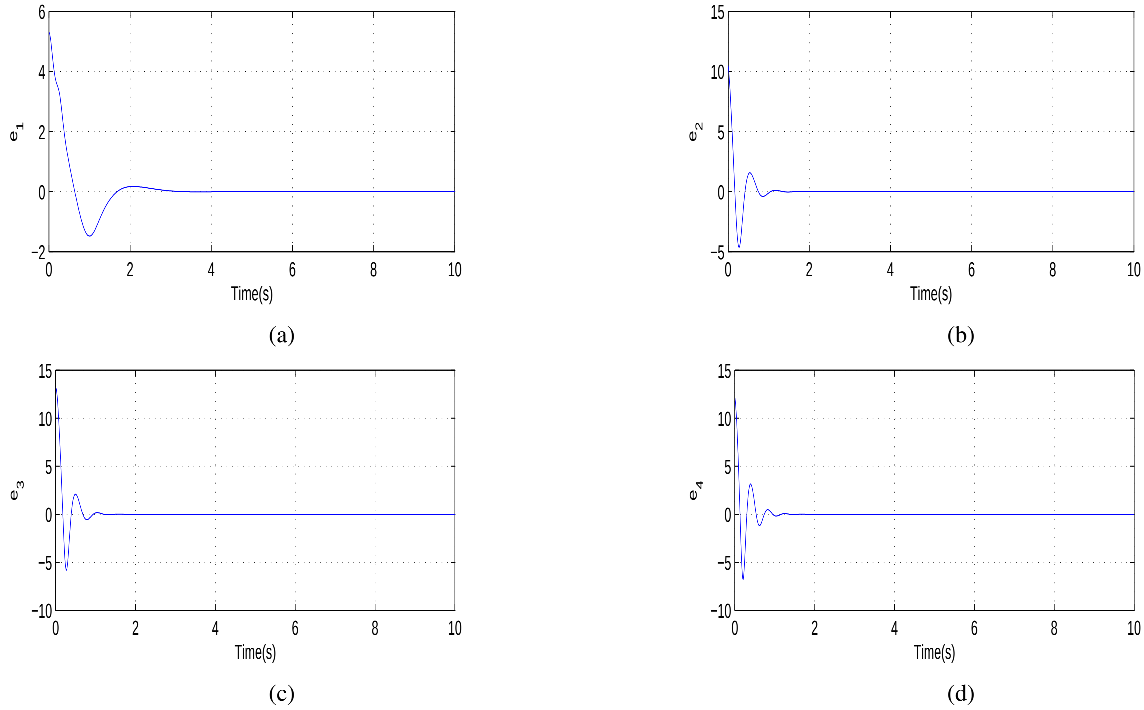


Figure 4: Dynamics of anti-synchronization errors (e_1, e_2, e_3, e_4) between hyperchaotic Lü system and hyperchaotic Chen system with time t via adaptive control.

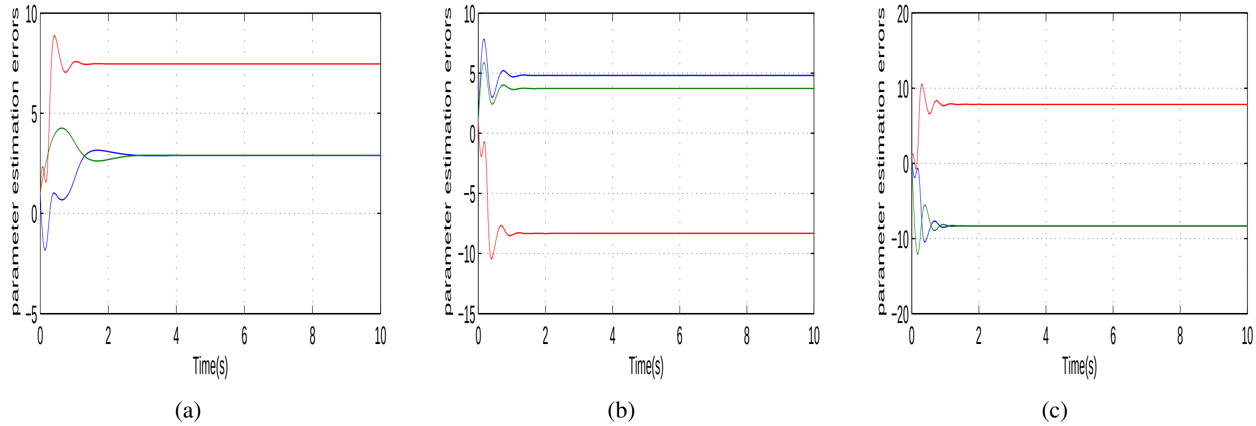


Figure 5: Adaptive parameters estimation errors:(a) $\hat{a}, \hat{a_1}, \hat{c}$;(b)$\hat{b}, \hat{d_1}, \hat{c_1}$; (c) $\hat{d}, \hat{b_1}, \hat{r_1}$.

7 Conclusion

In this paper, chaos anti-synchronization between two different chaotic systems with different structures and parameters via active and adaptive control is presented. Hyperchaotic Chen system and hyperchaotic Lü system are taken as an illustrative example to verify the effectiveness of the proposed method.

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