Generalized Projective Synchronization of the Energy Resource System

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Abstract: In this paper, an active control method is proposed to projective-synchronize two chaotic systems. The proposed technique is applied to achieve generalized projective synchronization for the energy resource system, where all state variables are in a proportional way. This property allows us to arbitrarily direct the scaling factor onto a desired value. Feasibility of the proposed control scheme is illustrated through the numerical examples.

Keywords: generalized projective synchronization; energy resource system; active control

1 Introduction

Since chaos control and synchronization has been observed by Pecora and Carroll in the early 1990s [1], different types of control and synchronization behaviors have been discovered because of potential applications in secure communications [3, 4]. Recently, projective synchronization has been first reported by Mainieri and Rehacek [5] in partially linear systems that the drive and response vectors evolve in a proportional scale—the vectors become proportional. Hence, identical synchronizations and anti-phase synchronizations are the special cases of projective synchronization where \( \alpha = 1 \) and \( \alpha = -1 \), respectively. This proportional feature can be used to extend binary digital to M-nary digital communication for achieving fast communication.

The early projective synchronization is usually observable only in a class of systems with partial-linearity [3, 4], but recently some researchers [6, 7] have achieved control of the projective synchronization in a general class of chaotic systems including non-partially-linear systems [8], and termed this projective synchronization as “generalized projective synchronization” (GPS).

In this paper, we first briefly introduced the energy resource chaotic system. Then we generalize active control to GPS, and demonstrate this technique by some typical chaotic systems, for example, the energy resource chaotic system such that GPS is achieved. The results from numerical simulations show that the method works well.

2 Energy resource chaotic system

Energy resource chaotic system [9] is described by the following system of differential equations:

\[
\begin{align*}
\frac{dx}{dt} &= a_1 x (1 - \frac{x}{N}) - a_2 (y + z) \\
\frac{dy}{dt} &= -b_1 y - b_2 z + b_3 x [N - (x - z)] \\
\frac{dz}{dt} &= c_1 z (c_2 x - c_3)
\end{align*}
\] (2.1)
where \( x(t) \) the energy resource shortage in \( A \) region, \( y(t) \) the energy resource supply increment in \( B \) region, \( z(t) \) the energy resource import in \( A \) region; \( a_i, b_i, c_i, M, N \) are positive real constants. This system has three equilibria: \( O(0,0,0) \), \( S_1 = (x_1, y_1, z_1) \), \( S_2 = (x_2, y_2, z_2) \), where

\[
\begin{align*}
    x_1 &= \frac{a_2b_3MN - a_1b_1M}{a_2b_3M - a_1b_1}, \\
    y_1 &= \frac{a_1b_3(M - N)(a_2b_3MN - a_1b_1M)}{(a_2b_3M - a_1b_1)^2}, \quad z_1 = 0, \quad x_2 = \frac{c_3}{c_2} ,
\end{align*}
\]

\[
y_2 = \frac{a_1b_1x_2(1 - \frac{c_4}{M}) - b_3N x_2 + b_3x_2^2}{b_1 + b_3x_2 - b_2}, \quad z_2 = \frac{a_1b_1x_2(1 - \frac{c_4}{M}) - b_3N x_2 + b_3x_2^2}{b_1 + b_3x_2 - b_2}.
\]

From [10] when \( a_1 = 0.09, a_2 = 0.15, b_1 = 0.06, b_2 = 0.082, b_3 = 0.07, c_1 = 0.2 \quad c_2 = 0.5, c_3 = 0.4, M = 1.8, N = 1 \), we obtain the equilibrium point \( O(0, 0, 0) \), \( S_1(0.68, 0.2539, 0) \), \( S_2(0.8, 0.1255, 0.1412) \), which are unstable. This system has a chaotic attractor shown in Fig. 1. When \( a_1 = 0.09, a_2 = 0.15, b_1 = 0.06, b_2 = 0.082, b_3 = 0.07, c_1 = 0.2, c_2 = 0.5, c_3 = 0.4, M = 1.8, N = 1 \) and initial condition \( [0.82, 0.29, 0.48] \), the value of Lyapunov exponents of this system is obtained as \( (0.068, 0.016, -0.016) \). This energy resource attractor is different from the well-known Lorenz attractor, \( Rössler \) attractor, Chua’s attractor, Chen attractor, \( Lü \) attractor and so on.

In paper [11]-[13], Hopf bifurcation of the energy resource system is analyzed by using an analytical method. It is shown that the energy resource chaotic system has complex dynamics with some interesting characteristics. We achieved chaotic synchronization for the energy resource system by applied the modified adaptive synchronization Chaos in new energy resource chaotic system is controlled to equilibrium points or periodic orbits by using linear feedback control, non-autonomous feedback control, adaptive control and time-delayed feedback control.

![Energy resource attractor.](image)

**Figure 1: Energy resource attractor.**

### 3 GPS of two identical energy resource chaotic systems

The projective synchronization means that the drive and response vectors synchronize up to a scaling factor, that is, the vectors become proportional. First, we define the GPS below.

Consider the following chaotic system:

\[
\begin{align*}
    \dot{x}_m &= f(x_m), \\
    \dot{x}_s &= q(x_m, x_s)
\end{align*}
\]

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where \( n \)-dimensional state vector \( x_m, x_s \in \mathbb{R}^n \). The low subscripts ‘\( m \)’ and ‘\( s \)’ stand for the master and slave systems, respectively. \( f : \mathbb{R}^n \to \mathbb{R}^n \) and \( q : \mathbb{R}^n \to \mathbb{R}^n \) are vector fields in \( n \)-dimensional space. If there exists a constant \( \alpha (\alpha \neq 0) \), such that \( \lim_{t \to \infty} \| x_m - \alpha x_s \| = 0 \), then the GPS of the systems (3.1) is achieved, and we call \( \alpha \) a ‘scaling factor’.

Now, take two identical energy resource systems into consideration. The master system

\[
\begin{align*}
\dot{x}_m &= ax_m(1 - \frac{x_m}{15}) - 0.15(y_m + z_m), \\
\dot{y}_m &= -by_m - cz_m + 0.07x_m[1 - (x_m - z_m)], \\
\dot{z}_m &= 0.2z_m(0.5x_m - 0.4)
\end{align*}
\]  (3.2)

where \( a = 0.09, b = 0.06, c = 0.082 \) to ensure the chaotic attractor. In order to achieve the GPS, by means of techniques from active control theory [14], the following slave system is constructed:

\[
\begin{align*}
\dot{x}_s &= ax_s(1 - \frac{x_s}{15}) - 0.15(y_s + z_s) + u_1, \\
\dot{y}_s &= -by_s - cz_s + 0.07x_s[1 - (x_s - z_s)] + u_2, \\
\dot{z}_s &= 0.2z_s(0.5x_s - 0.4) + u_3
\end{align*}
\]  (3.3)

There are three control functions \( u_i, (i = 1, 2, 3) \) to be determined later.

Define the error vector as

\[
e_1 = x_m - \alpha x_s, \quad e_2 = y_m - \alpha y_s, \quad e_3 = z_m - \alpha z_s,
\]

where \( \alpha \) is a desired scaling factor. Then one obtains the error dynamical system by subtracting (3.3) from (3.2)

\[
\begin{align*}
\dot{e}_1 &= a e_1 - 0.15(e_2 + e_3) - \frac{a}{15}(x_m^2 - x_s^2) - \alpha u_1, \\
\dot{e}_2 &= 0.07 e_1 - be_2 - ce_3 - 0.07(x_m^2 - \alpha x_s^2) + 0.07(x_m z_m - \alpha x_s z_s) - \alpha u_2, \\
\dot{e}_3 &= 0.1(x_m z_m - \alpha x_s z_s) - 0.08e_3 - \alpha u_3
\end{align*}
\]  (3.4)

Referring to the original methods of active control, so we must define the three control functions \( u_i, (i = 1, 2, 3) \) as follows:

\[
\begin{align*}
\alpha u_1 &= -v_1 - \frac{a}{15}(x_m^2 - \alpha x_s^2) + (a + 1)e_1, \\
\alpha u_2 &= -v_2 - 0.07(x_m^2 - \alpha x_s^2) + 0.07(x_m z_m - \alpha x_s z_s), \\
\alpha u_3 &= -v_3 + 0.1(x_m z_m - \alpha x_s z_s)
\end{align*}
\]  (3.5)

Hence the error system (3.4) becomes

\[
\begin{align*}
\dot{e}_1 &= v_1 - e_1 - 0.15(e_2 + e_3), \\
\dot{e}_2 &= v_2 + 0.07 e_1 - be_2 - ce_3, \\
\dot{e}_3 &= v_3 - 0.08e_3
\end{align*}
\]  (3.6)

The system (3.6) to be controlled is a linear system with a control input \( v_1, v_2 \) and \( v_3 \) as function of the error \( e_1, e_2 \) and \( e_3 \). As long as these feedbacks stabilize the system (3.6), \( e_1, e_2 \) and \( e_3 \) converge to zero as time tends to infinity, which implies that GPS of two identical energy resource systems is achieved with a scaling factor \( \alpha \). There are many possible choices for the control \( v_1, v_2 \) and \( v_3 \). In order to make the error system (3.6) be stable, the proper choice of the control should guarantees that the feedback system must have all eigenvalues with negative real parts. For simplify, we choose:

\[
\begin{bmatrix}
  v_1 \\
  v_2 \\
  v_3
\end{bmatrix} = A
\begin{bmatrix}
  e_1 \\
  e_2 \\
  e_3
\end{bmatrix}
= \begin{bmatrix}
  0 & 0.15 & 0.15 \\
  -0.07 & 0 & c \\
  0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  e_1 \\
  e_2 \\
  e_3
\end{bmatrix}
\]

In this particular choice, the three eigenvalues of the system (3.6) are \(-1, -b \) and \(-0.08 \). Since the error system has all eigenvalues that are found to have negative real parts, the system will be convergence. In other words, this choice will result in a stable system and the GPS of two identical energy resource systems. What deserves to be mentioned is that the values of the eigenvalues play an important role in the stability of the error system. In order to quicken the rate of convergence, we should make them get smaller.
We give some numerical examples to illustrate our results. Fourth-order Runge–Kutta method is used to solve the systems of differential equations (3.2) and (3.3) with time step being equal to 0.001 unless specified otherwise. The initial states for the master system and for the slave system are arbitrarily given by 

\( (0.82, 0.29, 0.48) \) and \( (0.1, 0.8, 0.12) \), respectively. First, we take the scaling factor \( \alpha = 5 \). The numerical simulation results are shown in Fig. 2. Fig. 2(a) displays the time response of the synchronization error \( e = [e_1, e_2, e_3]^T \). As expected, one can observe that the error vector \( e \) converges to zero finally after the controller is activated which implies that all the state variables tend to be synchronized in a proportional, and the ratio of the amplitudes of the two systems tends to a constant scaling factor. Fig. 2(b) depicts the projection of the synchronized attractors, which shows that the state vectors of the master state and the slave state are synchronized in the same direction all the time. The same results are shown in Fig. 3 with \( \alpha = -2 \). From Fig. 3, we can see that the state vectors of two synchronized systems evolve in the opposite directions.

4 Conclusion

In this paper, an active control method is proposed for manipulating generalized projective synchronization in the energy resource chaotic system. It extends the GPS to two chaotic systems where all state variables are in a proportional way. This property allows us to adjust the desired scaling factor by controlling the slave system. Numerical experiments show that the present method works very well.

(a) The time evolution of the error vectors

(b) The projection of the synchronized attractors

Figure.2: GPS between two identical systems (3.2) and (3.3) with the scaling factor \( \alpha = 5 \).

(a) The time evolution of the error vectors

(b) The projection of the synchronized attractors

Figure.3: GPS between two identical systems (3.2) and (3.3) with the scaling factor \( \alpha = -2 \).
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