ISSN 1749-3889(print),1749-3897(online)
International Journal of Nonlinear Science
Vol.2(2006) No.2,pp.117-121

Numerical Simulation of Nonlinear Dynamical Behaviors of Active Magnetic Bearing-rotor System

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(Received 1 January 2006, accepted 15 June 2006)

Abstract: In this paper, the nonlinear dynamical behaviors of an active magnetic bearing-rotor system are investigated using numerical simulation. Firstly, the nonlinear magnetic force is obtained by Taylor series. Secondly, the primary-internal resonance of an AMBRS(active magnetic bearing) -rotor system is studied with the help of Lung-Kutta method. To analyze the motion characters of system, some Poincare maps are drawn and several Lyapunov exponents are calculated. The conclusion:the chaotical phenomenon by period-doubling bifurcation and ring surface break period-doubling bifure are respectively discovered .Especially, a chaotic attractor on Poincare map that is extremely sensitive to initial value is discovered.

Keywords: active magnetic bearing-rotor system; nonlinear dynamics; bifurcation; chaos

1 Preface

Active magnetic bearing-rotor system(AMBRS) is a result that has been synthesized by mechanics,machinary,electronics and control science.The superiorities of AMBRS are having no power consumption caused by friction and material fatigue by wearing([1]).It has been used in many rotational machines([1, 2]).

A number of research results of the dynamic behavior and its control of AMBRS have been already obtained,but the results were mainly based on linear theory([3]).If the rotator inclines or control quantum changes heavily,behavor of nonlinearity will be very important([3, 4, 5, 9]).In this paper,complicated phenomenon of nonlinear dynamic behavor in an eight-pole AMBRS is introduced.Our investigations consist in choosing electric current as control quantum;obtaining the third power of nonlinear magnetic force by Taylor series; considering the effect of the rotator; the primary-internal resonance of an AMBRS are studied with the help of Lung-Kutta method.

2 Magnetic force

The Fig1 is a schematic drawing of AMBRS.The stator has eight poles.They have the same structure and full circle numbers of coil.According to electromagnetism theory([2]), the force of each pole is given as (2.1).

$$F_i = \frac{1}{4}\mu_0 N^2 A_\alpha \left(\frac{I_i}{C_i}\right)^2 \cos\beta \qquad i = 1, 2, ..., 8 \tag{2.1}$$

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IJNS.2006.10.15/041

Here μ_0 is the magnetic inductivity in the vacuum ,N the full circle number of coil's, A_α the cross setioc area of magnetic pole, C_i the gap between pole and rotator; I_i current throw coil; (α,β seen in fig.1.)

$$\begin{array}{ll} C_{1,5} = C_0 \pm x\sin\alpha \mp y\cos\alpha & C_{4,8} = C_0 \pm x\sin\alpha \pm y\cos\alpha \\ C_{2,6} = C_0 \pm x\cos\alpha \mp y\sin\beta & C_{3,7} == C_0 \pm x\cos\alpha \pm y\sin\beta \end{array}$$

C_0 the mean gap between pole and rotator.

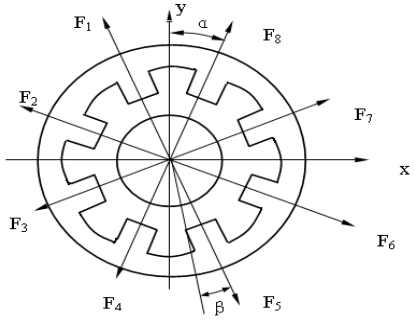

Figure 1: Simple map of active magnetic bearing structure

Current throw coil I_i equal to the sum of inclined current I_0 and the controlled one i. For the AMBRS arranged in fig1.the current passing throw each coil can be given as

$$\begin{array}{ll} I_1 = I_8 = I_0 - i_y, & I_4 = I_5 = I_0 + i_y \\ I_6 = I_7 = I_0 - i_x, & I_3 = I_2 = I_0 + i_x \end{array} \tag{2.2}$$

Choose the controlled current PD as Varity thus

$$i_x = k_p x + k_d \dot{x} \qquad i_y = k_p y + k_d \dot{y} \tag{2.3}$$

k_p is the propotional coefficiene and the k_d the differential one.

Substituting (2.2) an (2.3) into (2.1), the force of each pole is obtained, and the resultant force acting on the rotator is gotten by vector-sum.

$$\begin{array}{l} F_x = (F_6 + F_7 - F_2 - F_3)\cos\alpha + (F_5 + F_8 - F_1 - F_4)\sin\alpha \\ F_y = (F_1 + F_8 - F_4 - F_5)\cos\alpha + (F_2 + F_7 - F_3 - F_6)\sin\alpha \end{array} \tag{2.4}$$

3 Mathematic model

The differential equation of the AMBRS shown as fig.1 is (To simplify our discussion put the AMBRS relected).

$$\begin{array}{l} m\,\ddot{x} + c\,\dot{x} = F_x + me\Omega^2\cos\Omega t \\ m\,\ddot{y} + c\,\dot{y} = F_y + me\Omega^2\sin\Omega t \end{array} \tag{3.1}$$

Here, m is the half of rotator mass, c the damping ,e the length of the rotator's incline, Q the angular speed of the rotator incline, F_x is portion to x direction of the force acting on rotator,and F_y to y direction, $(\cdot)$ denotes differential coefficient with respect to time t.

In order to analyze the portion effect of the behavior of AMBRS, we extend F_x and F_y as Taylor series at point (0,0), and simplify the series by remaining the low power term. Put (2.1) into non-measure form $X = x/C_0, Y = y/C_0, i_x = I_0 I_x, i_y = I_0/I_y, t = B\tau$,$\Omega = \omega/B$, and $(\prime)$ denotes the derivative with respect to τ after some illation,(3.1)becomes

$$\begin{aligned} &X" + 2\mu X' + \omega_0^2 X - (\alpha_1 X^3 + \alpha_2 XY^2 + \alpha_3 X^2 X' + \alpha_4 X'Y^2 \\ &\quad + \alpha_5 XY'^2 + \alpha_6 XX'^2 + \alpha_7 XYY') = \rho\cos\omega\tau \\ &Y" + 2\mu Y' + \omega_0^2 Y - (\alpha_1 Y^3 + \alpha_2 X^2 Y + \alpha_3 Y^2 Y' + \alpha_4 X^2 Y' \\ &\quad + \alpha_5 X'^2 Y + \alpha_6 YY'^2 + \alpha_7 XX'Y) = \rho\sin\omega\tau \end{aligned} \tag{3.2}$$

$$2\mu = 8d\cos^3\alpha + c_1 \qquad \omega_0^2 = 8(p\cos\alpha - 1)$$

$$\alpha_1 = 16(\cos^4\alpha + \sin^4\alpha) - 24p\cos^3\alpha + 8p^2\cos^2\alpha$$

$$\alpha_2 = 96\cos^2\alpha\sin^2\alpha - 72p\cos\alpha\sin^2\alpha + 8p^2\sin^2\alpha$$

$$\alpha_3 = -24d\cos^3\alpha + 16pd\cos^2\alpha \qquad \alpha_4 = -24d\cos\alpha\sin^2\alpha \quad \alpha_5 = 8d^2\sin^2\alpha$$

$$\alpha_6 = 8d^2\cos^2\alpha, \quad \alpha_7 = 16pd\sin^2\alpha - 48d\sin^2\alpha\cos\alpha$$

$$p = \frac{k_p C_0}{I_0}, \quad d = \frac{k_d C_0}{I_0 B}, B^2 = \frac{4mC_0^3}{\mu_0 N^2 A_\alpha I_0^2}, c_1 = \frac{cC_0}{B}, \quad \rho = \frac{ew^2}{C_0} \tag{3.3}$$

Obviously, (3.2) is a two-degree system containing third power of variation and is under the situation of $1:1$ inner resonance.

4 Numerical simulation

Using the method of Lung-Kutta, we simulated (3.2),and non-menagere incline ρ is chosen as bifurcation. To simplify the calculation,we take $\omega = \omega_0 = 1$ and substitute $\alpha = \pi/8$ (the eight poles arranged evenly) into(7), and get $p = 1.217$. And let $d = 0.005, c_1 = 0.001$. We can get rest coefficient by taking them into eq.7.The (3.2) was integrated by taking time-step 0.01π, and using four power Lunge-Kutta.

The result shows that, when the system being in the situation of the one-third time second resonance and three time sup- resonance, it has very simple appearance, mainly indicating synchronous period motion. When the system is being in the primary-internal resonance,the behavior is very abundant.

From bifurcation maps Fig.2(a) and its partial-detail Fig2.(b) and Fig2.(c), we can know the motion regulation of the system which varies with the increase of the non-menageries incline ρ:synchronous periodic motion $\rightarrow$ form Hopf bifurcation to quasi-period motion $\rightarrow$ ring surface period-doubling bifurcation on Poincare sections $\rightarrow$ from ring surface break to chaos $\rightarrow$ $3T$ periodic motion on Poincare sections(including KT periodic window)$\rightarrow$ $7T$ periodic motion $\rightarrow$ from incessant period-doubling bifurcation to chaos $\rightarrow$ be off chaos from return period-doubling bifurcation $\rightarrow$ from period-doubling bifurcation to chaos motion $\rightarrow$ $13T$ periodic motion $\rightarrow$ from incessant period-doubling bifurcation to chaos$\rightarrow$ $6T$ periodic motion $\rightarrow$ from period-doubling bifurcation to chaos.

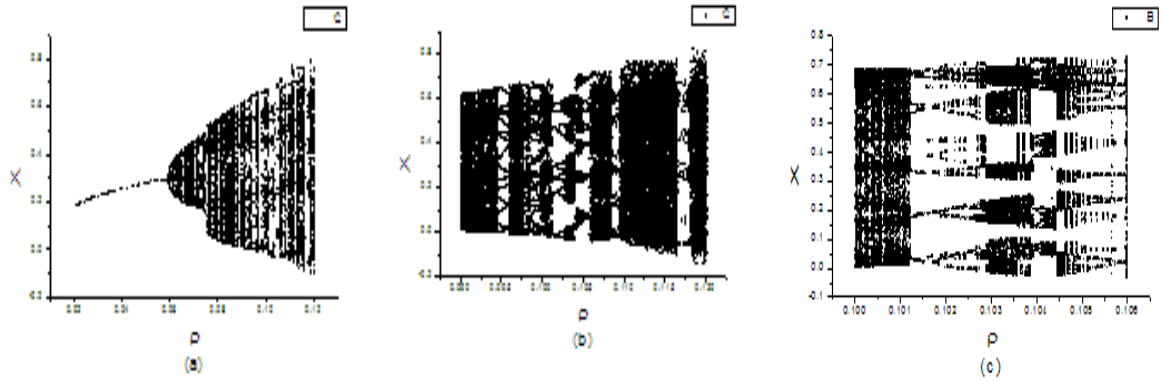

Figure 2: Bifurcation maps

To analyze the behavior of the system exactly,several different Poincare maps are made.Fig3(a)-Fig3(d) show that,when the non-menagere incline increase,the ring on Poincare sections varies from one to two,four,

even break,so,obviously it is from ring surface break(ring surface period-doubling bifurcation) to chaos. Fig3(e) indicates the system is quasi-period motion($3T$ periodic motion on Poincare sections) at this time. Fig3(d)-Fig3(f) are Poincare maps when $\rho = 0.07445$ and 0.114,which is typical chaotic attractor. The largest Lyapunov exponents are 0.0451 and 0.198 respectively, it is chaotical motion. Our investigation

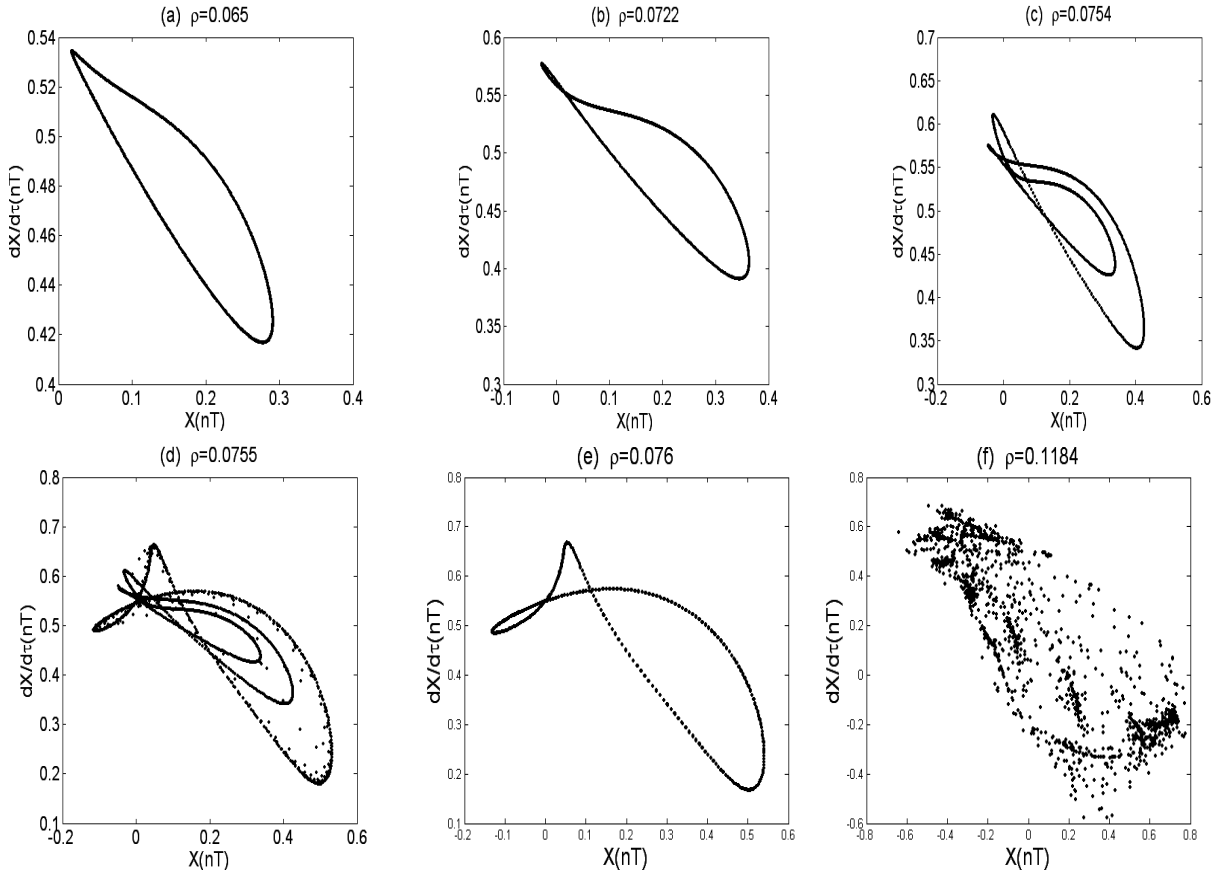

Figure 3: Poincare maps of several points

indicates that,when $\rho = 0.118$,chaotical attractor of Poincare section is very sensitive to initial value of ρ. In the region which radium is 0.8 on the (x, y) plane,we choose more than one hundred initial conditions, and obtain more than ten types of chaotic attractors (six different Poincare maps are given in fig.4).

5 Conclusions

(1)The nonlinear dynamical behaviors of an active magnetic bearing-rotor system are investigated using numerical simulation when the system being in the primary-internal resonance,and the chaotic phenomenon by ring surface break and period-doubling bifurcation is discovered with increase of the non-menageriee incline. Mostly the chaotic attractor of Poincare section is stable, but the chaotic attractor is very sensitive to initial value of ρ when it is with range of specificall parameter.

(2)The results discovered in this paper are useful to explain the nonlinear dynamical phenomenon and to diagnosis problems of an active magnetic bearing-rotor system.

(3)The bifurcation (chosen as non-menageries incline ρ)is effected by many factors. Though the nonlinear dynamical behaviors of a active magnetic bearing-rotor system is very complicated ,we can prevent its appearance by choosing suitable parameter .

Acknowledgements

This research is supported by the National Defuse Fundamental Science Foundation of China (No.A2520060236).

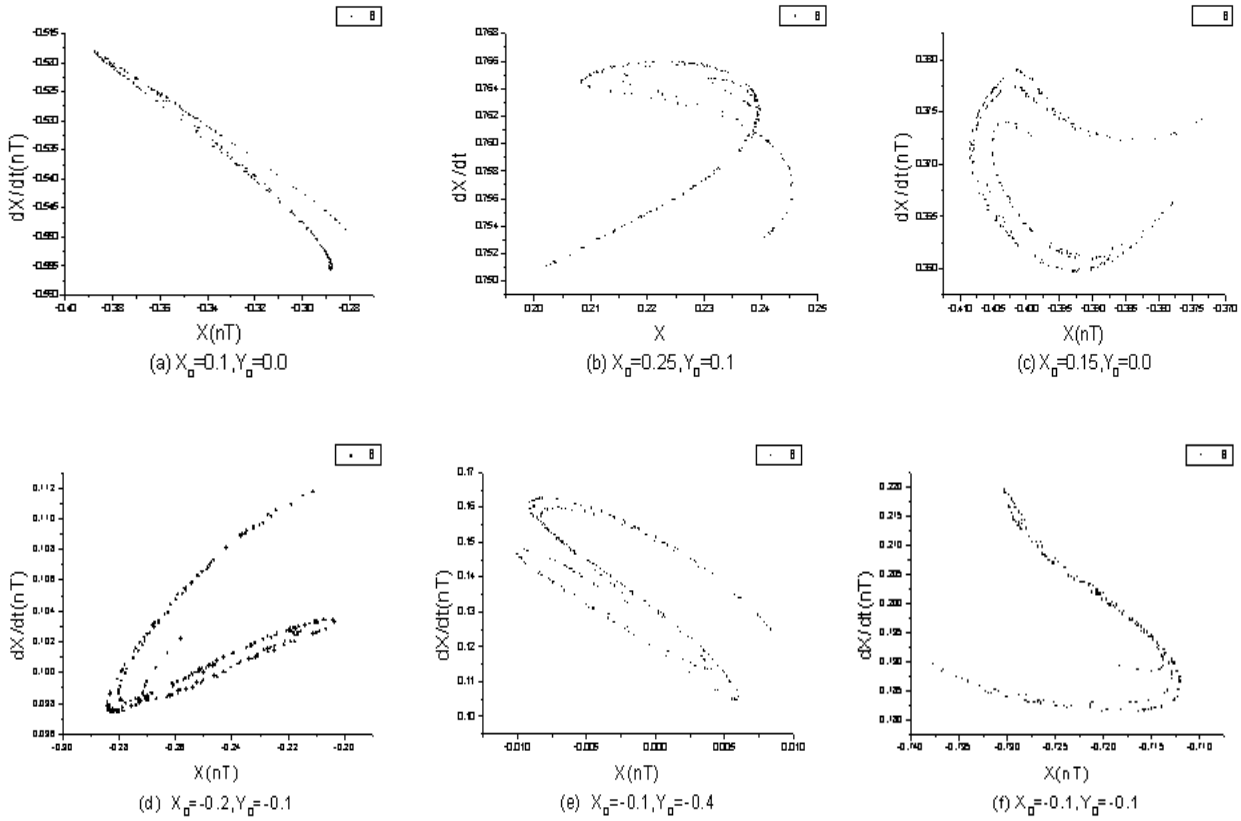

Figure 4: Several different Poincare Maps

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