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Nonclassical Symmetry Reductions for Coupled KdV Equations

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Abstract: In this paper, by using the nonclassical method, several new symmetries and solutions are obtained, which are unobtainable by Lie classical symmetries.

Key words: the coupled KdV equations; nonclassical symmetry; symmetry reduction

1 Introduction

The investigation of the exact solutions of nonlinear evolution equations play an important role in the study of nonlinear physical phenomena[1, 2, 3, 4]. The most famous and established method for finding exact solutions of differential equations is the classical symmetries method, also called group analysis, which originated in 1881 from the pioneering work of Lie [5]. Many good books have been dedicated to this subject and its generalizations [6, 7, 8, 9]. The nonclassical method of reduction was devised originally by Bluman and Cole in 1969, to find new exact solutions of the heat equation[10]. The nonclassical symmetries method consists of adding the invariant surface condition to the given equation, and then applying the classical symmetries method. The main difficulty of this approach is that the determining equations are no longer linear. On the other hand, the nonclassical symmetries method may yield more solutions than the classical symmetries method. This approach has been successfully applied to various equations [11, 12], for the purpose of finding new exact solutions.

The Painlevé classification of coupled Korteweg-de Vries (KdV) equations was made recently in [13], and the following new system possessing the Painlevé property was found there:

$$\begin{array}{l}\Delta_1^{(3)} \equiv u_t + (u_{xx} + u^2 + uv)_x = 0,\\ \Delta_2^{(3)} \equiv v_t + (v_{xx} + v^2 + uv)_x = 0.\end{array} \tag{1.1}$$

In this paper, for a new integrable coupled KdV system (1.1), by using the nonclassical method, one obtain several new solutions which are not invariant under any Lie group admitted by the equations and consequently which are not obtainable through the classical Lie method.

2 Nonclassical symmetries

In this section, we derive the determining equations for the nonclassical symmetries of the coupled KdV equation system (1.1) via compatibility. The basic idea of the method is to require that the Eqs.(1.1) and the invariance surface condition:

$$\begin{array}{l}\Delta_3^{(1)} \equiv \xi(x,t,u,v)u_x + \tau(x,t,u,v)u_t - \phi(x,t,u,v),\\ \Delta_4^{(1)} \equiv \xi(x,t,u,v)v_x + \tau(x,t,u,v)v_t - \varphi(x,t,u,v),\end{array} \tag{2.1}$$

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which is associated with the vector field:

$$\underline{v} = \xi(x,t,u,v)\frac{\partial}{\partial x} + \tau(x,t,u,v)\frac{\partial}{\partial t} + \phi(x,t,u,v)\frac{\partial}{\partial u} + \varphi(x,t,u,v)\frac{\partial}{\partial v} \tag{2.2}$$

are both invariant under the transformation with infinitesimal generator (2.2). By requiring that both (1.1) and (2.1) are invariant under the transformation with infinitesimal generator (2.2) one obtains an overdetermined, nonlinear system of equations for the infinitesimals $\xi(x,t,u,v), \tau(x,t,u,v), \phi(x,t,u,v)$ and $\varphi(x,t,u,v)$. The number of determining equations arising in the nonclassical method is smaller than for the classical method. Consequently, the set of solutions is in general, larger than for the classical method as in this method one requires only the subset of solutions of (1.1) and (2.1) to be invariant under the infinitesimal generator (2.2).

To obtain nonclassical symmetries of (1.1), we apply the algorithm described in [15] for calculating the determining equations. The determining equations for the nonclassical symmetries of the system(1.1) are obtained by requiring the governing equation as follows:

$$pr^{(1)}\Delta_i^{(1)} \mid_{\Delta_j^{(1)}=0, \Delta_k^{(3)}=0} = 0 (i,j=3,4)(k=1,2), \tag{2.3}$$

$$pr^{(3)}\Delta_i^{(3)} \mid_{\Delta_j^{(1)}=0, \Delta_k^{(3)}=0} = 0 (i,k=1,2)(j=3,4). \tag{2.4}$$

The equations in (2.3) are satisfied identically. The remaining equations lead to a set of equations that must then be solved. For further discussions, we consider some different subcases to partially solve the equations.

We can distinguish two different cases (in this paper, $t_i (i=0,1,2,3,...)$ are arbitrary constants):

Case 1: $\tau(x,t,u,v) \neq 0$. In this case, without loss of generality, we may set $\tau(x,t,u,v) = 1$.

Case 1(A):

$$\xi = \frac{x+3t_1}{3(t-t_0)}, \phi = \frac{(1+3t_2)u - 3(1+t_3)v}{3(t-t_0)}, \varphi = \frac{-3(1+t_2)u + (1+3t_3)v}{3(t-t_0)},$$

i.e.

$$\underline{v} = [\frac{x+3t_1}{3(t-t_0)}]\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + [\frac{(1+3t_2)u-3(1+t_3)v}{3(t-t_0)}]\frac{\partial}{\partial u} + [\frac{-3(1+t_2)u+(1+3t_3)v}{3(t-t_0)}]\frac{\partial}{\partial v}.$$

This subcase is just the result of classical Lie approach. The corresponding reduction coincides with $c_2 \neq 0$ of [14] for $t_0 = \frac{c_3}{3c_2}, t_1 = \frac{c_1}{3c_2}, t_2 = -\frac{1}{3}(2+\frac{c_4}{c_2}), t_3 = \frac{1}{3}(\frac{c_5}{c_2} - 2)$. Then, the group invariant solution of the coupled equation system (1.1) takes the form:

$$\begin{aligned} u &= \frac{(1+t_3)}{(1+t_2)(t-t_0)^{2/3)}} U(\zeta) - (t-t_0)^{(t_2+t_3+4/3)} V(\zeta), \\ v &= \frac{1}{(t-t_0)^{2/3)}} U(\zeta) + (t-t_0)^{(t_2+t_3+4/3)} V(\zeta), \\ \zeta &= \frac{x+3t_1}{(t-t_0)^{1/3}}, \end{aligned}$$

where $U(\zeta), V(\zeta)$ satisfy the following ODEs:

$$\begin{aligned} U''' &= (-\frac{2(t_2+t_3+2)}{1+t_2}U + \frac{\zeta}{3})U' + \frac{2}{3}U, \\ V''' &= (-\frac{(t_2+t_3+2)}{1+t_2}U + \frac{\zeta}{3})V' - \frac{(t_2+t_3+2)}{1+t_2}VU' - (t_2+t_3+\frac{4}{3})V \end{aligned} \tag{2.5}$$

and prime denotes differentiation with respect to ζ. For first equation of Eqs.(2.5), let $U(\zeta) = -\frac{(1+t_2)W(\zeta)}{6(t_2+t_3+2)} + \frac{1+t_2\zeta}{2(t_2+t_3+2)}$ and integrates once, then

$$W'' = \frac{(W')^2}{2W} - \frac{\zeta W}{3} + \frac{W^2}{9} + \frac{C}{W}.$$

Case 1(B):

$$\xi = t_0, \phi = t_1 u - t_2 v, \varphi = -t_1 u + t_2 v,$$

i.e.

$$\underline{v} = t_0\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + (t_1 u - t_2 v)\frac{\partial}{\partial u} + (-t_1 u + t_2 v)\frac{\partial}{\partial v}.$$

This subcase is just the result of classical Lie approach. The corresponding reduction coincides with $c_2 = 0, c_1c_3 \neq 0$ of [14] for $t_0 = \frac{c_1}{c_3}, t_1 = -\frac{c_4}{c_3}, t_2 = \frac{c_5}{c_3}$, or $c_2 = c_1 = 0, c_3 \neq 0$ of [14] for $t_0 = 0, t_1 = -\frac{c_4}{c_3}, t_2 = \frac{c_5}{c_3}$. Then the solution of the coupled system (1.1) takes the form:

$$\begin{aligned} &u = U(\zeta) + \exp(\tfrac{(t_1+t_2)x}{t_0})V(\zeta),\\ &v = \tfrac{t_1}{t_2}U(\zeta) - \exp(\tfrac{(t_1+t_2)x}{t_0})V(\zeta),\\ &\zeta = x - t_0 t. \end{aligned}$$

The symmetry reduction equations are

$$\begin{aligned} U''' &= -(\tfrac{2(t_1+t_2)}{t_2}U - t_0)U',\\ V''' &= -\tfrac{3(t_1+t_2)}{t_0}V'' - (\tfrac{3(t_1+t_2)^2}{t_0^2} - t_0)V' - \tfrac{t_1+t_2}{t_2}(VU)' - \tfrac{(t_1+t_2)^2}{t_0t_2}UV - \tfrac{(t_1+t_2)^3}{t_0^3}V. \end{aligned}$$

Case 2: $\tau(x,t,u,v) = 0$. In this case, without loss of generality, we may set $\xi(x,t,u,v) = 1$. Similar to Case 1, substituting all the extensions into Eqs.(2.4) , we obtain the determining equation for the infinitesimal ϕ, φ, but the complexity of this equation is the reason why we cannot solve it in general. Thus we proceed. By making ansatz on the form of functions $\phi(x,t,u,v)$ and $\varphi(x,t,u)$ to solve it, ten different subcases are obtained:

Case 2(A):

$$\begin{aligned} \phi &= (t_1 + \tfrac{1}{T})u + t_1 v + \tfrac{t_1t_2x - 2t_4}{T^2},\\ \varphi &= -(t_1 + \tfrac{1}{T})u - t_1 v - \tfrac{t_1t_2x - 2t_4 + t_2}{T^2}, \end{aligned}$$

i.e.

$$\begin{aligned} \underline{v} = &\tfrac{\partial}{\partial x} + [(t_1 + \tfrac{1}{T})u + t_1 v + \tfrac{t_1t_2x-2t_4}{T^2}]\tfrac{\partial}{\partial u}\\ &+[-(t_1 + \tfrac{1}{T})u - t_1 v - \tfrac{t_1t_2x-2t_4+t_2}{T^2}]\tfrac{\partial}{\partial v}, \end{aligned}$$

where $T = (-2t_2t - 2t_3)^{1/2}$. Let $t_2 = 0, t_4 = 0$, then this subcase is just the result of classical Lie approach. The corresponding reduction coincides with $c_2 = c_3 = 0, c_1 \neq 0$ of [14] for $t_1 = -c_5/c_1, t_3 = -c_1^2/[2(c_5 - c_4)^2]$.

Then the solution of the coupled system (1.1) takes the form:

$$\begin{aligned} &u = T\exp(x/T)U(t) + V(t) - \tfrac{t_2 x}{T^2},\\ &v = -T[\exp(x/T)U(t) + \tfrac{t_1V(t)}{t_1T+1} - \tfrac{t_2-2t_4}{T^2(tT+1)}]. \end{aligned}$$

So the reduced equations of the coupled KdV equation system (1.1) reads:

$$\begin{aligned} U' &= -(\tfrac{1}{T(t_1T+1)}V(t) - \tfrac{2t_1t_2T^2 - (t_1-t_2-2t_4)T - 1}{T^3(t_1T+1)})U(t),\\ V' &= -\tfrac{(t_1T+2)t_2}{T^2(t_1T+1)}V(t) + \tfrac{(t_2-2t_4)t_2}{T^3(t_1T+1)}. \end{aligned}$$

Case 2(B):

$$\begin{aligned} \phi &= \tfrac{t_3(t_3t+t_4)(u+v)}{t_3x+t_2-t_1t_3(t_3t+t_4)} - \tfrac{t_3(t_3x+t_2)}{2(t_3x+t_2-t_1t_3(t_3t+t_4))},\\ \varphi &= -\tfrac{t_3(t_3t+t_4-1)(u+v)}{t_3x+t_2-t_1t_3(t_3t+t_4)} + \tfrac{t_3(t_3x+t_2-t_1t_3)}{2(t_3x+t_2-t_1t_3(t_3t+t_4))}, \end{aligned}$$

i.e.

$$\begin{aligned} \underline{v} = &\tfrac{\partial}{\partial x} + [\tfrac{t_3(t_3t+t_4)(u+v)}{t_3x+t_2-t_1t_3(t_3t+t_4)} - \tfrac{t_3(t_3x+t_2)}{2(t_3x+t_2-t_1t_3(t_3t+t_4))}]\tfrac{\partial}{\partial u}\\ &+[-\tfrac{t_3(t_3t+t_4-1)(u+v)}{t_3x+t_2-t_1t_3(t_3t+t_4)} + \tfrac{t_3(t_3x+t_2-t_1t_3)}{2(t_3x+t_2-t_1t_3(t_3t+t_4))}]\tfrac{\partial}{\partial v}. \end{aligned}$$

Then the solution of the coupled system (1.1) takes the form:

$$\begin{aligned} &u = \tfrac{(t_3x+t_2)-t_3(x+t_1)(t_3t+t_4)}{t_3(t_3t+t_4)}U(t) - V(t) + \tfrac{t_3x+t_2}{2(t_3t+t_4)},\\ &v = xU(t) + V(t). \end{aligned}$$

Then the reduced equation of coupled KdV equation system (1.1) become:

$$\begin{aligned} U' &= -\tfrac{(2U+t_3)U}{t_3t+t_4},\\ V' &= (t_1 - \tfrac{t_2}{t_3(t_3t+t_4)})U^2 - \tfrac{2+t_3}{2(t_3t+t_4)}V - \tfrac{t_2}{2(t_3t+t_4)}. \end{aligned}$$

So the coupled KdV equation system (1.1) possesses the exact solutions as follow:

$$u=\frac{t_6}{(t_3^2t_5t+t_3t_4t_5-2)^{1/2}}+\frac{t_5(t_3x+t_2)-2t_1}{t_5(t_3^2t_5t+t_3t_4t_5-2)},$$
$$v=-\frac{t_6}{(t_3^2t_5t+t_3t_4t_5-2)^{1/2}}+\frac{(t_3t_5-2)[t_5(t_3x+t_2)-2t_1]}{2t_5(t_3^2t_5t+t_3t_4t_5-2)}.$$

Case 2(C):

$$\phi=\frac{t+t_1+t_2}{t_2x+t_3}u+\frac{t+t_1-t_2}{t_2x+t_3}v-\frac{1}{2c_2},$$
$$\varphi=-\frac{t+t_1}{t_2x+t_3}u-\frac{t+t_1-2t_2}{t_2x+t_3}v+\frac{1}{2c_2}.$$

i.e.

$$\underline{v}=\frac{\partial}{\partial x}+[\frac{t+t_1+t_2}{t_2x+t_3}u+\frac{t+t_1-t_2}{t_2x+t_3}v-\frac{1}{2c_2}]\frac{\partial}{\partial u}$$
$$+[\frac{t+t_1+t_2}{t_2x+t_3}u+\frac{t+t_1-t_2}{t_2x+t_3}v-\frac{1}{2c_2}]\frac{\partial}{\partial v}.$$

The solution of the coupled system (1.1) takes the form:

$$u=\frac{(t_2x+t_3)^2}{t_2^2}U(t)+\frac{(t_2x+t_3)}{t_2}V(t),$$
$$v=-\frac{(t_2x+t_3)^2}{t_2^2}U(t)-\frac{(t_2x+t_3)(t+t_1)}{t_2(t+t_1-t_2)}V(t)+\frac{t_2x+t_3}{2t_2(t+t_1-t_2)}.$$

The symmetry reduction equations are

$$U'=(\frac{3t_2}{t+t_1-t_2}V-\frac{3}{2(t+t_1-t_2)})U,$$
$$V'=\frac{2t_2}{t+t_1-t_2}V^2-\frac{1}{t+t_1-t_2}V.$$

The exact solution of the system (1.1):

$$u=\frac{t_4(t_2x+t_3)^2}{t_2^2[t_5t+2t_2+t_5(t_1-t_2)]^{3/2}}+\frac{t_2x+t_3}{t_2[t_5t+2t_2+t_5(t_1-t_2)]},$$
$$v=-\frac{t_4(t_2x+t_3)^2}{t_2^2[t_5t+2t_2+t_5(t_1-t_2)]^{3/2}}-\frac{(t_2x+t_3)(t+t_1)}{t_2(t+t_1-t_2)[t_5t+2t_2+t_5(t_1-t_2)]}+\frac{t_2x+t_3}{2t_2(t+t_1-t_2)}.$$

Case 2(D):

$$\phi=t_1(u+v)-\frac{t_1t_2x+t_1t_3-t_2}{2t_2t+t_1},$$
$$\varphi=-t_1(u+v)-\frac{t_1t_2x+t_1t_3}{2t_2t+t_1}.$$

i.e.

$$\underline{v}=\frac{\partial}{\partial x}+[t_1(u+v)-\frac{t_1t_2x+t_1t_3-t_2}{2t_2t+t_1}]\frac{\partial}{\partial u}$$
$$+[-t_1(u+v)-\frac{t_1t_2x+t_1t_3}{2t_2t+t_1}]\frac{\partial}{\partial v}.$$

The solution of the coupled system (1.1) takes the form:

$$u=xU(t)+V(t),v=-\frac{t_1x-1}{t_1}U(t)-V(t)+\frac{t_1(t_2x+t_3-t_2)}{2t_1(t_2t+t_1)}.$$

The symmetry reduction equations are

$$U'=-\frac{2t_2}{2t_2t+t_1}U,V'=-\frac{1}{t_1}U^2-\frac{t_1t_3-t_2}{t_1(2t_2t+t_1)}U-\frac{t_2}{2t_2t+t_1}V.$$

The exact solutions of the coupled KdV equation system (1.1) are obtained:

$$u=\frac{t_4x}{2t_2t+t_1}+\frac{t_4(t_4+t_1t_3-t_2)}{t_1t_2(2t_2t+t_1)}+\frac{t_5}{(2t_2t+t_1)^{1/2}},$$
$$v=\frac{(t_2-t_4)x}{2t_2t+t_1}-\frac{t_4(t_4+t_1t_3-2t_2)}{t_1t_2(2t_2t+t_1)}-\frac{t_5}{(2t_2t+t_1)^{1/2}}+\frac{t_1t_3-t_2}{t_1(2t_2t+t_1)},$$

Case 2(E):

$$\phi=\frac{(1+t_1)(u+v)}{t_1x+t_2},\varphi=-\frac{u+v}{t_1x+t_2}.$$

i.e.

$$\underline{v}=\frac{\partial}{\partial x}+\frac{(1+t_1)(u+v)}{t_1x+t_2}\frac{\partial}{\partial u}-\frac{u+v}{t_1x+t_2}\frac{\partial}{\partial v}.$$

The solution of the coupled system (1.1) takes the form:

$$u=xU(t)+V(t),v=-\frac{x-t_2}{1+t_1}U(t)-V(t).$$

The symmetry reduction equations are

$$U' = -\frac{2t_1U^2}{1+t_1}, V' = -\frac{(t_1V+t_2U)U}{1+t_1}.$$

The exact solutions of the coupled KdV equation system (1.1) are obtained:

$$u = \frac{(1+t_1)x}{2t_1t+(1+t_1)t_3} + \frac{t_4}{[2t_1t+(1+t_1)t_3]^{1/2}} + \frac{(1+t_1)t_2}{t_1[2t_1t+(1+t_1)t_3]},$$
$$v = -\frac{x}{2t_1t+(1+t_1)t_3} - \frac{t_4}{[2t_1t+(1+t_1)t_3]^{1/2}} - \frac{t_2}{t_1[2t_1t+(1+t_1)t_3]}.$$

Case 2(F):

$$\phi = \frac{u-v}{x-t_0}, \varphi = \frac{2v}{x-t_0},$$

i.e.

$$\underline{v} = \frac{\partial}{\partial x} + \left(\frac{u-v}{x-t_0}\right)\frac{\partial}{\partial u} + \left(\frac{2v}{x-t_0}\right)\frac{\partial}{\partial v}.$$

The solution of the coupled system (1.1) takes the form:

$$u = (x-t_0)(U(t) - xV(t)), v = (x-t_0)^2V(t).$$

The symmetry reduction equations are

$$U' = -2U^2 + t_1UV + t_1^2V^2, V' = 3t_1V^2 - 3UV.$$

The exact solutions of the coupled KdV equation system (1.1) are obtained:

$$u = \frac{3\sqrt{6}(x-t_0)^2}{4(t_1t+t_2)^{3/2}} + \frac{t_1(x-t_0)}{2(t_1t+t_2)},$$
$$v = \frac{3\sqrt{6}(x-t_0)^2}{4(t_1t+t_2)^{3/2}}.$$

Case 2(G):

$$\phi = \frac{u}{x-t_1}, \varphi = \frac{v}{x-t_1}.$$

i.e.

$$\underline{v} = \frac{\partial}{\partial x} + \frac{u}{x-t_1}\frac{\partial}{\partial u} + \frac{v}{x-t_1}\frac{\partial}{\partial v}.$$

the solution of the coupled system (1.1) takes the form:

$$u = (x-t_1)U(t), v = (x-t_1)V(t).$$

The symmetry reduction equations are

$$U' = -2U^2 - 2UV, V' = -2V^2 - 2UV.$$

The exact solution of the system (1.1):

$$u = -\frac{x-t_1}{t_2t+t_3}, v = \frac{(2+t_2)(x-t_1)}{2(t_2t+t_3)}.$$

Case 2(H):

$$\phi = -\frac{1}{t_1t+t_2}, \varphi = \frac{t_1+2}{2(t_1t+t_2)},$$

i.e.

$$\underline{v} = \frac{\partial}{\partial x} - \frac{1}{t_1t+t_2}\frac{\partial}{\partial u} + \frac{t_1+2}{2(t_1t+t_2)}\frac{\partial}{\partial v}.$$

The solution of the coupled system (1.1) takes the form:

$$u = U(t) - \frac{x}{t_1t+t_2}, v = V(t) + \frac{(t_1+2)x}{2(t_1t+t_2)}.$$

The symmetry reduction equations are

$$U' = -\frac{(t_1-2)U}{2(t_1t+t_2)} + \frac{V}{t_1t+t_2}, V' = -\frac{(t_1+2)U}{2(t_1t+t_2)} - \frac{(t_1+1)V}{t_1t+t_2}.$$

The exact solution of the system (1.1):

$$u=-\frac{x}{t_1t+t_2}+\frac{t_1t_3}{t_1t+t_2}+\frac{\sqrt{t_1}t_4}{(t_1t+t_2)^{1/2}},$$
$$v=\frac{(t_1+2)x}{2(t_1t+t_2)}-\frac{t_1t_3(t_1+2)}{2(t_1t+t_2)}-\frac{\sqrt{t_1}t_4}{(t_1t+t_2)^{1/2}}.$$

Case 2(I):

$$\phi=\frac{u}{x+t_1t+t_2}+\frac{v}{x+t_1t+t_2}+\frac{t_1}{2(x+t_1t+t_2)},\varphi=0,$$

i.e.

$$\underline{v}=\frac{\partial}{\partial x}+\frac{u}{x+t_1t+t_2}+\frac{v}{x+t_1t+t_2}+\frac{t_1}{2(x+t_1t+t_2)}\frac{\partial}{\partial u}.$$

The solution of the coupled system (1.1) takes the form:

$$u=(x+t_1t+t_2)U(t)-V(t)-t_1/2, v=V(t).$$

The symmetry reduction equations are

$$U'=-2U^2, V'=-UV.$$

The exact solution of the system (1.1):

$$u=\frac{x+t_1t+t_2}{2t+t_3}-\frac{t_4}{(2t+t_3)^{1/2}}-\frac{t_1}{2}, v=\frac{t_4}{(2t+t_3)^{1/2}}$$

Case 2(J):

$$\phi=0,\varphi=\frac{1}{2(t-t_0)}.$$

i.e.

$$\underline{v}=\frac{\partial}{\partial x}+\frac{1}{2(t-t_0)}\frac{\partial}{\partial v}.$$

The solution of the coupled system (1.1) takes the form:

$$u=U(t), v=V(t)+\frac{x}{2(t-t_0)}.$$

The symmetry reduction equations are

$$U'=-\frac{U}{2(t-t_0)}, V'=-\frac{2V+U}{2(t-t_0)}.$$

The exact solution of the system (1.1):

$$u=\frac{x}{2(t-t_0)}-\frac{t_1}{2(t-t_0)}-\frac{t_2}{\sqrt{2(t-t_0)}}, v=\frac{t_2}{\sqrt{2(t-t_0)}}.$$

In summary, we have proved that for the coupled KdV equation system (1.1) the nonclassical method yields to symmetry reductions which are unobtainable by using the Lie classical method and the exact solutions obtained are not group invariant solutions. Consequently, we have proved that the nonclassical method is effective for PDEs.

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