ACADIMIC
World Academic Union

ISSN 1749-3889 (print), 1749-3897 (online)
International Journal of Nonlinear Science
Vol.6(2008) No.1,pp.21-28

Non-wandering Property of Differentiation Operator

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(Received 5 March 2008, accepted 16 June 2008)

Abstract:we provide a sufficient condition for an unbounded operator to be non-wandering operator, and then apply the condition to the differentiation operator on the Bargmann space F and the Hardy space H^2. Finally, we give a sufficient condition for the operator $g(D)$ defined by means of a functional calculus to be non-wandering operator.

Keywords:non-wandering operator, unbounded operator, Bargmann space, Hardy space

1 Introduction

It is well known that linear operators in finite-dimensional linear spaces can't be chaotic but the nonlinear operator may be. Only in infinite-dimensional linear spaces can linear operators have chaotic properties. This has attracted widely attention (see [1-5]). Lixin Tian and other researchers introduced non-wandering operators in infinite-dimensional Banach space, which are the generalization of Axiom A dynamic system but different from it. They are new linear chaotic operators and relative to hypercyclic operators, but different from them (see [2]). In recent years, Jiangbo Zhou discussed the hereditayily hypercyclic decomposition of non-wandering operators in infinite dimensional Frechet space (see [3]); Shaoguang Shi obtained non-wandering operator sequences on Banach space (see[4]) ; Lihong Ren studied n-multiple non-wandering operator (see [5])and Minggang Wang studied the pseudo orbit tracing property of invertible non-wandering operator (see[6]).

In this paper the Bargmann space is denoted by F and the Hardy space is denoted by H^2. These spaces have been studied by many authors(see[9-14]). Bargmann space's roots can be found in mathematical problem of relativistic physics (see[9]) or in quantum optics (see[10]). In physics the Bargmann space contains the canonical coherent states, so it is the main tool for studying the bosonic coherent state theory of radiation field (see[11]) and for other application (see[12]). In the same, the Hardy space is also the important space (see[13][14]).

In finite-dimensional separable Banach space, for the bounded linear operators, Lixin Tian and other researchers have given the definition of non-wandering operator (see[2]). However, this definition is restricted for the bounded linear operators. In this paper, we consider the non-wandering property of the unbounded operators. Let T be an unbounded operator on a separable infinite dimensional Banach space X. It may happen that a vector x is in the domain of T, but Tx fails to be in the domain of T. For this reason, in order to consider the non-wandering property of the unbounded operator, we should firstly suppose that if x in the domain of T then for every integer $n \geq 1$ the vector $T^n x$ is in the domain of T.

On the basis of the above research, in this paper, we first provide a sufficient condition for an unbounded operator to be non-wandering operator(see Theorem 1), and then apply the condition to the differentiation operator on the Bargmann space F (see Theorem 3) and the Hardy space H^2 (see Theorem 5). Finally, we give a sufficient condition for the operator $g(D)$ defined by means of a functional calculus to be non-wandering operator (see Theorem 6).

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IJNS.2008.08.15/158

2 Basic notation and definitions

Definition 1 *([2]) Let $(X, \|\cdot\|)$ be an infinite dimensional separable $Banach$ space. Suppose $T \in L(X)$*
(1) Assume that there exists a closed subspace $E \subset X$, which has hyperbolic structure:$E = E^u \oplus E^s$, $TE^u = E^u, TE^s = E^s$,where E^u, E^s are closed subspaces. In addition, there exists constants $\tau\,(0 < \tau < 1)$ and $C > 0$, such that for any $\xi \in E^u, k \in N$, $\left\|T^k \xi\right\| \geq C\tau^{-k} \|\xi\|$, and for any $\eta \in E^s, k \in N$, $\left\|T^k \eta\right\| \leq C\tau^k \|\eta\|$;
(2) Assume also that $Per\,(T)$ is dense in E. Then T is said to be a non-wandering operator relative to E.

Definition 2 *Suppose $T \in L(X)$ and $\{e_i\}_1^\infty$ is a basis in X, then T is called a unilateral backward shift operator relative to $\{e_i\}_1^\infty$ if $Te_n = e_{n-1}\,(n > 1)$ and $Te_1 = 0$.*

3 Main results

Theorem 1 *Let $(X, \|\cdot\|)$ be an infinite dimensional separable $Banach$ space. T is an unbounded operator, if for $\forall n \geq 1$, T^n is the closed operator and T satisfy (1) there exists a closed subspace $E \subset X$, which has hyperbolic structure; (2) $Per\,(T)$ is dense in E. Then T is a non-wandering operator relative to E.*

Proof. By the Closed Graph Theorem, we can easily obtain this result. ■

Remark 1 *In fact, from Theorem 1, if an unbounded operator T has non-wandering property, then T need to satisfy: (1) T^n is the closed operator, $\forall n \geq 1$; (2) there exists a closed subspace $E \subset X$, which has hyperbolic structure relative to T; (3) $\overline{Per\,(T)} = E$*

In the following, we will apply Theorem 1 to discuss the Non-wandering property of differentiation operator.

3.1 Non-wandering property of differentiation operators in Bargmann space

Let $\{w_n\}_{n\in N}$ be an arbitrary weight sequence, we define the iterated unbounded back-ward shift T^n in Bargmann space by

$$T^n\left(\sum_{k\geq 0} C_k \frac{x^k}{\sqrt{k!}}\right) = \sum_{k\geq 0}\left(\prod_{j=k}^{n+k-1} w_j\right) C_{n+k}\frac{x^k}{\sqrt{k!}}$$

with its domain in F, and we define

$$D\left(T^n\right) = \left\{ f(x) = \sum_{k\geq 0} C_k \frac{x^k}{\sqrt{k!}} \left| \sum_{k\geq 0} |C_k|^2 < \infty; \sum_{k\geq 0} \left|\prod_{j=k}^{m+k-1} w_j\right|^2 |C_{k+m}|^2 < \infty \right.\right\}$$

for all $m \in N, 1 \leq m \leq n$.

Theorem 2 *A linear unbounded backward shift operator T: $F \to F$ is non-wandering operator if the positive series*

$$\sum_{n=0}^{\infty}\prod_{j=0}^{n-1}\frac{1}{|w_j|^2}$$

converges.

Proof. From Theorem 1, we need three steps to proof the theorem:

(1)We proof that for $\forall n \in N$, T^n is closed.

By the definition, we choose $\{f_j\} \in D\left(T^n\right)$,since F is a Hilber space, then $\{f_j\} \to f_0$ in F, so $f_j(x) = \sum_{k\geq 0} C_{k,j} \frac{x^k}{\sqrt{k!}} \quad \to f_0(x) = \sum_{k\geq 0} C_k^0 \frac{x^k}{\sqrt{k!}} \quad j \to \infty$

$$\Rightarrow \left(\prod_{j=k}^{n+k-1} w_j\right) C_{n+k,j} \to \left(\prod_{j=k}^{n+k-1} w_j\right) C_{n+k}^0,$$

Let $T^n f_j \to g_0$, then $g_0(x) = \sum\limits_{k\geq 0} \xi_k^0 \frac{x^k}{\sqrt{k!}}$

We can conclude that

$$\xi_k^0 = \left(\prod_{j=k}^{n+k-1} w_j\right) C_{n+k}^0$$

This proves that $f_0 \in D(T^n)$ and $T^n f_0 = g_0$.

(2) Since the positive series $\sum\limits_{n=0}^{\infty}\prod\limits_{j=0}^{n-1}\frac{1}{|w_j|^2}$ converges, then $\prod\limits_{j=0}^{n-1}\frac{1}{|w_j|^2} \to 0$, that is, $\prod\limits_{j=0}^{n} w_j \to \infty \quad n \to \infty$

We choose w_j is an increasing sequence.

For $\forall\lambda \in C$, let $f_\lambda(x) = \sum\limits_{k\geq 0}\left(\prod\limits_{j=0}^{k-1}\frac{\lambda}{w_j}\right)\frac{x^k}{\sqrt{k!}}$, then for $\forall 0 < \upsilon < 1$ and $n \in N$ large enough, we have $|\lambda| \leq \upsilon\,|w_n|$, so

$$\sum_{k\geq 0}\prod_{j=0}^{k-1}\left|\frac{\lambda}{w_j}\right|^2 = \sum_{k=0}^{n}\prod_{j=0}^{k-1}\left|\frac{\lambda}{w_j}\right|^2 + \sum_{k=n+1}^{\infty}\prod_{j=0}^{k-1}\left|\frac{\lambda}{w_j}\right|^2$$

$$\leq \sum_{k=0}^{n}\prod_{j=0}^{k-1}\left|\frac{\lambda}{w_j}\right|^2 + \left(\frac{1}{1-\upsilon}\right)\prod_{j=0}^{n}\left|\frac{\lambda}{w_j}\right|^2 < \infty$$

Thus we get $f_\lambda(x) \in F$ and

$$Tf_\lambda(x) = \sum_{k\geq 0} w_k\left(\prod_{j=0}^{k}\frac{\lambda}{w_j}\right)\frac{x^k}{\sqrt{k!}} = \lambda\sum_{k\geq 0}\left(\prod_{j=0}^{k-1}\frac{\lambda}{w_j}\right)\frac{x^k}{\sqrt{k!}} = \lambda f_\lambda(x)$$

Therefore, $f_\lambda(x)$ is the eigenvector corresponding to the eigenvalue λ, furthermore, by the arbitrarily of λ, we can construct the sets:

$$V_1 = \{\lambda: \quad |\lambda| > 1 \quad \lambda \in \delta_P(T)\}$$

$V_2 = \{\lambda: \quad 0 < |\lambda| < 1 \quad \lambda \in \delta_P(T)\}$, where $\delta_p(T)$ is the spectrum of T.

Let $E^u = \overline{span\ \{f_\lambda; \quad \lambda \in V_1\}}$ for $\forall\xi \in E^u$, then $\xi = \sum\limits_{i=1}^{\infty}\alpha_i f_{\lambda_i} = \sum\limits_{i=1}^{\infty}\alpha_i\sum\limits_{k\geq 0}\left(\prod\limits_{j=0}^{k-1}\frac{\lambda_i}{w_j}\right)\frac{x^k}{\sqrt{k!}}$ and for $\forall k \in N$, we have

$$\left\|T^k(\xi)\right\| = \left\|T^k\left(\sum_{i=1}^{\infty}\alpha_i f_{\lambda_i}\right)\right\| = \left\|\sum_{i=1}^{\infty}\alpha_i\lambda_i^k\sum_{k\geq 0}\left(\prod_{j=0}^{k-1}\frac{\lambda_i}{w_j}\right)\frac{x^k}{\sqrt{k!}}\right\|$$

$$\geq \mu^k\left|\sum_{i=1}^{\infty}\sum_{k\geq 0}\left(\prod_{j=0}^{k-1}\frac{\lambda_i}{w_j}\right)\frac{x^k}{\sqrt{k!}}\right| = \mu^k\,|\xi| \tag{1}$$

where $\mu = \min\ \{|\lambda_i| \quad \lambda_i \in V_1\} \ > 1$. Let $\tau = \frac{1}{\mu}$, then we can easily get $0 < \tau < 1$. So by (1) we have $\left\|T^k(\xi)\right\| = \left\|T^k\left(\sum\limits_{i=1}^{\infty}\alpha_i f_{\lambda_i}\right)\right\| = \left\|\sum\limits_{i=1}^{\infty}\alpha_i\lambda_i^k f_{\lambda_i}\right\| \geq \tau^{-k}\,|\xi|$.

Next, we will prove E^u is the invariant subspace of T.

Since for $\forall \xi \in E^u$, then $\xi = \sum\limits_{i=1}^{\infty} \alpha_i f_{\lambda_i} = T \sum\limits_{i=1}^{\infty} \frac{\alpha_i}{\lambda_i} f_{\lambda_i} = TE^u \Rightarrow \xi \in TE^u$, so $E^u \subset TE^u$. In the other hand, for $\forall \eta \in TE^u$, then there exists $\varphi \in E^u \Rightarrow \varphi = \sum\limits_{i=1}^{\infty} \beta_i f_{\lambda_i}$, such that, $\eta = T\varphi = \sum\limits_{i=1}^{\infty} \lambda_i \beta_i f_{\lambda_i} \in E^u$. So we can get $TE^u \subset E^u$, therefore, $TE^u = E^u$.

Similarly, let $E^s = \overline{span\ \{f_\lambda; \quad \lambda \in V_2\}}$, then $TE^s = E^s$, for $\forall \eta \in E^s$,we have $\eta = \sum\limits_{i=1}^{\infty} \beta_i f_{\lambda_i} = \sum\limits_{i=1}^{\infty} \beta_i \sum\limits_{k \geq 0} \left(\prod\limits_{j=0}^{k-1} \frac{\lambda}{w_j} \right) \frac{x^k}{\sqrt{k!}}$, so for $\forall k \in N$,$\left\| T^k(\eta) \right\| = \left\| T^k \left(\sum\limits_{i=1}^{\infty} \beta_i f_{\lambda_i} \right) \right\| = \left\| \sum\limits_{i=1}^{\infty} \beta_i \lambda_i^k f_{\lambda_i} \right\| \leq \tau^k |\eta|$ where $0 < \tau = \max\ \{|\lambda_i| \quad \lambda_i \in V_2\} \ < 1$ and $TE^s = E^s$

Let $E = E^u \oplus E^s$, then we can easily get E has hyperbolic structure.

(3) From the definition, we have $T^n \left(\sum\limits_{k \geq 0} C_k \frac{x^k}{\sqrt{k!}} \right) = \sum\limits_{k \geq 0} \left(\prod\limits_{j=k}^{n+k-1} w_j \right) C_{n+k} \frac{x^k}{\sqrt{k!}}$, if T has the $N-$periodic point, then we have $\left(\prod\limits_{j=k}^{n+k-1} w_j \right) C_{n+k} = C_k$,$\forall k \geq 0$, so for $\forall l = 0, 1 \cdots n-1$, $k \geq 1$, we have $C_{kn+l} = \left(\prod\limits_{j=l}^{kn+l-1} \frac{1}{w_j} \right) C_l$, thus for $\ \forall \upsilon \geq 0 \quad n \geq \upsilon$,$n \in N$, we can construct

$$g_{v,n}(x) = \frac{x^v}{\sqrt{v!}} + \sum_{k=1}^{\infty} \left(\prod_{j=v}^{kn+v-1} \frac{1}{w_j} \right) \frac{x^{kn+v}}{\sqrt{(kn+v)!}} \quad .$$

Since the series $\sum\limits_{n=0}^{\infty} \prod\limits_{j=0}^{n-1} \frac{1}{|w_j|^2}$ converges, then we have $\sum\limits_{k=1}^{\infty} \prod\limits_{j=v}^{kn+v-1} \frac{1}{w_j} < \infty$, so, we get $g_{v,n} \in F$ and

$$T^N g_{v,n}(x) = T^N \left(\frac{x^v}{\sqrt{v!}} + \sum_{k=1}^{\infty} \left(\prod_{j=v}^{kn+v-1} \frac{1}{w_j} \right) \frac{x^{kn+v}}{\sqrt{(kn+v)!}} \right) = \frac{x^v}{\sqrt{v!}} + \sum_{k=1}^{\infty} \left(\prod_{j=v}^{kn+v-1} \frac{1}{w_j} \right) \frac{x^{kn+v}}{\sqrt{(kn+v)!}}.$$

Thus $g_{v,n}(x)$ is the $N-$periodic point of T.

Let $E_0 = span\,\{g_{v,n}(x)\}$, in the following we will prove that E_0 is dense on F.

Since for $\forall f(x) \in F$, let $f(x) = \sum\limits_{v=0}^{m} C_v \frac{x^v}{\sqrt{v!}}$, by the definition of Bargmann space, we have

$$|C_v \prod_{j=0}^{v-1} w_j| < \infty.$$

Suppose $|C_v \prod_{j=0}^{v-1} w_j| < 1$, then there exists $g(x) \in E_0$ and

$$g(x) = \sum_{v=0}^{m} C_v g_{v,n}(x)$$

so that

$$\|g - f\| = \|\sum_{v=0}^{m} C_v (g_{v,n}(x) - \frac{x^v}{\sqrt{v!}})\| = \|\sum_{v=0}^{m} (C_v \prod_{j=0}^{v-1} w_j) \sum_{k=1}^{\infty} (\prod_{j=0}^{kn+v-1} \frac{1}{w_j} \frac{x^k v + n}{\sqrt{(kn+v)!}}\|$$

$$\leq \sum_{v=0}^{m} \|\sum_{k=1}^{\infty} (\prod_{j=0}^{kn+v-1} \frac{1}{w_j} \frac{x^k v + n}{\sqrt{(kn+v)!}}\| \tag{2}$$

Furthermore, from the series $\sum\limits_{n=0}^{\infty} \prod\limits_{j=0}^{n-1} \frac{1}{|w_j|^2}$ converges, the there exists $n \geq m$ such that for $\forall \varepsilon > 0$, we have

$$\sum_{k\geq n+1}(\prod_{j=0}^{k}\frac{1}{w_j}\varepsilon_k\frac{x^k}{\sqrt{k!}}\| < \frac{\varepsilon}{m+1}$$

where ε_k taking values 0 or 1 , so we can get when $n \geq m$, then $(2) < \varepsilon$.

Therefore, E_0 is dense on F. ■

Theorem 3 *The operator of differentiation* $D: \ f \ \rightarrow \ f'$ *defined on*

$$\delta = \left\{ f \in F \middle| f' \in F \right\}$$

is the non-wandering operator on F.

Proof. Since $\frac{x^k}{\sqrt{k!}}$ is an orthonormal basis in F, then we have

$$D\left(\frac{x^k}{\sqrt{k!}}\right) = k \cdot \frac{x^{k-1}}{\sqrt{k!}} = \sqrt{k}\frac{x^{k-1}}{\sqrt{(k-1)!}} = w_{k-1}\frac{x^{k-1}}{\sqrt{(k-1)!}}$$

where $w_k = \sqrt{k+1}$,

So, we can get the operator of differentiation $D: \ f \ \rightarrow \ f'$ is the weighted backward shift operator, therefore,

$$D\left(\sum_{k\geq 0} C_k\frac{x^k}{\sqrt{k!}}\right) = \sum_{k\geq 0} w_k C_{k+1}\frac{x^k}{\sqrt{k!}} \quad , \quad w_k = \sqrt{k+1}$$

so the series

$$\sum_{n=0}^{\infty}\prod_{j=0}^{n-1}\frac{1}{|w_j|^2} = \sum_{n=0}^{\infty}\prod_{k=0}^{n-1}\frac{1}{k+1}$$

converges in F, thus by the Theorem 2, we have D is the non-wandering operator on F. ■

3.2 Non-wandering property of differentiation operators in Hardy space

Denote Hardy space as H^2, then

$$H^2 = \{f \in H(D) : \|f\| < \infty\}$$

where D is the unit disk which centers at zero and $H(D)$ is made of all the analytic function which defined in D. Suppose $f(x) \in H(D)$, then we have $f(x) = \sum\limits_{n=0}^{\infty} \overline{f(n)} x^n$

Lemma 4 *([8]) For each positive integer* n*, the operator* D^n *is the closed operator on its domain* $\varepsilon_r = \left\{f \in H^2; \quad f^{(n)} \in H^2\right\}$.

Theorem 5 *The operator* D *of differentiation defined on* $\varepsilon = \left\{ f \in H^2 \middle| f' \in H^2 \right\}$ *of* H^2 *by* $D: \ f \ \rightarrow \ f'$ *is the non-wandering operator.*

Proof. Since D is the chaotic operator on ε(see[8]), so D has the dense set of periodic points on ε. Thus , from Theorem 1, we only need to construct the hyperbolic structure.

In the following, we will construct the hyperbolic structure.

Firstly, we pick a weighted sequence (w_j) on the unit disk. Suppose (w_j) is the decreasing sequence which satisfy:

$1 > w_1 > w_2^{>} \cdots > 0$ and $\lim\limits_{n\to\infty} w_n = 0$

Let $r_n = \frac{1}{n!}\prod_{i=1}^{n}\frac{1}{w_i}$,$n \geq 1$, we construct $\varepsilon = \{f(x): f(x) = \sum_{n=0}^{\infty} a_n x^n\}$and define $\|f\| = \left(\sum_{n=0}^{\infty}\frac{|a_n|^2}{r_n^2}\right)^{1/2} < \infty.$

It is simple to check that ε is a Hilbert space having norm $\|\cdot\|$ and orthonormal basis $e_n = r_n x^n$.

Since $De_n = D(r_n x^n) = nr_n x^{n-1} = n \cdot \frac{1}{n!}\prod_{i=1}^{n}\frac{1}{w_i}x^{n-1} = \frac{1}{(n-1)!}\prod_{i=1}^{n}\frac{1}{w_i}x^{n-1} = \frac{1}{w_n}e_{n-1}$, so, the operator of differentiation D is the weighted backward shift operator on ε.

For $\forall\lambda_i \in C$, we construct $f_{\lambda_i}(x) = \sum_{n=0}^{\infty}\prod_{i=0}^{n-1}\lambda_i w_i e_n$,then $\lambda_i w_i \to 0 \quad n \to \infty$,

So, when n large enough, we have

$\left\|\sum_{n=0}^{\infty}\prod_{i=0}^{n-1}\lambda_i w_i\right\| < \infty$,so $f_{\lambda_i}(x) \in \varepsilon$ and

$$Df_{\lambda_i}(x) = \sum_{n=0}^{\infty}\frac{1}{w_n}\prod_{i=0}^{n}\lambda_i w_i e_n = \lambda_i\sum_{n=0}^{\infty}\prod_{i=0}^{n-1}\lambda_i w_i e_n = \lambda f_{\lambda_i}(x)$$

Therefore , $f_{\lambda_i}(x)$ is the eigenvector corresponding to the eigenvalue λ_i. So we can construct

$$E^u = \overline{span\ \{f_{\lambda_i}(x): \quad |\lambda_i| > 1\}}$$

For $\forall\xi \in E^u$, then $\xi = \sum_{i=1}^{\infty}\alpha_i f_{\lambda_i} = \sum_{i=1}^{\infty}\alpha_i\sum_{n=0}^{\infty}\prod_{i=0}^{n-1}\lambda_i w_i e_n$ and for $\forall k \in N$, we have

$$\begin{aligned}\|D^k(\xi)\| &= \left\|D^k\left(\sum_{i=1}^{\infty}\alpha_i f_{\lambda_i}\right)\right\| = \left\|\sum_{i=`}^{\infty}\alpha_i\lambda_i^k\sum_{n=0}^{\infty}\prod_{i=0}^{\infty}\lambda_i w_i e_n\right\|, \\ &\geq \mu^k|\xi|\end{aligned}$$

where $\mu = \min\ \{|\lambda_i| \quad > 1\}$, let $\tau = \frac{1}{\mu}$, then $0 < \tau < 1$, so we get

$$\left\|D^k(\xi)\right\| = \left\|D^k\left(\sum_{i=1}^{\infty}\alpha_i f_{\lambda_i}\right)\right\| = \left\|\sum_{i=`}^{\infty}\alpha_i\lambda_i^k\sum_{n=0}^{\infty}\prod_{i=0}^{\infty}\lambda_i w_i e_n\right\| \geq \tau^{-k}|\xi|$$

Next, we prove E^u is the invariant subspace of D.

For $\forall\eta \in E^u$, then $\xi = \sum_{i=1}^{\infty}\alpha_i f_{\lambda_i} = D\sum_{i=1}^{\infty}\frac{\alpha_i}{\lambda_i}f_{\lambda_i} = DE^u \Rightarrow \xi \in DE^u$.

So $E^u \subset DE^u$, otherwise, for $\forall\eta \in DE^u$, there exists $\varphi \in E^u \Rightarrow \varphi = \sum_{i=1}^{\infty}\beta_i f_{\lambda_i}$ such that $\eta = D\varphi = \sum_{i=1}^{\infty}\lambda_i\beta_i f_{\lambda_i} \in E^u$, thus $DE^u \subset E^u$, therefore, $DE^u = E^u$.

Similarly, let $E^s = \overline{span\ .\{f_{\lambda_i}(x): \quad |\lambda_i| < 1\}}$

For $\forall\eta \in E^s$, then $\eta = \sum_{i=1}^{\infty}\beta_i f_{\lambda_i} = \sum_{i=1}^{\infty}\beta_i\sum_{n=0}^{\infty}\prod_{i=0}^{n-1}\lambda_i w_i e_n$and for $\forall k \in N$

$$\left\|D^k(\xi)\right\| = \left\|D^k\left(\sum_{i=1}^{\infty}\alpha_i f_{\lambda_i}\right)\right\| = \left\|\sum_{i=1}^{\infty}\alpha_i\lambda_i^k\sum_{n=0}^{\infty}\prod_{i=0}^{\infty}\lambda_i w_i e_n\right\| \leq \tau^k|\xi|,$$

where $\tau = \max_{i=1\cdots n}\{|\lambda_i| < 1\}$, $0 < \tau < 1$ and $DE^s = E^s$.

Let $E = E^u \oplus E^s$, then E has hyperbolic structure. ■

Theorem 6 *If ∂U is the boundary of the open unit disk U and if g is a nonconstant analytic function for which $g(U) \cap \partial U \neq \phi$, then the operator $T: f \to g(D)f$ is the non-wandering operator on H^2.*

Proof. Since $g(U) \cap \partial U \neq \phi$, then there exists $x_0 \in U$ such that $|g(x_0)| = 1$, furthermore, since g is nonconstant and analytic on U, there exists $0 < \varepsilon < \frac{1-|x_0|}{4}$ so that the sets V_1, V_2 and V_3 given by

$$V_1 = \{\lambda; \quad |\lambda - x_0| < \varepsilon \quad |g(\lambda)| < 1\}$$

$$V_2 = \{\lambda; \quad |\lambda - x_0| < \varepsilon \quad |g(\lambda)| > 1\}$$

and

$$V_3 = \{\lambda \in U; \quad \exists n \geq 1 \qquad s.t. \quad g(\lambda)^n = 1\}$$

are nonempty open subsets of U. Next , let $\tau = |x_0| + 2\varepsilon$, and pick a sequence of posive numbers (w_n) with $1 > w_1 > w_2^{>} \cdots > \tau$ and $\lim\limits_{n\to\infty} w_n = \tau$.

Also, let $r_0 = 1$ and $r_n = \frac{1}{n!}\prod\limits_{i=1}^{n} w_i \ (n \geq 1)$. We consider the space ε of analytic functions on U of the form $f(z) = \sum\limits_{n=0}^{\infty} a_n z^n$ that satisfy

$$\|f\| = \left(\sum_{n=0}^{\infty} \frac{|a_n|^2}{r_n^2}\right)^{1/2} < \infty.$$

It is simple to check that ε is a Hilbert space having norm $\|\cdot\|$ and orthonormal basis $e_n = r_n x^n$.

Since $De_n = D(r_n x^n) = n r_n x^{n-1} = n \cdot \frac{1}{n!}\prod\limits_{i=1}^{n} w_i x^{n-1} = \frac{1}{(n-1)!}\prod\limits_{i=1}^{n} w_i x^{n-1} = w_n e_{n-1}$, so the operator of differentiation D is the weighted backward shift operator on ε . Since $\|D\| = \sup\limits_{n\geq 1}(w_n) = w_1 < 1$, and furthermore, g is analytic, so $g(D)$ is an absolutely convergent power series in D , and hence the operator $T : f \to g(D) f$ is bounded on ε . Also, notice that for each $|\lambda| < \tau$, we have $e^{\lambda x} \in \varepsilon$ and is an eigenfunction of T with eigenvalue $g(\lambda)$. Let $E^s = \overline{span\left(e^{\lambda x}; \quad \lambda \in V_1\right\}}$, then we have $TE^s = E^s$ and for $\forall \xi \in E^s$ then $\xi = \sum\limits_{i=1}^{n} \alpha_i e^{\lambda_i x}$

$$\left\|T^k(\xi)\right\| = \left\|T^k \sum_{i=1}^{n} \alpha_i e^{\lambda_i x}\right\| = \left\|\sum_{i=1}^{n} \alpha_i \left(g(\lambda)^k\right) e^{\lambda_i x}\right\| \leq \tau^k |\xi|$$

where $0 < \tau = \max\limits_{i=1\cdots n} |g(\lambda_i)| < 1$.

Similarly, let $E^u = \overline{span\left(e^{\lambda x}; \quad \lambda \in V_2\right\}}$, then $TE^u = E^u$ and for $\forall \eta \in E^u \quad \eta = \sum\limits_{i=1}^{n} \beta_i e^{\lambda_i x} \quad \left\|T^k(\eta)\right\| = \left\|T^k \sum\limits_{i=1}^{n} \beta_i e^{\lambda_i x}\right\| = \left\|\sum\limits_{i=1}^{n} \beta_i \left(g(\lambda)^k\right) e^{\lambda_i x}\right\| \geq \tau^{-k} |\eta|$, where $\mu = \min\limits_{i=1\cdots n} |g(\lambda_i)| > 1$, $0 < \tau = \frac{1}{\mu} < 1$.

Let $E = E^u \oplus E^s$, then E has hyperbolic structure.

Finally we prove that the periodic points of T are dense on E .

We consider $V_3 = \{\lambda \in U; \quad \exists n \geq 1 \qquad s.t. \quad g(\lambda)^n = 1\}$, then there exists the accumulation point of λ in U .

Denote $E_0 = span \ \left\{e^{\lambda_i x}; \quad \lambda_i \in V_3\right\}$, then E_0 is set which made of all the periodic points of $g(D)$. So, we can get E_0 is dense in H^2 , otherwise, E_0 is not dense in H^2 , then by $H - B$ Theorem, there exists $\varphi \in \left(H^2\right)^* = H^2$, for $\forall x \in E_0$, we have $\langle\varphi \quad x\rangle \ = 0$, so for $\forall x \in E_0$, $g_\varphi(\lambda) = \ \langle\varphi \quad x\rangle \ = 0$, furthermore, there exists the accumulation point of λ in U and $g_\varphi(\lambda)$ is analytic on U , so $g_\varphi(\lambda) \equiv 0$ on U . Thus it is contrary to the condition. ■

Acknowlegement

The author would like to express great gratitude to Professor Lixin Tian of Nonlinear Scientific Research Center of Jiangsu University for useful discussions and valuable suggestions.

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