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N-multiple Nonwandering Unilateral Weighted Backward Shift Operators and the Property of Direct Sum Operators in Banach Space

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Abstract: In this paper,we take the methods of differential dynamics and basic theories of the operator as tools on the basis of hypercyclicity chaos of operator,and do further research on the nonwandering property of the operator.In infinite dimensional separable Banach space,we have known the nonwandering operators,which are linear ones of chaotic character.We introduce n-multiple non- wandering operators.Firstly,we give the definition of n-multiple nonwandering operator.Then,by the method of function analysis,we prove that in separable Banach space unilateral weighted backward shifts operators are n-multiple nonwandering operators.Also,we obtain that the direct sum operator of the finite nonwandering operators are n-multiple nonwandering operators.

Key words: nonwandering operator; Banach space; direct sum operators; n-multiple nonwandering operator

1 Introduction

In the research field of operator,hypercyclic operators and linear chaotic operators have been intensively studied recently.The first observation of hypercyclic operators was by Birkho([1]).Since then,many researchers discussed this kind of operators ([2]-[8]).In 1991,for the first time, Godefroy and Shapiro ([7]) connected the research of hypercyclic operators and linear chaotic operators and pointed out that some hypercyclic operators are chaotic under the definition of Devaney .From then on,most hypercyclic operators in the literature have been proved to be chaotic.This implies that almost all hypercyclic operators are linear chaotic.It is well known that linear operators in finite-dimensional linear spaces can not be chaotic but the nonlinear operators may be.Only in infinite dimensional linear spaces, linear operators have chaotic properties([9, 10]). This has attracted wide attention.In ([11, 12]),Lixin Tian and other researchers introduce a new type of linear chaotic operator in infinite dimensional space, which is called nonwandering operator.This new operator is relative to the linear chaotic operator,hypercyclic operator in infinite dimensional space and Axiom A dynamic system because it has hyperbolic structure and dense period point.While in the research field of differential dynamical system, Axiom A system is an important subject. It requires that the nonwandering set possess hyperbolic structure and density of periodic points,where hyperbolic structure is based on Whitney's decomposition and the hyperbolic property of the tangent cluster at each point. However,Axiom A system is restricted in finite-dimensional compact Riemann space.Due to the linear property of operators,its tangent bundle at each point is linear operator itself.On the basis of the above work,Lixin

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Tian and other researchers introduce nonwandering operators in infinite dimensional Banach space,which are the generalization of Axiom A dynamic system but different from it.They are new linear chaotic operators and are relative to hypercylic operators,but different from them. Jiangbo Zhou discussed the hereditayily hypercyclic decomposition of nonwandering operators in infinite dimensional in Frechet space [13].Xun Liu discussed nonwandering semigroup in [14]. Shaoguang Shi obtained nonwandering operator sequences on Banach space in [15, 16]. In this paper,we go on studying the nonwandering operator,i.e.n-multiple nonwandering operator.We discuss n-multiple nonwandering unilateral weighted Backward shifts operators and direct sum operators in Banach space.

2 Basic notations and definitions

Let $(X, \|\cdot\|)$ be an infinite-dimensional separable Banach space in real number field or complex number field K. Let $L(X)$ be the set of all bounded linear operators over X. By N, Z, Q, R and C. We will refer to the sets of positive integers, integers, rational numbers, and to the real and complex scalar fields.

Definition 2.1 ([2]-[6],[9]-[15]) Suppose $T \in L(X)$. If there is a vector $x \in X$ such that Orb $(T, x) = \left\{ x, Tx, T^2x, \cdots \right\}$ is dense in X, then we call x a hypercyclic vector and T a hypercyclic operator.

Definition 2.2 ([1, 4]) Suppose $E \subset X$ is a closed linear subspace of $X$and $E_1 \subset E$ and $E_2 \subset E$ are also closed linear subspaces in X. For arbitrary $x \in E$,if there is a unique decomposition such that $x = x_1 + x_2$,$x_1 \in E_1$, $x_2 \in E_2$, $E_1 \cap E_2 = \{0\}$, then E is called direct sum of E_1 and E_2 , and written as $E = E_1 \oplus E_2$, where $\oplus$ represents direct sum.

Definition 2.3 ([11]-[17]) Suppose $T \in L(X)$,

(1)Assume that there exists a closed subspace $E \subset X$, which has hyperbolic structure: $E = E^s \oplus E^u$, $TE^u = E^u, TE^s = E^s$, where E^u, E^s are closed subspaces. In addition, there exist constants $c > 0$ and $\tau(0 < \tau < 1)$ such that

$$\left\|T^k\xi\right\| \geq c\tau^{-k}\left\|\xi\right\|, \forall\xi \in E^u, \forall k \in N,$$

$$\left\|T^k\eta\right\| \leq c\tau^{k}\left\|\eta\right\|, \forall\eta \in E^s, \forall k \in N;$$

(2)Assume also that $Per(T)$ is dense in E,i.e.$\overline{Per(T)} = E$,

Then T is said to be a nonwandering operator relative to E.

Definition 2.4 ([17]) Suppose $T \in B(X)$,

(1) Assume that there exists finite closed subspaces $E_i \subset X$, each one having hyperbolic structure:

$$E_i = E_i^u \oplus E_i^s, TE_i^u = E_i^u, TE_i^s = E_i^s,$$

where E_i^u, E_i^s are closed subspaces.In addition,there exist constants $c_i > 0$ and $\tau_i(0 < \tau_i < 1)$ such that

$$\left\|T^k\xi_i\right\| \geq c_i\tau_i^{-k}\left\|\xi_i\right\|, \forall\xi_i \in E_i^u, \forall k \in N,$$

$$\left\|T^k\eta_i\right\| \leq c_i\tau_i^{k}\left\|\eta_i\right\|, \forall\eta_i \in E_i^s, \forall k \in N;$$

(2)Assume also that $Per(T)$ is dense in $\bigcup\limits_{i=1}^{n} E_i$,i.e. $Per(T) = \bigcup\limits_{i=1}^{n} E_i$.

Then T is said to be a n-multiple nonwandering operator relative to E_i.

3 N-multiple nonwandering unilateral weighted backward shift operators

Shapiro thought,when a bounded linear operator on Banach space X has enough eigenvalues, the research of it's hypercyclic propertywill become direct in [7]; so the same is true with the nonwandering operators and n-multiple nonwandering operators.

Lemma 3.1 *Let $T \in B(X)$, the spectral of T : $\sigma_p(T) = \{\lambda : Tx = \lambda x, 0 \neq x \in X\}, V_+ = \{\lambda : \lambda \in \sigma_p(T), |\lambda| > 1\}$ and $V_- = \{\lambda : \lambda \in \sigma_p(T), 0 < |\lambda| < 1\}$, $E_0 = span\{x|x \in X : Tx = \lambda x, \lambda^n = 1, \lambda \neq 1\}$, if V_+, V_- are finite and nonempty, E_0 is dense in X, then T is a n-multiple nonwandering operator on X.*

Proof: V_+,V_- are finite and nonempty, so we can divide V_+ and V_- into finite zones as follows:

$$\begin{array}{l} V_{1+} = \{\lambda : 1 < |\lambda| < 2, \lambda \in \sigma_p(T)\}, V_{2+} = \{\lambda : 2 \leq |\lambda| < 3, \lambda \in \sigma_p(T)\}, \\ V_{3+} = \{\lambda : 3 \leq |\lambda| < 4, \lambda \in \sigma_p(T)\}, \cdots, V_{n+} = \{\lambda : n \leq |\lambda| < n+1, \lambda \in \sigma_p(T)\}, \\ V_{1-} = \left\{\lambda : \frac{1}{2} < |\lambda| < 1, \lambda \in \sigma_p(T)\right\}, V_{2-} = \left\{\lambda : \frac{1}{3} < |\lambda| \leq \frac{1}{2}, \lambda \in \sigma_p(T)\right\}, \\ V_{3-} = \left\{\lambda : \frac{1}{4} < |\lambda| \leq \frac{1}{3}, \lambda \in \sigma_p(T)\right\}, \cdots, V_{n-} = \left\{\lambda : \frac{1}{n+1} < |\lambda| \leq \frac{1}{n}, \lambda \in \sigma_p(T)\right\} \end{array}$$

Let $E_{1+} = span\{x_{\lambda_{1+}}|\lambda_{1+} \in V_{1+}\}$, then $TE_{1+} = E_{1+}$. For any $\xi \in E_{1+}$, let $\xi = \sum_{i=1}^{n} \alpha_i x_{\lambda_{1+i}}$, then for any $K \in N$,

$$\left\|T^k \xi\right\| = \left\|\sum_{i=1}^{n} \alpha_i (\lambda_{1+i})^k x_{\lambda_{1+i}}\right\| \geq \tau_1^{-k} \|\xi\|.$$

Let $E_{1-} = span\{x_{\lambda_{1-}}|\lambda_{1-} \in V_{1-}\}$, then $TE_{1-} = E_{1-}$. For any $\eta \in E_{1-}$, let $\eta = \sum_{i=1}^{n} \alpha_i x_{\lambda_{1-i}}$, then for any $K \in N$,

$$\left\|T^k \eta\right\| = \left\|\sum_{i=1}^{n} b_i (\lambda_{1-i})^k x_{\lambda_{1-i}}\right\| \leq \tau_1^{k} \|\eta\|.$$

Above τ_1 is : $\tau_1 = \max_{1 \leq i \leq n} \left\{|\lambda_{1+i}|^{-1}, |\lambda_{1-i}|\right\} < 1$.

Let $E_1^u = E_{1+}, E_1^s = E_{1-}$, then $E_1 = E_1^u \oplus E_1^s$ is a hyperbolic closed invariant subspace.

Similarly, let $E_{2+} = span\{x_{\lambda_{2+}}|\lambda_{2+} \in V_{2+}\}, TE_{2+} = E_{2+}$, for any $\xi \in E_{2+}$, let $\xi = \sum_{i=1}^{n} \alpha_i x_{\lambda_{2+i}}$, then for any $K \in N$,

$$\left\|T^k \xi\right\| = \left\|\sum_{i=1}^{n} \alpha_i (\lambda_{2+i})^k x_{\lambda_{2+i}}\right\| \geq \tau_2^{-k} \|\xi\|,$$

Let $E_{2-} = span\{x_{\lambda_{2-}}|\lambda_{2-} \in V_{2-}\}, TE_{2-} = E_{2-}$, for any $\eta \in E_{2-}$, let $\eta = \sum_{i=1}^{n} b_i x_{\lambda_{2-i}}$, then for any $k \in N$,

$$\left\|T^k \eta\right\| = \left\|\sum_{i=1}^{n} b_i (\lambda_{2-i})^k x_{\lambda_{2-i}}\right\| \leq \tau_2^{k} \|\eta\|.$$

τ_2 is : $\tau_2 = \max_{1 \leq i \leq n} \left\{|\lambda_{2+i}|^{-1}, |\lambda_{2-i}|\right\} < 1$.

Let $E_2^u = E_{2+}, E_2^s = E_{2-}$, then $E_2 = E_2^u \oplus E_2^s$ is a hyperbolic closed invariant subspace.

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Similarly, let $E_{n+} = span\{x_{\lambda_{n+}} | \lambda_{n+} \in V_{n+}\}, TE_{n+} = E_{n+}$, for any $\xi \in E_{n+}$, let $\xi = \sum_{i=1}^{n} \alpha_i x_{\lambda_{n+i}}$, then for any $K \in N$,

$$\left\| T^k \xi \right\| = \left\| \sum_{i=1}^{n} \alpha_i (\lambda_{n+i})^k x_{\lambda_{n+i}} \right\| \geq \tau_n^{-k} \left\| \xi \right\|,$$

Let $E_{n-} = span\{x_{\lambda_{n-}} | \lambda_{n-} \in V_{n-}\}$, then $TE_{n-} = E_{n-}$, for any $\eta \in E_{n-}$, let $\eta = \sum_{i=1}^{n} b_i x_{\lambda_{n-i}}$, then for any $k \in N$,

$$\left\| T^k \eta \right\| = \left\| \sum_{i=1}^{n} b_i (\lambda_{n-i})^k x_{\lambda_{n-i}} \right\| \leq \tau_n^{k} \left\| \eta \right\|.$$

Let $\tau_n = \max_{1 \leq i \leq n} \left\{ |\lambda_{n+i}|^{-1}, |\lambda_{n-i}| \right\} < 1$.

Let $E_n^u = E_{n+}, E_n^s = E_{n-}$, then $E_n = E_n^u \oplus E_n^s$ is a hyperbolic closed invariant subspace.

Obviously, $E_0 \subset Per(T)$, E_0 is dense in X, and $Per(T)$ is dense in X, $Per(T)$ is dense in $E_1, E_2, \cdots, E_n$. According to the definition 2.4. We know T is a n-multiple nonwandering operator relative to E.

Theorem 3.2 *Suppose T is n-multiple nonwandering unilateral weighted backward shift operators on $l^p(\mathrm{N})$, i.e. $(Tx_i)_i = (w_{i+1}x_{i+1})_i$ $(i \geq 1)$, where $\{|w_i|\}_{i \geq 2}$ is nonzero bounded increasing sequence, if $\lim_{n \to \infty} \sup_{j \in \{1, \cdots n\}} \left\{ \prod_{i=1}^{n} |w_{i+j}| \right\} = \infty$, T is n-multiple nonwandering operator.*

Proof: Let $y \in l^p(N)$,$y = (y_1, y_2, \cdots)$,$\|y\|_p = (\sum_{i=1}^{\infty} |y_i|^p)^{\frac{1}{p}} < \infty$. Define the vector

$$x_n = (y_1, \cdots, y_n, a_{n+1}y_1, \cdots, a_{2n}y_n, a_{2n+1}y_1, \cdots, a_{3n}y_n, a_{3n+1}y_1, \cdots),$$

where $a_{kn+j} = \prod_{i=1+j}^{kn+j} \frac{1}{w_i} (k \in \mathrm{N}, 1 \leq j \leq n)$, then we can improve $x_n \in l^p(N)$.

In fact , because $|w_i| \leq |w_i + 1| (i = N)$, $\frac{1}{|w_i|} \geq \frac{1}{|w_i + 1|}$,

$$\|x_n\|_p^p = |y_1|^p + \cdots + |y_n|^p + |a_{n+1}y_1|^p + \cdots + |a_{2n}y_n|^p + |a_{2n+1}y_1|^p + \cdots + |a_{3n}y_n|^p + \cdots$$

$$= (1 + \frac{1}{\prod_{i=2}^{n+1} |w_i|^p} + \frac{1}{\prod_{i=2}^{2n+1} |w_i|^p} + \cdots) |y_1|^p + \cdots + (1 + \frac{1}{\prod_{i=n+1}^{3n} |w_i|^p} + \frac{1}{\prod_{i=n+1}^{4n} |w_i|^p} + \cdots) |y_n|^p + \cdots$$

$$\leq \sup_{j=(0, \cdots (n-1))} \{(1 + \prod_{i=2}^{n+1} \frac{1}{|w_{j+i}|^p} + \prod_{i=2}^{2n+1} \frac{1}{|w_{j+i}|^p} + \prod_{i=2}^{3n+1} \frac{1}{|w_{j+i}|^p} + \cdots)\} \sum_{j=1}^{n} |y_j|^p$$

$$\leq \sup_{j=(0, \cdots (n-1))} \{(1 + \prod_{i=2}^{n+1} \frac{1}{|w_{j+i}|^p} + \prod_{i=2}^{2n+1} \frac{1}{|w_{j+i}|^p} + \prod_{i=2}^{3n+1} \frac{1}{|w_{j+i}|^p} + \cdots)\} \|y\|_p^p$$

$$\leq \sup_{j=(0, \cdots (n-1))} \{\frac{1}{1 - \prod_{i=2}^{n+1} \frac{1}{|w_{j+i}|^p}}\} \|y\|_p^p \to \|y\|_p^p \quad (n \to \infty)$$

Thereby , $x_n \in l^p(N)$. We have $x_n \stackrel{n \to \infty}{\longrightarrow} y$, $T^n x_n = x_n$ easily, so the periods of T is dense in $l^p(N)$.

Assume that $h_\mu = (1, \frac{\mu}{w_2}, \frac{\mu^2}{w_2 w_3}, \cdots, \frac{\mu^n}{\prod_{i=2}^{n+1} w_i}, \frac{\mu^{n+1}}{\prod_{i=2}^{n+2} w_i}, \cdots)$ satisfies the equation $Bh_\mu = \mu h_\mu$, and let $a_n = \prod_{i=2}^{n+1} \frac{1}{w_i}(n \geq 1), a_0 = 1$. When $R = \varliminf_{n\to\infty} |w_{n+2}|$, the power series $\sum_{n=0}^{\infty} |a_n|^p |\mu|^{np}$ is convergence.

Even from the supposition, We can prove: $\varliminf_{n\to\infty} |w_{n+2}| > 1$ from the suppose. When $|\mu| \in (0, R)$, $h_\mu \in l^p(\mathrm{N})$, the spectra of T-$\sigma_p(T)$ contains opening sets $v_1 = \{\mu : 0 < |\mu| < 1\}, v_2 = \{\mu : R > |\mu| > 1\}$. According to Lemma 2.1, we can construct a hyperbolic closed invariant subspace on $l^p(\mathrm{N})$,in advance. The operator T is n-multiple nonwandering operator.

4 The n-multiple nonwandering property of direct sum operators

If the condition is suitable, H Salas accepted that the finite direct sum of hypercyclic weighted operators is also hypercyclic . We'll show that the finite direct sum of n-multiple nonwandering operators is also n-multiple nonwandering.

Theorem 4.1 *Suppose* $(X, \|\cdot\|)$*is a infinite dimensional separable Banach space.* T_1, T_2 *are n-multiple nonwandering operators relative to closed invariant subspaces* $E_i, F_i \subset X, (i = 1, 2, \cdots, n)$. $E_i \cap F_i = \phi$. *then* $T_1 \oplus T_2$ *is a n-multiple nonwandering operator.*

Proof : Since T_1 is n-multiple nonwandering relative to E_i $(i = 1, 2, \cdots, n)$,

(1) there exist finite closed subspaces $E_i \subset X$ which has hyperbolic structure:

$$E_i = E_i^u \oplus E_i^s T E_i^u = E_i^u, T E_i^s = E_i^s,$$

where E_i^u, E_i^s are closed subspaces. In addition, there exist constants $a_i > 0$ and $\lambda_i(0 < \lambda_i < 1)$ such that

$$\left\|T^k \xi_i\right\| \geq a_i \lambda_i^{-k} \|\xi_i\|, \ \forall \xi_i \in E_i^u, \ \forall K \in N,$$

$$\left\|T^k \eta_i\right\| \leq a_i \lambda_i^k \|\eta_i\|, \ \forall \eta_i \in E_i^s, \ \forall k \in N;$$

(2) $Per(T)$ is dense in $\bigcup_{i=1}^{n} E_i$;

Since T_2 is n-multiple nonwandering relative to F_i $(i = 1, 2, \cdots, n)$,

(3) there exist finite closed subspaces $F_i \subset X$ which has hyperbolic structure:

$$F_i = F_i^u \oplus F_i^s, T F_i^u = F_i^u, T F_i^s = F_i^s,$$

where F_i^u, F_i^s are closed subspaces. In addition, there exist constants $b_i > 0$ and $\tau_i(0 < \tau_i < 1)$ such that for any

$$\left\|T^k \xi_i\right\| \geq b_i \tau_i^{-k} \|\xi_i\|, \ \forall \xi_i \in E_i^u, \ \forall k \in N,$$

$$\left\|T^k \eta_i\right\| \leq b_i \tau_i^k \|\eta_i\|, \ \forall \eta_i \in E_i^s, \ \forall k \in N;$$

(4) $Per(T)$ is dense in $\bigcup_{i=1}^{n} F_i$,

Let $X_i^u = E_i^u \oplus F_i^u$, we define a new norm $\|\cdot\|_0$ in X^u.

For each $x \in X^u$, $x = x_1 + x_2$, $x_1 \in E_i^u$, $x_2 \in F_i^u$,$\| x \|_0 = \max\{ \|x_1\| , \|x_2\| \}$, we can prove $\|\cdot\|_0$ is equivalent to $\|\cdot\|$ easily, namely , there exist $p_1, p_2 > 0$, such that $p_2 \| x \| \leq \| x \|_0 \leq p_1 \| x \|, \forall x \in X_i^u$. For each $x \in X_1^u$, $x = x_1 + x_2$, $x_1 \in E_1^u$, $x_2 \in F_1^u$,$k \in N$, we have

$$\begin{aligned}\left\| (T_1 \oplus T_2)^k x \right\| &= \left\| (T_1 \oplus T_2)^k (x_1 + x_2) \right\| \\ &\geq p_1^{-1} \left\| (T_1 \oplus T_2)^k (x_1 + x_2) \right\|_0 \geq \frac{p_1^{-1}}{2} (\left\| T_1^k x_1 \right\| + \left\| T_2^k x_2 \right\|) \\ &\geq \frac{p_1^{-1}}{2} (a_1 \lambda_1^{-k} \| x_1 \| + b_1 \tau_1^{-k} \| x_2 \|) \\ &\geq \frac{p_1^{-1}}{2} c_1 y^{-k} \|x\|_0 \geq \frac{p_1^{-1}}{2} p_2 c_1 y^{-k} \|x\|, \quad c_1 = \min\{a_1, b_1\},\ z_1 = \max\{\lambda_1, \tau_1\}.\end{aligned}$$

Similarly, let $X_1^s = E_1^s \oplus F_1^s$, then for each$y \in E^s$, $y = y_1 + y_2$, $y_1 \in E_1^s$, $y_2 \in F_1^s$, $k \in N$, we have

$$\begin{aligned}\left\| (T_1 \oplus T_2)^k y \right\| &\leq \left\| T_1^k y_1 \right\| + \left\| T_2^k y_2 \right\| \\ &\leq a_1 \lambda_1^k \| y_1 \| + b_1 \tau_1^k \| y_2 \| \leq 2 p_1 c_1' z^k \|y\|,\end{aligned}$$

where $c_1' = \max\{p_1, p_2\},\ z_1 = \max\{\tau_1, \tau_2\}$.
Let $X_1 = X_1^u \oplus X_1^s = E_1 \oplus E_2 = (E_1^u \oplus E_1^s) \oplus (F_1^u \oplus F_1^s)$,then

$$\begin{aligned}(T_1 \oplus T_2) X_1^u &= T_1 E_1^u \oplus T_2 F_1^u = E_1^u \oplus F_1^u \\ &= X_1^u, (T_1 \oplus T_2) X_1^s = T_1 E_1^s \oplus T_2 F_1^s = E_1^s \oplus F_1^s = X_1^s\end{aligned}$$

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Similarly ,let $X_n^u = E_n^u \oplus F_n^u$, for each $x \in X_n^u$, $x = x_1 + x_2$,$x_1 \in E_n^u$,$x_2 \in F_n^u$,$k \in N$, we have

$$\begin{aligned}\left\| (T_1 \oplus T_2)^k x \right\| &= \left\| (T_1 \oplus T_2)^k (x_1 + x_2) \right\| \\ &\geq p_1^{-1} \left\| (T_1 \oplus T_2)^k (x_1 + x_2) \right\|_0 \geq \frac{p_1^{-1}}{2} (\left\| T_1^k x_1 \right\| + \left\| T_2^k x_2 \right\|) \\ &\geq \frac{p_1^{-1}}{2} (a_n \lambda_n^{-k} \| x_1 \| + b_n \tau_n^{-k} \| x_2 \|) \geq \frac{p_1^{-1}}{2} c_n y^{-k} \|x\|_0 \\ &\geq \frac{p_1^{-1}}{2} p_2 c_n y^{-k} \|x\|\end{aligned}$$

where $c_n = \min\{a_n, b_n\}$, $z_n = \max\{\lambda_n, \tau_n\}$.
Similarly, let $X_n^s = E_n^s \oplus F_n^s$, for each $y \in X_n^s$, $y = y_1 + y_2$, $y_1 \in E_n^s$, $y_2 \in F_n^s$, $k \in N$, we have

$$\begin{aligned}\left\| (T_1 \oplus T_2)^k y \right\| &\leq \left\| T_1^k y_1 \right\| + \left\| T_2^k y_2 \right\| \\ &\leq a_n \lambda_n^k \| y_1 \| + b_n \tau_n^k \| y_2 \| \leq 2 c_n c_n' z_n^k \|y\|,\end{aligned}$$

where $c_n' = \max\{a_n, b_n\}$,$z_n = \max\{\lambda_n, \tau_n\}$.
Let $X_n = X_n^u \oplus X_n^s = E_n \oplus F_n = (E_n^u \oplus E_n^s) \oplus (F_n^u \oplus F_n^s)$,

$$(T_1 \oplus T_2) X_n^u = T_1 E_n^u \oplus T_2 F_n^u = E_n^u \oplus F_n^u = X_n^u,$$

$$(T_1 \oplus T_2) X_n^s = T_1 E_n^s \oplus T_2 F_n^s = E_n^s \oplus F_n^s = X_n^s.$$

Because $PerT_1$ is dense in E_i,$PerT_2$ is dense in F_i. $Per(T_1 \oplus T_2)$ is dense in $\bigcup_{i=1}^{n} X_i$, so $Per(T_1 \oplus T_2)$ is dense in $\bigcup_{i=1}^{n} E_i \oplus F_i$.
Consequently, $T_1 \oplus T_2$ is n-multiple nonwandering operator relative to $E_i \oplus F_i$ $(i = 1, 2, \cdots, n)$.
This result can be extended to finite n-multiple nonwandering operators .

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