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The Finite Element Analysis on Reinforced Concrete Short Pier Shear Wall Structures

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Abstract: The paper utilizes computer langue compiling calculational program and applies the plain-plate finite element model and integrate model to the structures of short-leg walls. The calculated results manifest that the new method is valid and reliable. Based on plain-plate finite element model, the ductility analysis of the short-leg shear walls with different flange width is studied.

Key words: plain-plate finite element; integrate model; the finite element analysis; short-leg shear wall

1 Introduction

When length is 5-8 times of thickness of the pier, and the mechanical properties are intervenient between abnormity section frame pillar and common shear wall, the shear wall is called short pier shear wall[1]. In recent years, this structure system has been gradually applied to tier dwellings and high-rises in our country widely. However, the study on theory of this structure system has been lagged compared with the practice[1-3]. The research of theory and test about this structure system at home and abroad center in unitary performance analysis, and most of the calculational methods are based on H –W –B (thin walled bar) or T-P (thin plates) theory which results have large errors[4,5] .In this paper , the author puts forward calculational method of short-pier shear walls structure utilizing plain-plate finite element based on P-S (plates and shells) theory.

2 Combination of rectangle plain-plate finite element

The finite element analysis on reinforced concrete shear walls can show the developing process of the structures deformation. It can describe or simulate the crack-forming and growing process, the destructive process, the modality and ductility of structure. It can also evaluate the ultimate bearing capacity of shear walls and open out the weak place and links, which is propitious to optimum structural design. This analytic procedure is applied widely to different shear wall structural types enduring different loads.

The loads not only lying in plane but also in the outside plane for the shear walls of abnormity section ,so plain-plate finite element made up of planar stress element and bending element which is adopted as the corresponding finite element in this paper.

The x-axis and y-axis under local coordinate system are set in the plane which the element lines in, length of side of the rectangular plate along x and y direction is expressed respectively as “2a” and “2b”, “t” represents the plate’s thickness.

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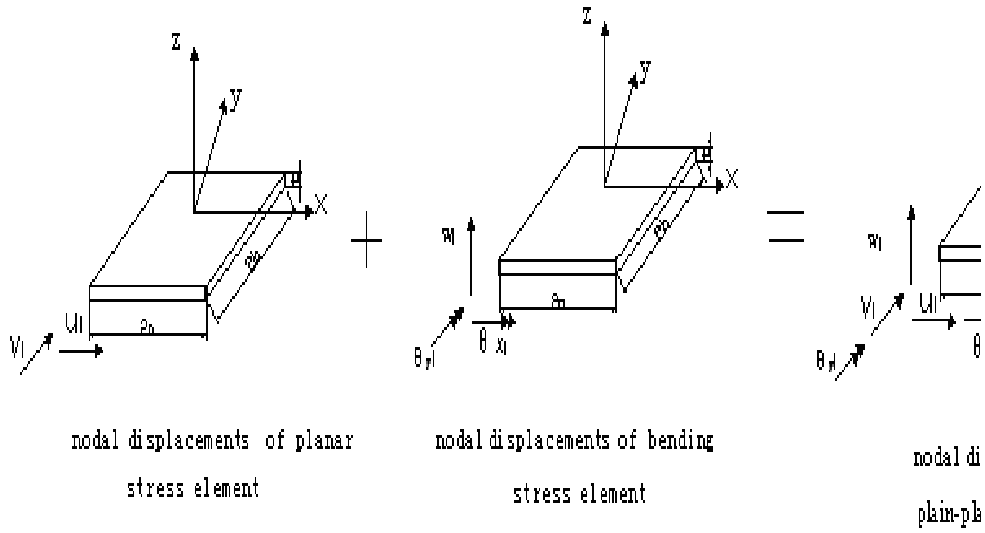


Figure 1: Plain-plate finite element in the local coordinate system

Two kinds of stress element are fitted together, and nodal displacements constitute the elements freedom degrees[6].In order to transform the stiffness matrix from local coordinate system to global coordinate system and aggregate it expediently in the later work, the corner θ_{zi} is included into the nodal displacement in spite of θ_{zi} not effecting the element's state of stress, while, a dummy bending moment $\mathrm{M}_{\theta\mathrm{zi}}$ is also included accordingly into the nodal forces.

$$\{\delta_i\} = \begin{bmatrix} \mathrm{u}_i & \mathrm{v}_i & w_i & \theta_{\mathrm{xi}} & \theta_{\mathrm{yi}} & \theta_{\mathrm{zi}} \end{bmatrix}^T (i = 1, 2, 3, 4) \tag{2.1}$$

$$\{\mathrm{F}_i\} = \begin{bmatrix} \mathrm{U}_i\mathrm{V}_i & \mathrm{W}_i\mathrm{M}_{\theta\mathrm{xi}} & \mathrm{M}_{\theta\mathrm{yi}}\mathrm{M}_{\theta\mathrm{zi}} \end{bmatrix}^T (i = 1, 2, 3, 4) \tag{2.2}$$

3 The model of reinforced concrete plain-plate finite element

Interrelation of reinforcing steels and concrete should be considered when we constitute a finite element model of R.C (Reinforced Concrete). Here, integrate finite element model is adopted according to the characteristic that the reinforcing steels equably distributing generally in the shear walls[6-8]. As shown in Fig. 2.

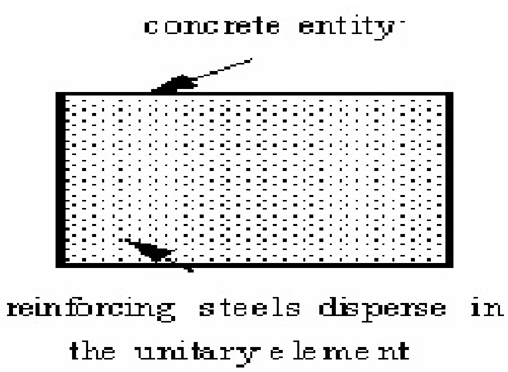


Figure 2: Integrate model

The reinforcing steels are considered dispersing in the unitary finite element, and the element is considered as homogeneous material. The stiffness matrix are made up of reinforcing steel and concrete, while, assuming that there are not relative slips between reinforcing and concrete. The elastic stiffness matrix of the integrate element could be calculated by the underhood:

$$[\mathrm{D}] = [\mathrm{D}_c] + \sum_i [\mathrm{D}_s]_{\mathrm{i}} \tag{3.1}$$

Where $[\mathrm{D}_c]$ is the stiffness matrix of concrete and $[\mathrm{D}_s]_{\mathrm{i}}$ is the stiffness matrix of reinforcing steels distributing equably along the "i_{th}" direction.

4 The constitutive relation of concrete under two-way stress

Adopting the form of nonlinear elasticity-increment is reasonable for the Reinforced Concrete shear wall structure when the loads are exerted progressively and out of proportion.

Because of adopting the nonlinear elastic theory, here we assume all the same there being one-to-one correspondence between stress state and strain state, while the material parameters are the functions of stress state (or strain state). So, stress increments and strain increments are brought into relation by material constitutive matrix. This paper applies the nonlinear constitutive model of concrete brought forward by Darwin and Pecknold. The fundamental assumption of this model lies in: the concrete is orthogonal, anisotropic, and the relation between stress and strain is linear elastic under any load increments.

Relation between stress increments and strain interments of concrete is expressed by Eq.(4.1)[9].

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{Bmatrix} = \frac{1}{1-\upsilon^2} \begin{bmatrix} E_1 & \upsilon\sqrt{E_1E_2} & 0 \\ \upsilon\sqrt{E_1E_2} & E_2 & 0 \\ 0 & 0 & (1-\upsilon^2)G \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} \tag{4.1}$$

in which: E_1, E_2 is the equivalent tangent modulus along direction of principal stress when one grade load is inflicted on the structure; $d\sigma_1, d\sigma_2, d\tau_{12}$ is stress increments brought by load increments; $d\varepsilon_1, d\varepsilon_2, d\gamma_{12}$ is strain increments brought by load increments; G is the shear modulus and υ is the Poisson's ratio of concrete.

(a) Assuming the value of the shear modulus G does not relate to the choice of coordinate axis, and this request can be proved by the following formulate using substituting method[10].

$$G = \frac{1}{4(1-\upsilon^2)}(E_1 + E_2 - 2\upsilon\sqrt{E_1E_2}) \tag{4.2}$$

(b) Calculation of the Poisson's ratio υ

According to the fundamental equations of elasticity about anisotropic orthogonal materials:

$\upsilon_1 E_2 = \upsilon_2 E_1$,let $\upsilon = \sqrt{\upsilon_1\upsilon_2}$($\upsilon_1\upsilon_2$ is the Poisson's ratio of principal stress direction);

Adopting the following suggestion of Darwin and Pecknold[9]:

(i)pressed or pulled under two-way stress: $\nu = 0.2$

(ii)pressed and pulled under one-way stress:

$$\upsilon = 0.2 + 0.6(\frac{\sigma_1}{f_c'})^4 + (\frac{\sigma_2}{f_c'})^4 \tag{4.3}$$

Where ${f_c}'$ is the cylinder crushing strength of the concrete.

(c) Calculation of the tangent modulus (E_1, E_2) of the concrete along the principal stress direction exerted first-grade load.

Equivalent monoaxial stress-strain relation put forward by Darwin and Pecknold is used in this paper. If the concrete of element is not crazed, E_i is calculated in the pressure direction by Eq. (4.4) in the two cases: pressed under two-way stress, pressed and pulled under one-way stress[9].

$$E_i = \frac{d\sigma_i}{d\varepsilon_{iu}} = \frac{E_0[1-(\frac{\varepsilon_{iu}}{\varepsilon_{ic}})^2]}{[1+[\frac{E_0}{E_s}-2](\frac{\varepsilon_{iu}}{\varepsilon_{ic}})+(\frac{\varepsilon_{iu}}{\varepsilon_{ic}})^2]^2} \tag{4.4}$$

In which, E_0 is the modulus of elasticity passing origin; E_s is the secant modulus corresponding the equivalent maximal compressive stressσ_{ic};

$$E_s = \frac{\sigma_{\text{ic}}}{\varepsilon_{\text{ic}}} \tag{4.5}$$

Where ε_{ic} is the compressional strain corresponding the equivalent maximal compressive stress σ_{ic};

$$\varepsilon_{iu} = \sum \frac{\Delta\sigma_i}{E_i} \tag{4.6}$$

in which, $\Delta\sigma_i$ is the stress increments caused by each-grade load increments in the principal stress direction "i"; E_i is the tangent modulus in the principal stress direction "i" when previous-grade load increments are exerted. The calculation of σ_{ic} and ε_{ic} bases on the formulate put forward by Kupfer and Gerstle.

5 Matrix expressing stress-strain relation of reinforcing steels in the integrate model

The reinforcing steels are regarded as unidimensional continuum dispersing in the unitary finite element(fig.3), it's relation of stress and strain along "i" direction is:

$$d\sigma_{si} = \rho_i E_{si} d\varepsilon_{si} \tag{5.1}$$

Where ρ_i is the reinforcement ratio along "i" direction.

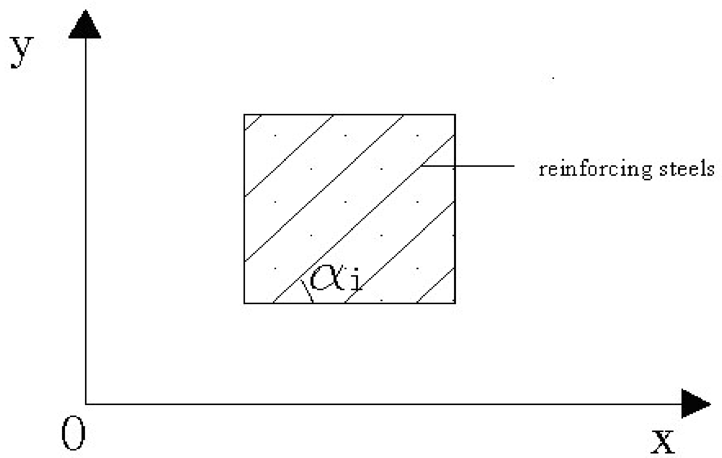


Figure 3: The model of reinforcing steel element

The stiffness matrix of "i" direction is expressed:

$$[D_s]_i^{,} = \begin{bmatrix} \rho_i E_{si} 0 & 0 \\ 00 & 0 \\ 00 & 0 \end{bmatrix} \tag{5.2}$$

The local coordinate of "i" direction is transformed to x and y coordinate,

$$[D_s]_i = [T_i]^T [D_s]' [T_i] \tag{5.3}$$

Where $[T]$ is the transition matrix from local coordinate to global coordinate.[9,11]

$$[T] = \begin{bmatrix} \cos^2\alpha \sin^2\alpha & \sin\alpha\cos\alpha \\ \sin^2\alpha \cos^2\alpha & -\sin\alpha\cos\alpha \\ -2\sin\alpha\cos\alpha 2\sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha \end{bmatrix} \tag{5.4}$$

the value of α in the above formulate is prescripted as positive sign when the revolution is clockwise from "i"direction to x-axis.

6 Nonlinear analytic procedure on the finite element of reinforced concrete shear wall

Here, with the computational model is integrated, the shear walls element is plain-plate finite-element of rectangle, D-P model is adopted as constitutive relations of concrete and reinforcing steels are assumed as ideal elastoplastic body.

6.1 To calculate the actual load increments and cumulative loads

1) The actual load increments

$$\{\Delta R_i\} = \{\Delta R_i\}^0 + \{\Delta Q_{i-1}\} \tag{6.1}$$

where, $\{\Delta R_i\}^0$is the load increment of signed originally, $\{\Delta Q_{i-1}\}$is lopsided load increment calculated of $(i-1)_{th}$.

2) The cumulative loads

$$\{R_i\} = \{R_0\} + \sum_{j=1}^{i} \{\Delta R_j\} \tag{6.2}$$

6.2 To calculate the element stiffness matrix and unitary stiffness matrix of the load increments according to its behavior of the $(i-1)_{th}$ at the end

1) Elastic stiffness matrix of the integrate element

Assumed that: the tangent modulus on the two principal stress direction of concrete element of $(i1)_{th}$ in the end is $E_{1,i-1}$ and $E_{2,i-1}$ respectively, the Poisson's ratio is μ_{i-1} and the tangent modulus of reinforcing steels element is$E_{s,i-1}$.We can first calculate the elastic stiffness matrix of reinforcing steels and the concrete element ——$D_{c,i-1}$,$D_{s,i-1}$, and then calculate the elastic stiffness matrix of the integrate element "D_{i-1}"[4]:

$$[D_{i-1}] = [T]^t [D_{c,i-1}] [T] + [L]^T [D_{s,i-1}] [L] \tag{6.3}$$

In which, $[T]$ and $[L]$ is the transition matrix of concrete and reinforcing steels element respectively.

2) To calculate the stiffness matrix of element "k_{i-1}" and transform it from local coordinate system to global coordinate system;

3) To calculate the unitary stiffness matrix K_{i-1} while giving the stiffness coefficient any fixed value $k_{z\theta}$ on the direction of θ_z at the special node.

6.3 To solve the unitary equations of equilibrium

Basing on the above work, the displacement increments $\{\Delta\delta_i\}$ of the structure can be calculated according to $[K_{i-1}]\{\Delta\delta_i\} = \{\Delta R_i\}$, and the displacements can be added up by:$\{\delta_i\} = \{\delta_{i-1}\} + \{\Delta\delta_i\}$.

6.4 To calculate the nodal displacements and strains of the element

After calculating the nodal displacement increments $\{\Delta\delta_i\}$ and the nodal displacement $\{\delta_i\}$ of the structure, $\{\Delta\delta_i\}^e$ and $\{\delta_i\}^e$ can be separated respectively. Then the strains of the element can be calculated too.

Where $\{\Delta\delta_i\}^e$,$\{\delta_i\}^e$ is the nodal displacement increments and displacements of the element respectively.

1) To calculate the strain increments of the element ——$\{\Delta\varepsilon_i\}^e$and add up the strains:

$$\{\Delta\varepsilon_i\}^e \{\varepsilon_i\}^e = \{\Delta\varepsilon_i\}^e + \{\varepsilon_{i-1}\}^e$$

2) To calculate the principal strain increments of the element——$\Delta\varepsilon^e_{1,i}$, $\Delta\varepsilon^e_{2,i}$ and the principal strains—— $\varepsilon^e_{1,i}$, $\varepsilon^e_{2,i}$.

6.5 To calculate the stresses of the element

1) The calculation of the stress increments of reinforcing steels and concrete of the element——$\{\Delta\sigma_{s,i}\}^e$,$\{\Delta\sigma_{c,i}\}^e$

$$\{\Delta\sigma_{s,i}\}^e = [L]^T [D_{s,i-1}]^e [L] \{\Delta\varepsilon_i\}^e , \{\Delta\sigma_{c,i}\}^e = [T]^t [D_{c,i-1}]^e [T] \{\Delta\varepsilon_i\}^e$$

2) To add up the stresses of reinforcing steels and concrete of the element respectively

$$\{\sigma_{s,i}\}^e = \{\Delta\sigma_{s,i}\}^e + \{\sigma_{s,i-1}\}^e \quad , \{\sigma_{c,i}\}^e = \{\Delta\sigma_{c,i}\}^e + \{\sigma_{c,i-1}\}^e$$

$$\{\sigma_i\}^e = \{\Delta\sigma_i\}^e + \{\sigma_{i-1}\}^e = \{\Delta\sigma_{s,i}\}^e + \{\Delta\sigma_{c,i}\}^e + \{\sigma_{i-1}\}^e$$

3) The calculation of the principal stresses and the principal stress's direction angle of the concrete element——$\sigma^e_{c1,i}, \sigma^e_{c2,i}$,$\theta^e_{c,i}$.

6.6 To calculate the tangent modulus of the concrete element——$E_{1s,i}, E_{2s,i}$

1) Calculation of the equivalent principal strains of concrete

The increments of equivalent principal strain of concrete:

$$\Delta\varepsilon_{1u,i} = \frac{\sigma_{c1,i} - \sigma_{c1,i-1}}{E_{1,i-1}}; \quad \Delta\varepsilon_{2u,i} = \frac{\sigma_{c2,i} - \sigma_{c2,i-1}}{E_{2,i-1}}$$

The equivalent principal strain of concrete: $\varepsilon_{1u,,i} = \varepsilon_{1u,i-1} + \Delta\varepsilon_{1u,i}$; $\varepsilon_{2u,,i} = \varepsilon_{2u,i-1} + \Delta\varepsilon_{2u,i}$

2) Calculation of $\varepsilon_{1c,i}$and $\varepsilon_{2c,i}$.

3) Calculation of $E_{1s,i}, E_{2s,i}$: $E_{js,i} = \frac{\sigma_{jc,i}}{\varepsilon_{jc,i}}$ (j=1,2).

6.7 The material behavior of i_{th} load increments in the end

The above calculation stresses are based on the assumption that the material behavior hold the line of the previous grade in the end, and the tangent matrix is utilized to the calculating. The actual stress value of current grade should be solved on the basis of analysis of element behavior and considering the actual relation between stress and strain.

1) Calculate the elastic modulus $E_{1,i}, E_{2,i}$ and elastic stiffness matrix $[D_i]$ of the concrete element;

2) Calculate the actual stresses of the concrete element;

3) Calculate the elastic modulus $E_{s,i}$ and elastic stiffness matrix $[D_{si}]$ of the reinforcing steel element.

6.8 Calculation of unbalanced stress and unbalanced nodal force

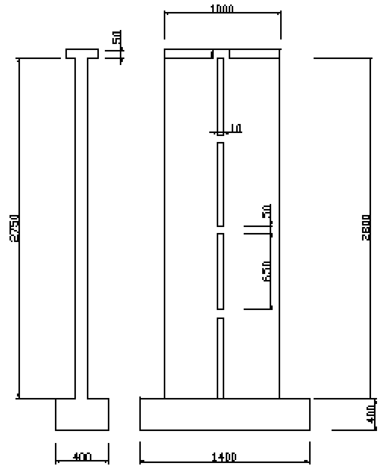
Releasing of stress caused by cracking concrete brought the unbalanced element stress. Its ' corresponding nodal forces is named unbalanced nodal force and the nodal loads corresponded to unbalanced nodal force is named unbalanced nodal loads.

1) Calculate the unbalanced nodal loads of element——$\{\Delta Q_i\}^e$;

2) Unbalanced nodal loads of each element formed unbalanced nodal loads $\{\Delta Q_i\}^e$ of structures though transforming the coordinate.

7 Analysis by examples

Example 1 *Reinforced concrete shear wall is analyzed on the basis of experimenting and calculating in the references[1,2]. Where, the strength and elastic modulus of the concrete is respectively* $f_c = 34.64MPa$ *and* $E_c = 3.6 \times 10^4 MPa$*; the strength and elastic modulus of the reinforcing steel is respectively* $f_y = 310MPa$ *and* $E_s = 2.0 \times 10^5 MPa$ *;ratio of the longitudinal reinforcement and hoop reinforcement is respectively 1% and 0.3% in the wall leg; ratio of the longitudinal reinforcement and hoop reinforcement is respectively 1% and 0.566%[4,9,12] in the lintel. The computational model is established by the shape of test-pieces as fig.5, and there are 68 elements in all.*



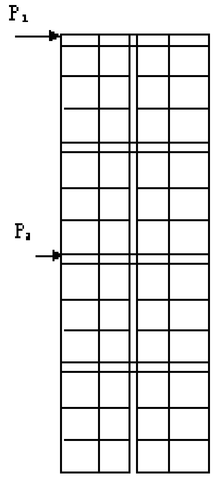
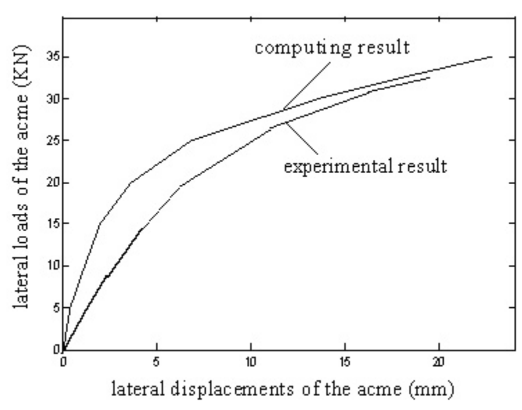


Figure 4: The experimental model Fig.5 The partition of the finite element.

Figure 5: The partition of the finiteelement.

According to the loading method and procedure, lateral loads P_1=2P/3 is inflicted on the top of structure, and P_2=P/3 on the half of the structure's height. During the experiment, the top load is $P_1 = 5kN, 10kN, 15kN, 20kN, 25kN, 30kN$ *in turn. The curve of load-displacement on the basis of experiment and calculating analysis is drawn as Fig.6.*

The calculated results manifest that it can put up perfectly the characteristic of reinforced concrete shear wall enduring force to utilize integrate model and plain-plate finite element during the analysis. So the calculated results are dependable. Certainly, there are some errors between calculating and experimentation because of the model of the shear wall adopted being crude comparatively; the stress of element caused by each grade loads being put on the next grade directly as external loads, which bring paranormal unitary stiffness of the shear wall and not detailed partition of element.



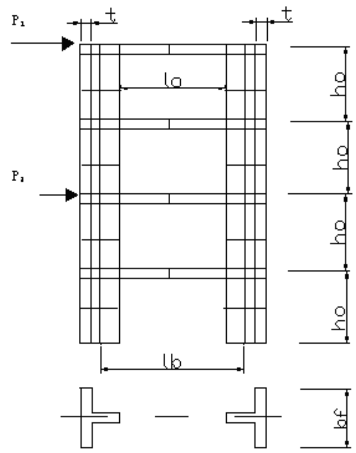


Figure 6: The load displacement curive of shear wall

Figure 7: The partition of the shear wall element

Example 2 *In this example, the ductility is analyzed the double symmetrical short-legs shear wall with 4 floors, here, the story height and thickness is* $2.8m$ *and* $0.2m$ *respectively.*

In the computational model, horizontal lateral loads P_1=2P/3 are inflicted on the top of structureand P_2=P/3 on the half of the structure height (here, P are the monotonic total loads).The short-leg shear wall is regard in ductility stage when the legs cracked enduring tensile stress. However, the structure is regard destroyed when the concrete of legs crushed enduring compression stress. The partition of the finite element is shown as Fig.7. The unitary shear wall is compartmentalized to 104 elements including 138 nodes. The *stresses are centralized relatively at the lintel of the model, amplitude of variation of the stresses are huge relatively, so the elements nearby are compartmentalized in detail.*

The concrete adopted is C50, the material parameter: $f_c = 26.5MPa, f_t = 2.45MPa$, $\nu = 0.2$, $E_c = 3.6 \times 10^4 MPa$. *The reinforcing steel adopted is the first grade and the material parameter is respectively:* $E_s = 2.0 \times 10^5 MPa, f_y = 310MPa$.*The ratio of the longitudinal reinforcement and hoop reinforcement is respectively 1% and 0.3% in the wall leg; ratio of the longitudinal reinforcement and hoop reinforcement is respectively 1.5% and 0.3% in the lintel.*

The ductility analysis of short-leg shear walls with different width of flange is based on the section height of shear wall's leg and lintel are fixed. The result of analysis and the curve of loads-lateral displacements are shown respectively as Table.1 and Fig.8.

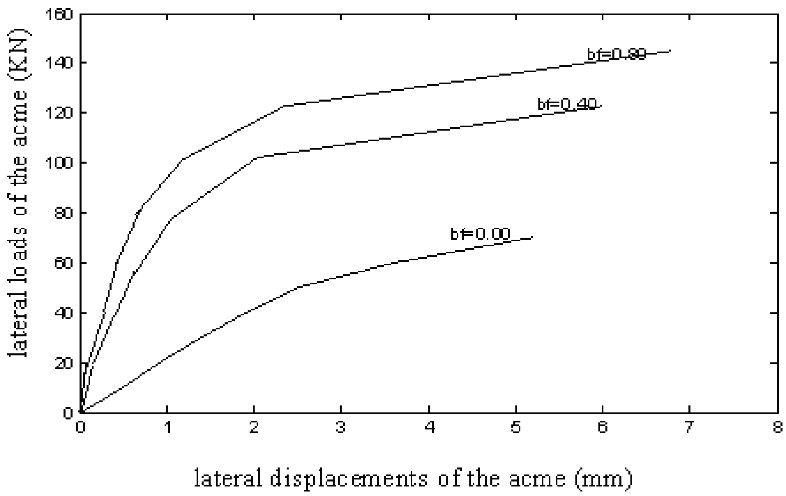


Figure 8: The load-displacement curve with different width of flange b_f

Table 1: Ductility of the short-leg shear wall with different width of flange b_f

Sequence number	distance between legs (m)	height of leg (m)	height of lintel (m)	width of the flange (m)	cracking loads of legs enduring tensile stress (KN)	cracking loads of legs enduring compression stress (KN)	ductility ratio
1	3.7	1.2	0.4	0.00	50	70	2.08
2	3.7	1.2	0.4	0.40	80	120	5.08
3	3.7	1.2	0.4	0.80	80	140	6.80

Thus it can be seen that the elastoplasticity of shear walls is affected largely by the width of flange; the cracking loads of wall-legs increase with the increasing of width of flange, but the amplitude is in a narrow range. However the carrying capacity of shear walls, ultimate loads and displacement ductility of wall-leg increased all by a big margin. It can be seen from the load-displacement curve that the power dissipation of wall-legs increased with the width of flange across the board too.

8 Conclusion

The paper utilized computer langue compiles calculating program and applies the plain-plate finite element model and integrates model to the structures of short-leg walls. Based on plain-plate finite element model, the ductility analysis of the short-leg shear walls with different flange width of wall-legs is studied.

The calculated results manifest : the new method is valid and reliable; the wall-legs of "—"section should be avoided when the sectional dimension and ratio of reinforcement is fixedthe ductility and ultimate bearing capacity of short-leg shear wall increase with the increasing of width of flange b_f. The flange of wall-legs can improve grate the ductility and the power dissipation under earthquake force of short-leg shear walls.

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